An inverse method of characteristics for analyzing multirate circuits

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Outline

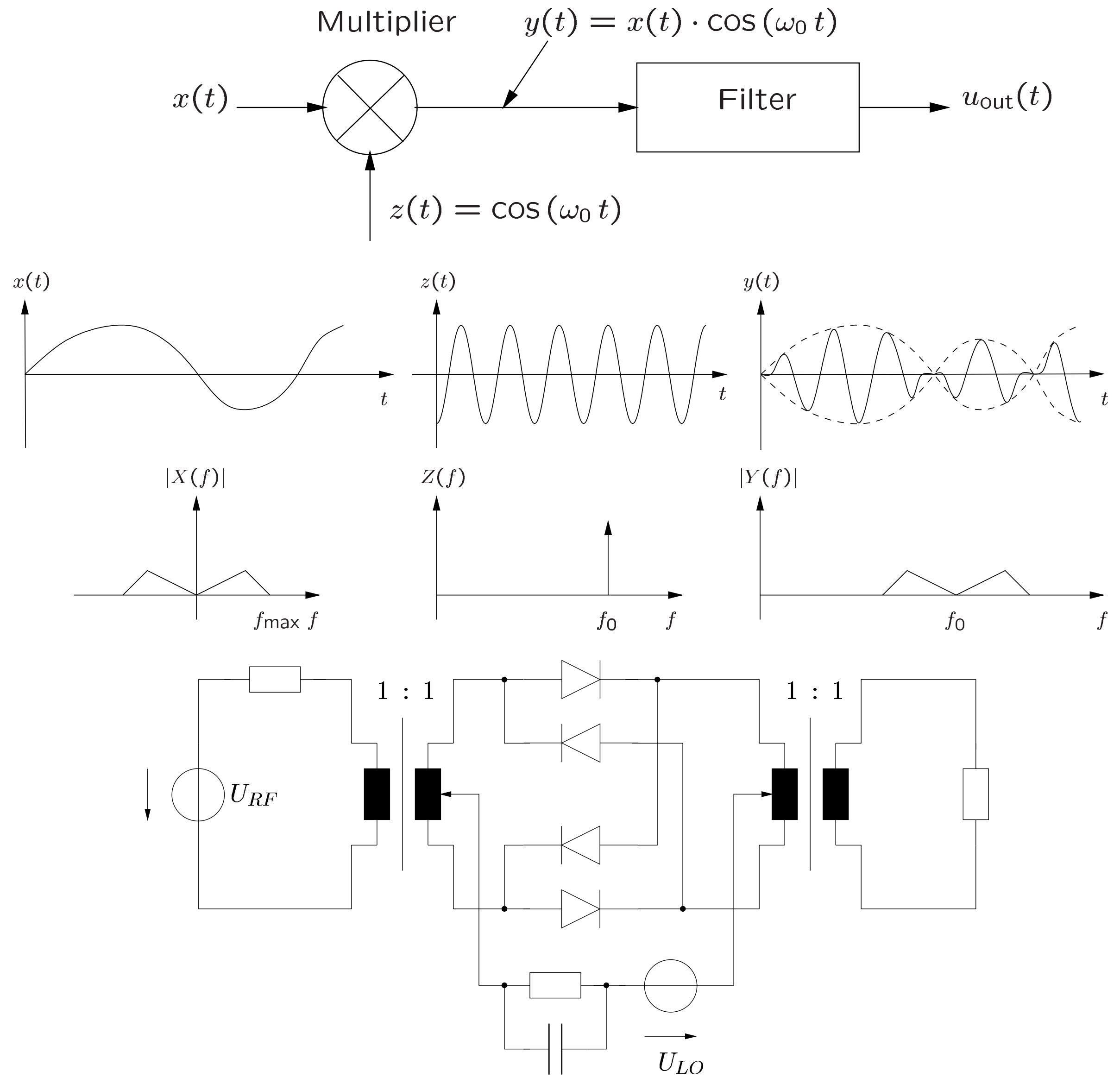
1. Motivation: Circuits exhibiting multi-rate behavior

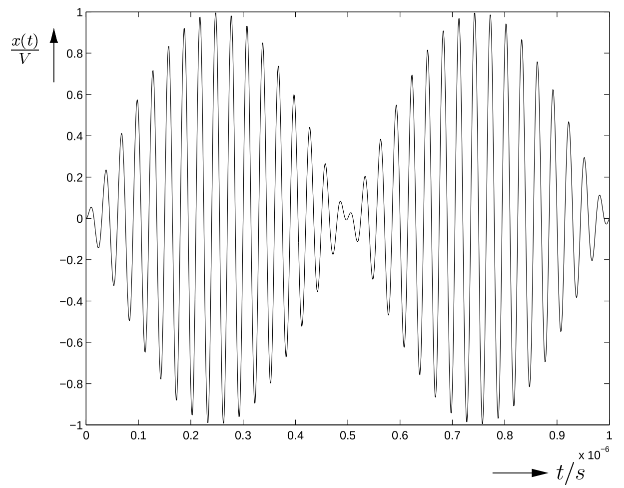
2. Basic concepts: Circuit DAEs

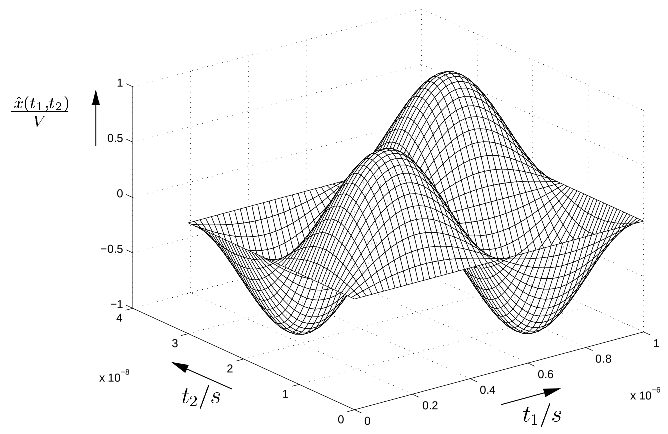
3. An inverse method of characteristics

4. Simulation Results

Motivation · Multi-rate circuits







Number of gridpoints K necessary for numerical solution:

Assumptions:

(i) Let $f_{LO} \gg f_{NF}$,

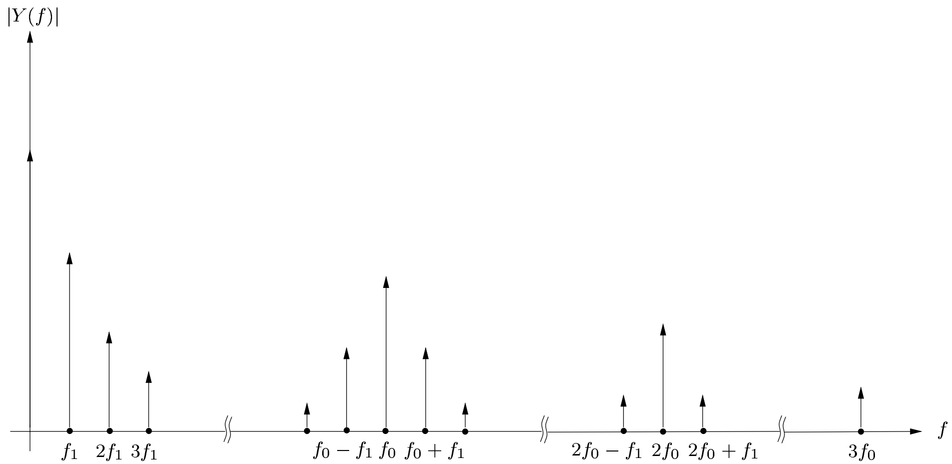
(ii) Mesh spacing for sufficient resolution a factor of 10 smaller than f_{LO},

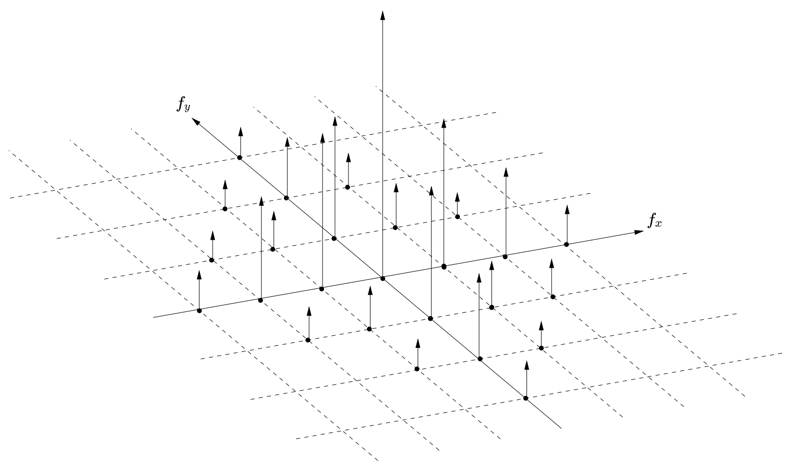
(iii) simulation interval $T \approx 1/f_{NF}$:

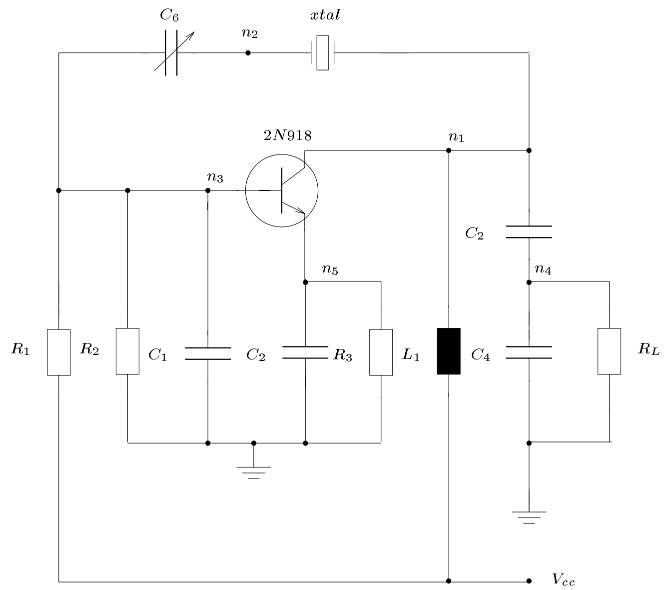
$\Rightarrow K \approx 10 \cdot \frac{f_{LO}}{f_{NF}}$

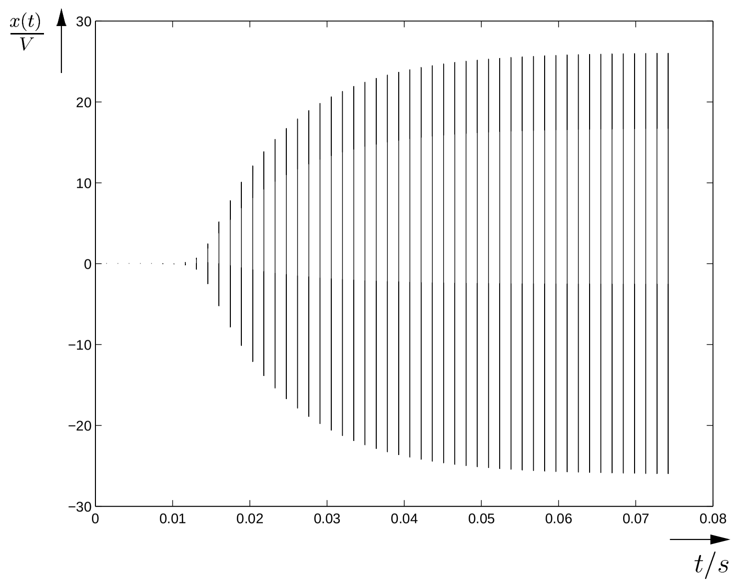
$K = 100$ gridpoints in the 2-dimensional plane independent of f

Interpretation in frequency domain









Stiff systems:

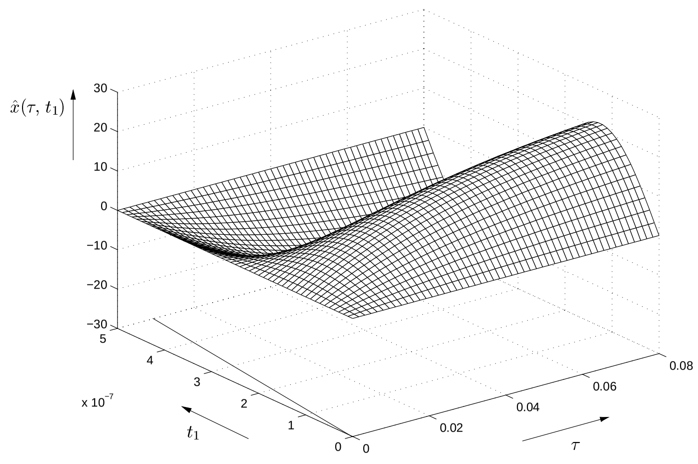
Quartz and cavity oscillators

Pierce quartz oscillator:

(i) $f_{LO} = 2MHz$

(ii) Transit time $T_r \approx 50ms$

$\Rightarrow K > 10 \cdot T_r * f_{LO} = 10^5$



Oscillatory transient response with smooth envelope:

$$x(t) = \sum_{k=-K}^{K} X_k(t) \exp\left(j\, k\, \omega(t)\, t\right)$$

Periodic waveform in t_1, smooth envelope in τ:

$$\hat{x}(\tau, t_1) := \sum_{k=-K}^{K} X_k(\tau) \exp\left(j\, k\, \omega(\tau)\, t_1\right)$$

Kirchhoff's law:

$$A\,i = 0, \qquad u = A^T\,v$$

device constitutive equations:

$$f(i,u,t) = i + g(u) + \frac{\mathrm{d}q(u)}{\mathrm{d}t} + b(t) = 0, \qquad f:\ \mathbb{R}^z \times \mathbb{R}^z \times \mathbb{R} \to \mathbb{R}^z$$

results into nodal formulation:

$$A\,g(A^T\,v) + A\,\frac{\mathrm{d}q(A^T\,v)}{\mathrm{d}t} + A\,b(t) = 0$$

i. e. system of $n-1$ differential-algebraic equations:

$$f(x,\dot{x},t) = i(x(t)) + \frac{\mathrm{d}}{\mathrm{d}t}q(x(t)) + b(t) = 0, \qquad \text{non-autonomous}$$

$$f(x,\dot{x}) = i(x(t)) + \frac{\mathrm{d}}{\mathrm{d}t}q(x(t)) = 0, \qquad \text{autonomous}$$

1. Example: Mixer in steady state, 2-tone case
Solve partial DAE on a set Ω
for periodic boundary conditions in t_1, t_2

$$f(\hat{x}(t_1,t_2), \nabla \hat{x}(t_1,t_2), (t_1,t_2))$$

$$= i(\hat{x}(t_1,t_2)) + \nabla q(\hat{x}(t_1,t_2)) + \hat{b}(t_1,t_2) = 0, \quad (t_1,t_2) \in \Omega$$

where

$$\nabla := \sum_{i=1}^{2} \omega_i \cdot \frac{\partial}{\partial t_i}$$

$$b(t) = \hat{b}(\omega_1 t, \omega_2 t)$$

and

$$\Omega := \{(t_1, t_2) \,|\, t_i \in [0, 2\pi[, \; i = 1,2\}$$

$$\hat{b}(t_1,t_2) = \hat{b}(t_1 + 2\pi, t_2), \quad \hat{b}(t_1,t_2) = \hat{b}(t_1, t_2 + 2\pi)$$

Solutions for a **Family of Initial Conditions** along characteristics:

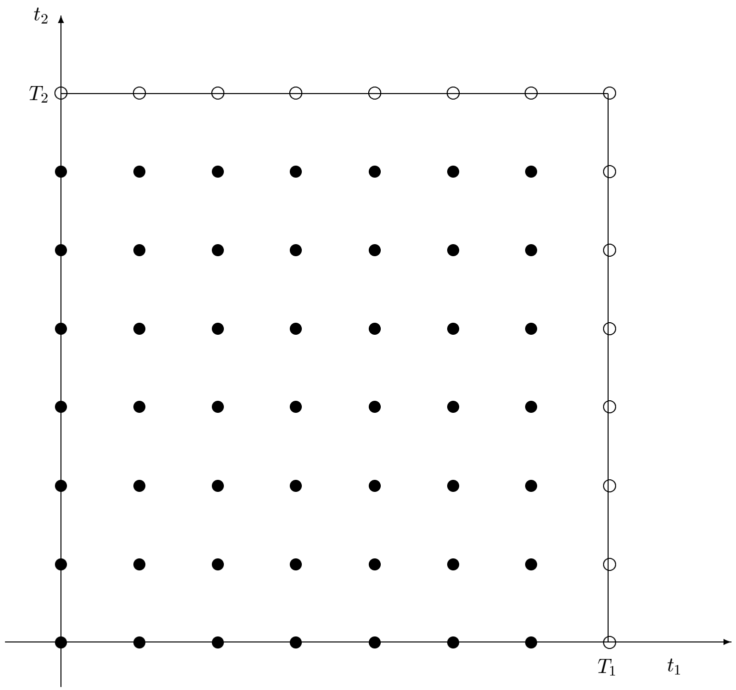
$$t_1 = \omega_1 t + \Theta_1$$

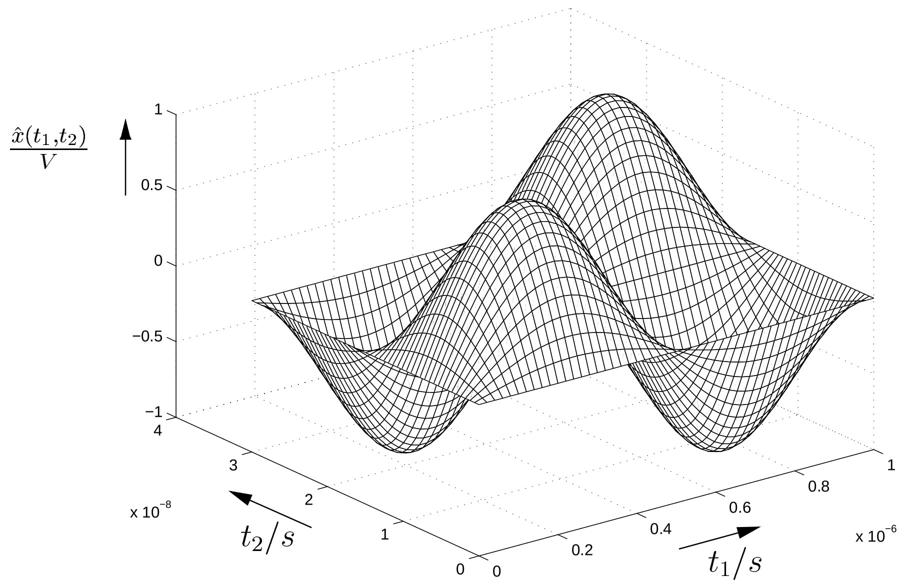
$$t_2 = \omega_2 t + \Theta_2, \quad 0 \leq \Theta_{1,2} < 2\pi$$

Special solution of the underlying ordinary DAE along characteristic

$$\Theta_1 = \Theta_2 = 0$$

Discretization on the semi-open set Ω:





Transient response of high-Q-oscillators: Solve on

$$\Omega := \{(\tau,\, t_1) \,|\, \tau \in \mathbb{R},\, t_1 \in [0,\, 2\,\pi[\}$$

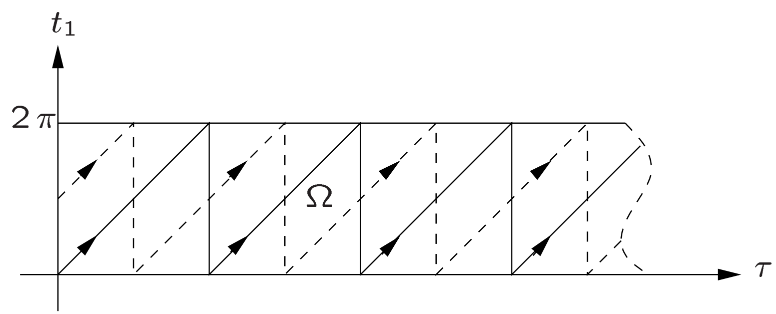
autonomous partial DAE

$$f(\widehat{x}(\tau, t_1), \nabla\, \widehat{x}(\tau, t_1))$$

$$= i(\widehat{x}(\tau, t_1)) + \nabla\, q(\widehat{x}(\tau, t_1)) = 0,\ \widehat{x}(0, t_1) = x_0(t_1)$$

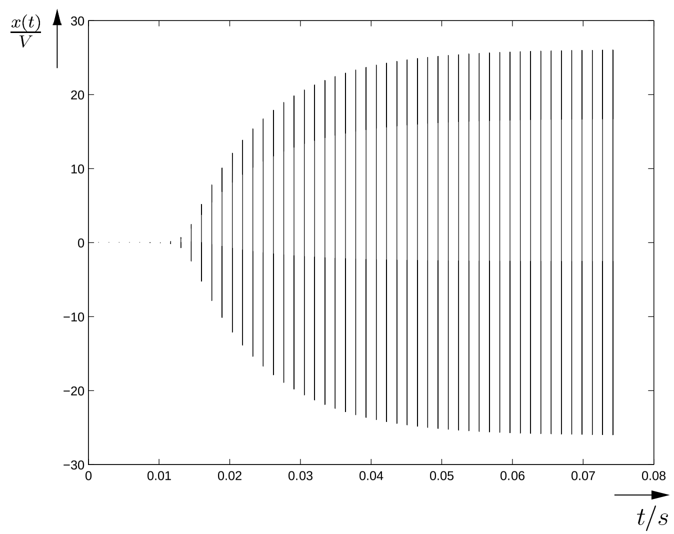
for periodic boundary conditions in t_1, where:

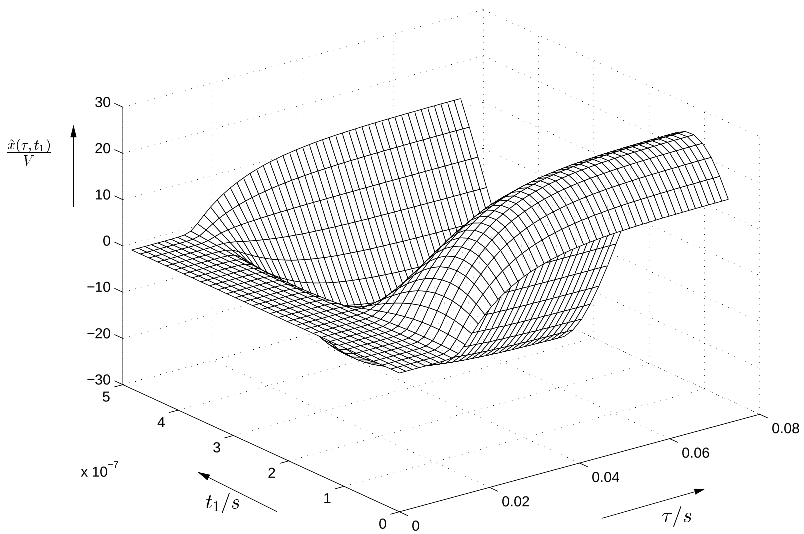
$$\nabla := \frac{\partial}{\partial \tau} + \frac{\mathrm{d}(\tau\,\omega(\tau))}{\mathrm{d}\tau} \cdot \frac{\partial}{\partial t_1}, \quad \omega \in C^1(\mathbb{R})$$



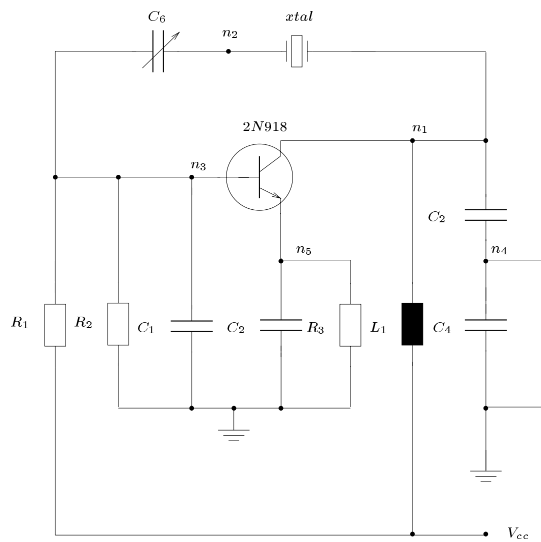
Solution along characteristic

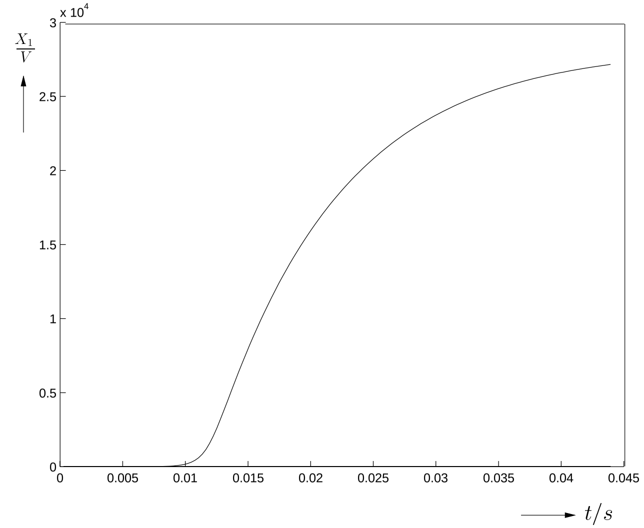
Time domain and partial DAE solution

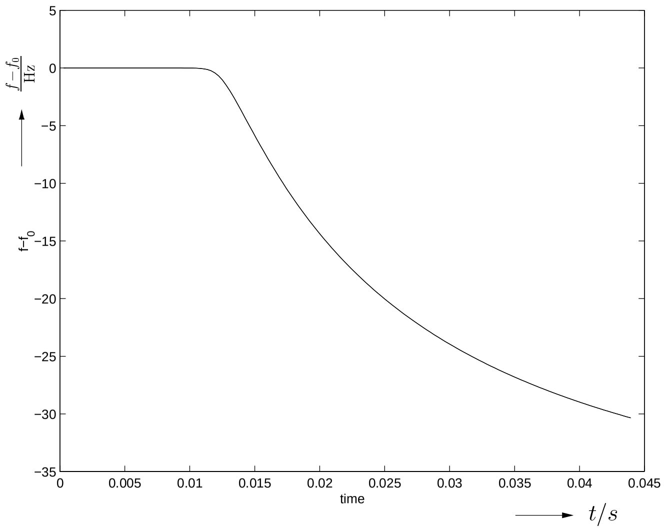


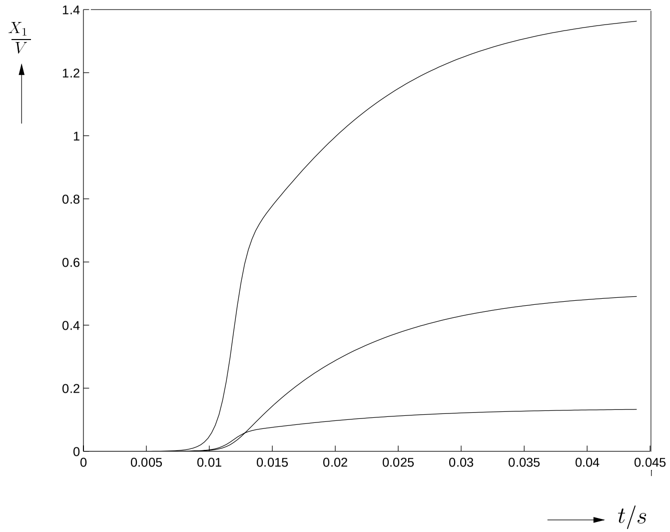


Results · 2MHz Pierce-oscillator

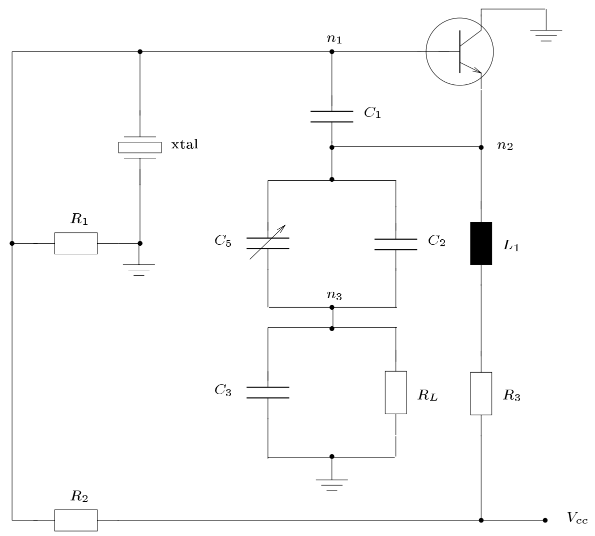


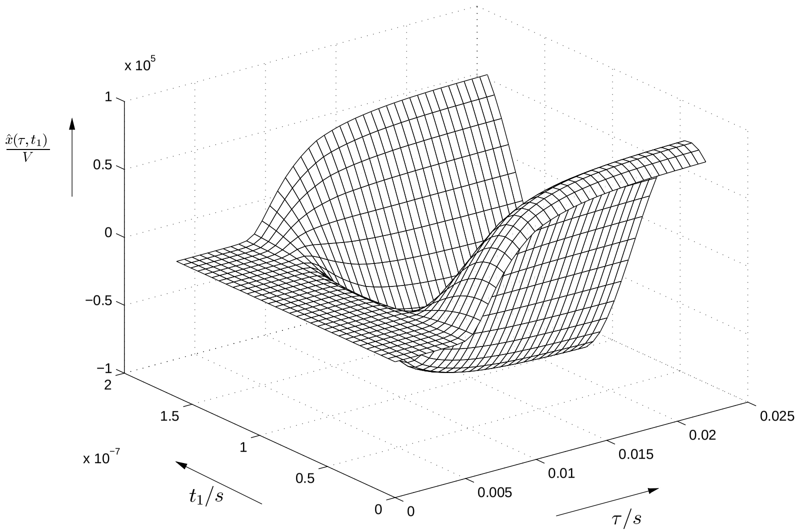


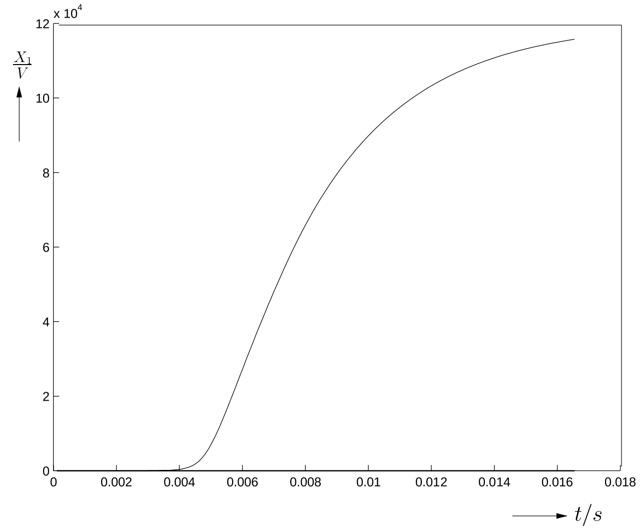


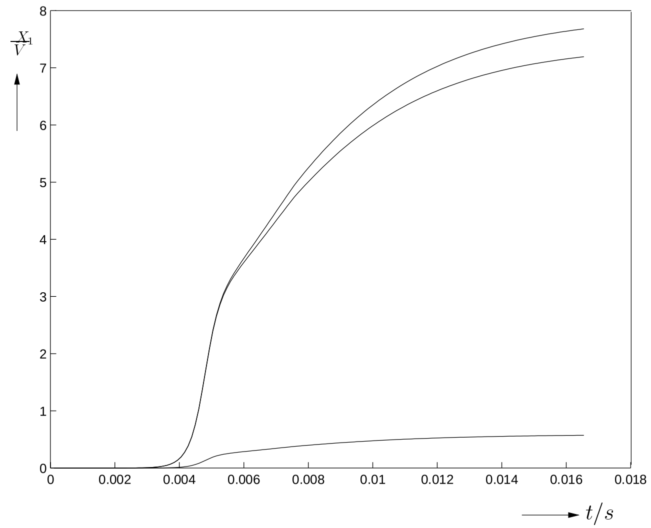


Results · 3MHz Colpitts-oscillator









Initial value problem for a **Parametrized Family of Initial Conditions**

$$f(x_\Theta(t), \dot{x}_\Theta(t), t) = i(x_\Theta(t)) + \frac{\mathrm{d}}{\mathrm{d}t} q(x_\Theta(t)) + b_\Theta(t) = 0,$$

$$x_\Theta(0) = x_0(\Theta), \quad \Theta \in \mathbb{R}^m$$

where $x_0(\Theta) : \mathbb{R}^m \to \mathbb{R}^N$, $b_\Theta(t) : \mathbb{R} \to \mathbb{R}^N$
(2π)-periodic in $\Theta_1, \ldots, \Theta_m$
Furthermore: semi-open set Ω

$$\Omega := \{(\tau, t_1, \ldots, t_m) \mid \tau \in \mathbb{R}, t_i \in [0, 2\pi[, 1 \le i \le m\}$$

Define a partial DAE on Ω

$$f(\hat{x}(\tau, t_1, \ldots, t_m), \nabla \hat{x}(\tau, t_1, \ldots, t_m), (\tau, t_1, \ldots, t_m))$$

$$= i(\hat{x}(\tau, t_1, \ldots, t_m)) + \nabla q(\hat{x}(\tau, t_1, \ldots, t_m)) + \hat{b}(\tau, t_1, \ldots, t_m) = 0,$$

Remarks

- Bra'94 et al, Simulation des eingeschwungenen Verhaltens elektronischer Schaltungen, PhD thesis
- Bra et al '96, Numerical steady state analysis of electronic circuits driven by multi-tone signals, Electronic Engineering, Vol. 79, No. 2, Apr. 1996, pp. 103-112.
- Roy'97, Efficient methods for simulating highly nonlinear multirate circuits, Proc. DAC, pp. 269-274.
- Bra'97 et al, A novel time-frequency method for the simulation of the steady state of circuits driven by multi-tone signals, Proc. ISCAS, pp. 1508-1511.
- Roy'01, Analyzing Circuits with Widely Separated Time Scales Using Numerical PDE Methods, IEEE Trans. CAS-I, Vol. 48, No. 5, pp. 578-594.

- Bra'98 et al, A time-frequency algorithm for the simulation of the initial transient response of oscillators, Proc. ISCAS, 1998.
- Narayan'99 et al, Multi-time simulation of voltage-controlled oscillators, Proc. DAC.
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