Uncertainty Analysis for Multiple Penetration Depth Dielectrometry

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Dielectric spectroscopy is a powerful tool for estimation of electrical insulation properties, including moisture concentration, density, and aging status. Particularly promising applications that involve evaluation of spatial distribution of dielectric properties often suffer from a lack of accuracy, even when low-noise equipment is used. This paper continues to explore algorithmic approaches to dielectric imaging. Propagation of uncertainty is shown to have a critical effect in sequential material profiling algorithms.

Key words: dielectrometer sensors, uncertainty

1. INTRODUCTION

Interdigital dielectrometry offers the ability to measure non-destructively, from one side, the dielectric permittivity, conductivity, and dependent physical properties of dielectric materials [1]. Profiling and imaging of such properties as moisture concentration, aging status, and density of dielectric materials has numerous applications in power engineering, thin-film technology, and polymer industry [2]. Use of multiple penetration depth dielectrometry has remained a promising methodology for non-destructive testing for more than a decade [3].

When using the fringing fields of a three-wavelength sensor to detect the dielectric constant of a material distributed in separate layers, the uncertainty for the deepest layer grows to very high values as preceding layers are added. In general, the dielectrometry measurements deal with both dielectric permittivity and conductivity. The analytical treatment in this paper is reduced to permittivity only in order to keep equations manageable and not represent complex capacitance.

Experimental results with transformer oil and corn oil, in combination with several types of solid dielectrics, confirm the derivations presented in this paper and will be included in future publications. The dramatic increase of the measurement error arises from heaving to rely on the measurements for preceding layers, with each having their own uncertainty, when computing a dielectric value for the current layer.

2. GENERAL UNCERTAINTY

The general uncertainty equation is

\[ U_r^2 = \sum_{i=1}^{J} \left( \frac{\partial r}{\partial X_i} U_{X_i} \right)^2 \]  

(1)

where \( r \) is the variable with the uncertainty that varies depending on the \( J \) independent variables \( X_i \):

\[ r = r(X_1, X_2, ..., X_J) \]  

(2)

\( U_r \) is the uncertainty in the variable \( r \), and the variables \( U_{X_i} \) represent the independent uncertainties in the variables \( X_i \) [4]. Uncertainties are typically measured in a region of 95% confidence. For example, if a variable \( X \) has a region of 95% confidence that spans \( \pm 3\% \) of the nominal value, then \( U \) is \( 2\times 3\% \approx 6\% \).

A convenient way to calculate uncertainties is as the ratio \( U_r / r \), which is the format that this paper uses. Here the equation is reorganized for clarity:

\[ \left( \frac{U_r}{r} \right)^2 = \sum_{i=1}^{J} \left( \frac{1}{r} \frac{\partial r}{\partial X_i} U_{X_i} \right)^2 \]  

(3)

3. APPLICATION OF SIMULATED DATA

The data presented in this paper was obtained numerically using finite-element modelling software. The example equations presented will be linear approximations derived from the non-linear simulated results. The linearized analysis is accurate for small-signal perturbations around each measurement point.

The generic experimental setup with a multiple penetration depth sensor is shown in Figure 1. Each layer is assumed to be nonconductive and homogeneous with a given relative dielectric permittivity, \( \varepsilon \). The separation of the electrodes in the interdigital sensor determines the sensing depth. In this case 1.0 mm, 2.5 mm, and 5.0 mm separations correspond to depths of about 300 \( \mu \)m, 750 \( \mu \)m, and 1500 \( \mu \)m respectively. This setup allows monitoring fluids behind an isolating layer of 1 mm thick material and estimation of moisture content in the cross-section.

3.1. LABELING CONVENTIONS

In this paper, relative dielectric permittivities are labelled with a subscript \( i \) as in \( \varepsilon_i \) where \( i \) is an integer that represents the layer. The numbering starts with the layer closest to the sensor, \( i = 1 \), and increases
sequentially for more remote layers. Capacitances are labeled with two subscripts, \(a\) and \(b\) as in \(C_{a,b}^{+}\); the prime shows that the value is in units of farads per meter instead of just farads. The subscript \(a\) specifies the two nodes that the capacitance spans in the circuit (for example, in this paper \(a = 12\) because the material is assumed to be between nodes 1, the driven electrode, and 2, the sensing electrode); \(b\) specifies the sensor wavelength being used. In this case, 1, 2, and 3 correspond with the 1.0 mm, 2.5 mm, and 5.0 mm spatial wavelengths, respectively. The variables can also have an extra superscript: \(n\) for nominal (\(n\) is left off in equations for simplicity), \(h\) for the highest value of uncertainty, or \(l\) for the lowest value of uncertainty. For example, \(C_{12,2}^{+h}\) would be the high value of uncertainty seen between the nodes 1 (the driven electrode) and 2 (the sensing electrode) from the second sensor (2.5 mm separation).

![Diagram](image)

Fig. 1. Experimental and simulated setup for a three-wavelength dielectrometry sensor. The driven electrode is assumed to be node 1, and the sensing electrode is node 2.

3.2. FIRST CASE

In the first (and most simple) case, there is only one layer, which extends from 0 \(\mu\)m to infinity, with a relative dielectric permittivity of \(\varepsilon = 2.1\) (Teflon) (which applies to all remaining figures in this paper). Figures 2, 3, and 4 show the simulated functions and the actual uncertainties \(c^{h} - c^{l}\) for the 1.0 mm, 2.5 mm, and 5.0 mm cases, respectively. Notice that the relationship between capacitance and permittivity is not linear, which is characteristic, of fringing field capacitors. Because electric field lines change shape with the change the change of material properties, the ratio of energy amounts stored where \(a\) is the slope and \(h\) is the \(C^{'}\) intercept for a line approximating the nonlinear function around the point \((\varepsilon_{1},C_{12,1}^{n})\). The following equations are then generated for the sensor with a spacing of 1.0 mm using the general uncertainty equations

\[
\left( \frac{U_{c}}{\varepsilon_{1}} \right)^{2} = \left( \frac{U'_{12,1}}{\varepsilon_{1}} \frac{\partial \varepsilon_{1}}{\partial C'_{12,1}} \right)^{2}
\]

(5)

\[
\left( \frac{U_{c}}{\varepsilon_{1}} \right)^{2} = \left[ \frac{aU'_{12,1}}{aC'_{12,1} + b} \right]^{2}
\]

(6)

![Graph](image)

Fig. 2. Simulated function curve of the sensor with 1.0 mm wavelength, and a capacitance uncertainty of 6%; the uncertainty shown for \(\varepsilon_{1}\) is approximately 7.2%

![Graph](image)

Fig. 3. Simulated function curve of the sensor with 2.5 mm wavelength and a capacitance uncertainty of 6%; the uncertainty shown for \(\varepsilon_{1}\) is approximately 5.4%

Using the plots, the following values are estimated:

\[
\frac{U_{C_{12,1}^{n}}}{C_{12,1}} \equiv 6.0\%, \quad \frac{U_{\varepsilon_{1}}}{\varepsilon_{1}} \equiv 7.2\%
\]

Thus a capacitance uncertainty of 6.0% produces approximately 7.2% output uncertainty in the permittivity. Following these same steps with the other two sensors produces
for the 2.5 mm separation and
\[
\frac{U_{c_{12,2}}}{C_{12,2}} \equiv 6.0\%, \quad \frac{U_{e_2}}{e_2} \equiv 4.9\%
\]
for the 5 mm separation.

Notice how the uncertainty goes down as the separation between the electrodes increase. This is because the increase in separation spreads more of the electric field through the object, which increases the sensor’s sensitivity.

3.3. SECOND CASE

In the second case, another layer is added. The first layer extends from 0 to 300 \( \mu \text{m} \) and the second extends from 300 \( \mu \text{m} \) to infinity. The simulation assumes that both layers have an \( \varepsilon \) of 2.1, but this does not affect the uncertainty because the layers would be unknown in an actual experiment. For this case, two simulations were performed with the understanding that the results from Figure 2 were used to estimate the permittivity of the first layer: the first simulation, shown in Figure 5, uses the sensor with the 2.5 mm separation to calculate the second layer; and the second simulation, shown in Figure 6, uses the sensor with the 5.0 mm separation to calculate the second layer. Now, the functional dependence of capacitance on permittivity is no longer a line, but rather a region, with the top boundary based on the \( \varepsilon_1^a \) and the bottom boundary based on \( \varepsilon_1^b \).

Now, the calculated permittivity depends on measurements from both sensor elements instead of just one. The following equation is the approximation used for the function given in Figure 5.
\[
\varepsilon_2 = a_1 e_{12,1} + a_2 e_{12,2} + b
\]

\[
\left( \frac{U_{e_2}}{e_2} \right)^2 = \left( \frac{U_{c_{12,1}}}{e_2} \left( \frac{\partial e_2}{\partial C_{12,1}} \right) \right)^2 + \left( \frac{U_{c_{12,2}}}{e_2} \left( \frac{\partial e_2}{\partial C_{12,2}} \right) \right)^2
\]

\[
\left( \frac{U_{e_2}}{e_2} \right)^2 = \left( \frac{a_1 U_{c_{12,1}}}{a_1 C_{12,1} + a_2 C_{12,2} + b} \right)^2 + \left( \frac{a_2 U_{c_{12,2}}}{a_1 C_{12,1} + a_2 C_{12,2} + b} \right)^2
\]
The uncertainty values for the figure 5 case are:
\[
\frac{U_{C_{12,1}^n}}{C_{12,1}} = 6.00\%, \quad \frac{U_{C_{12,2}^n}}{C_{12,2}} = 6.00\%, \quad \frac{U_{\varepsilon_2}}{\varepsilon_2} = 31.32\%
\]
and the values for the Figure 6 case are:
\[
\frac{U_{C_{12,1}^n}}{C_{12,1}} = 6.00\%, \quad \frac{U_{C_{12,2}^n}}{C_{12,2}} = 6.00\%, \quad \frac{U_{\varepsilon_2}}{\varepsilon_2} = 11.06\%
\]

3.4. THIRD CASE

The third (and final) case is shown in Figure 7. This curve is generated by using the values retrieved from Figures 2 and 5 where \(\varepsilon_1\) is generated by using the 1.0 mm wavelength, which is then used with the 2.5 mm wavelength to generate \(\varepsilon_2\). Finally both \(\varepsilon_1\) and \(\varepsilon_2\) are used with the 5.0 mm sensor to generate \(\varepsilon_3\). Thus, all three capacitance measurements are needed in our linear approximation of this curve:
\[
\varepsilon_3 = a_1 C_{12,1} + a_2 C_{12,2} + a_3 C_{12,3} + b \quad (10)
\]

![Graph](image)

Fig. 7. Simulated function curve or the 5.0 mm sensor when determining \(\varepsilon_3\) for the third layer. We use the \(\varepsilon\) found with the 1.0 mm and 2.5 mm wavelengths, Figures 2 and 5 respectively. The capacitance uncertainty is 6%; the uncertainty shown for \(\varepsilon_3\) is approximately 166%

Notice that the nonlinear effects are becoming large enough to cause asymmetry of high and low estimates with respect to the nominal values.

The basic uncertainty equation for this case is:
\[
\left( \frac{U_{\varepsilon_3}}{\varepsilon_3} \right)^2 = \left( \frac{U_{C_{12,1}}}{C_{12,1}} \left( \frac{\partial \varepsilon_3}{\partial C_{12,1}} \right) \right)^2 + \left( \frac{U_{C_{12,2}}}{C_{12,2}} \left( \frac{\partial \varepsilon_3}{\partial C_{12,2}} \right) \right)^2 + \left( \frac{U_{C_{12,3}}}{C_{12,3}} \left( \frac{\partial \varepsilon_3}{\partial C_{12,3}} \right) \right)^2 \quad (11)
\]

and the numerical estimates are:
\[
\frac{U_{C_{12,1}^n}}{C_{12,1}} = 6.00\%, \quad \frac{U_{C_{12,2}^n}}{C_{12,2}} = 6.00\%
\]

Notice how the uncertainty has grown. The uncertainty of the first layer was 7.2%, then grew to 31.3% for the second layer, and finally became a huge 87.6%. The same parameters calculated using a graphical method and taking into account the non-linearity are 7.6%, 51%, and 166%, respectively.

4. CONCLUSIONS AND FUTURE WORK

The analysis in the paper complements experimental work with multiple penetration depth dielectricometry sensors. Even though the capacitance associated with each penetration pattern is evaluated accurately, experiments and theory show that even slight variations of raw sensor output can cause a large variation in measured dielectrics for each layer of increasing depth. Given examples show how relying on a single interdigital sensor per layer becomes increasingly less accurate with layer depth. As a result, to improve the certainty for the final layer, superior parameter estimation algorithms should be developed to provide accurate profiles of material properties. It is recommended that generalized uncertainty analysis should be a priority when considering this method for measurement of stratified material samples.

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