Comparison of Scenario Reduction Techniques for the Stochastic Unit Commitment

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***Abstract*—A number of scenario reduction techniques have been proposed to make possible the practical implementation of stochastic unit commitment formulations. These scenario-reduction techniques aggregate similar scenarios based on their metrics, such as their probability, hourly magnitudes, or the cost resulting from each scenario. This paper compares these different scenario reduction techniques in terms of the resulting operating cost and the amount of time required to complete computation of the stochastic UC. This comparison is based on Monte Carlo simulations of the resulting generation schedules for a modified version of the 24-bus IEEE-RTS.**

I. Introduction

Stochastic formulations of the unit commitment (UC) problem consider a set of scenarios representing possible realizations of the uncertainty associated with renewable generation. These scenarios can be generated using statistical models [1]. Statistical models used to generate these scenarios include support vector regression (SVR) [2], regularized neural network (NN) [3], [4], random forest (RF) and bagging decision tree [5]. Scenarios generated using different statistical models can be combined into a single set of scenarios using an ensemble approach [3], which weighs the predictions of different statistical models in a way that preserves the special features of the scenarios obtained with different models.

The stochastic UC can be implemented using either a scenario formulation [6] or an interval formulation [7].

A. Scenario-Based Stochastic UC

The uncertainty model used in the scenario-based stochastic UC consists of a set of scenarios, as illustrated in Fig. 1a. Each scenario represents a possible realization of uncertainty and has a certain probability. The scenario-based stochastic UC produces a single schedule for a given scenario set. All scenarios are, therefore, bound by the same on/off status of generators. However, the output of generators would be different for each scenario. The objective function minimizes the expected operating cost over all scenarios in this set. Since the extreme scenarios may have low probabilities, the schedule of the scenario-based stochastic UC sacrifices optimality for these scenarios to minimize the operating cost for the more probable (and less extreme) scenarios.

An accurate representation of possible realizations of renewable generations uncertainty in scenario-based stochastic UC requires the consideration of a large number of scenarios. However, the computational burden increases rapidly with the number of scenarios considered. Scenario reduction techniques are, therefore, used to manage the amount of computing time required. Such a reduction unavoidably results in a less accurate representation of uncertainty and may produce generation schedules that require expensive corrective actions for some uncertainty realizations. This paper analyzes how the choice of a particular scenario reduction technique affects the cost of operating a power system with a substantial penetration of renewable generation.

Scenario-reduction techniques aggregate similar scenarios based on a particular metrics, such as their probability, hourly magnitudes, or the cost resulting from each scenario. An unsupervised clustering method, k-means [8], can be used to partition a given set of scenarios into a given number of clusters. As a result of this partition, scenarios with similar features are assigned to the same cluster. The centroid of each cluster represents a somewhat average pattern of all the scenarios included in a cluster. Since this centroid is an artificial scenario, the original scenario with the lowest probability distance from the centroid, is used to represent the cluster. This scenario is known as the medoid of the cluster. The k-means++ method [8] is an enhancement of the k-means method that relies on an initial cluster partitions. Dupacova *et al* [9] proposed to reduce the original set of scenarios in a way that minimizes the Kantorovich distance between the scenarios in the original set and in the reduced set. This method is implemented in the forward scenario selection and backward scenario reduction approaches. The forward scenario selection approach [9] iteratively adds one scenario from the given set to the reduced set until the reduced set contains the desired number of scenarios. On the

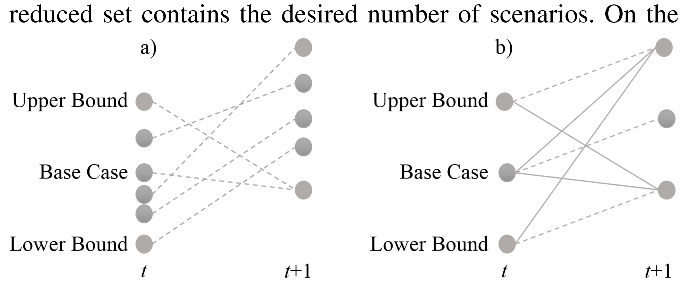

Fig. 1. Schematic representation of a scenario set in the (a) scenario-based with six scenarios, and (b) interval stochastic UC with three scenarios. The dashed lines represent scenarios and the solid lines represent the deterministic constraints on rampings.

other hand, the backward reduction approach [9] iteratively eliminates one scenario from the original set until the desired number of scenarios remains and thus constitutes the reduced set. Heitsch *et al* [10] improved the work of Dupacova *et al* [9] by implementing computationally more effective forward scenario selection and backward scenario reduction approaches. Morales *et al* [11] modified the fast forward scenario selection approach to make it compatible with the two-stage stochastic programming problems encountered in electricity markets. Although this compatibility comes at the cost of a larger computational burden as compared to [11], the approach proposed in [11] remains tractable. Papavasiliou *et al* [12] proposed an importance sampling technique to select scenarios that best represent the monetary impact of uncertainty on the operating cost.

B. Interval Stochastic UC

The interval stochastic UC [7] can be thought of as a particular case of the scenario reduction procedure. Fig. 1 illustrates the difference between the uncertainty models in the scenario-based and in the interval stochastic UC. In the interval formulation, the scenario set contains only three scenarios: the central forecast, an upper bound and a lower bound. The bounds for each hour are set in such a way that a given percentile of scenario magnitudes remains within the bounds. Additionally, the inter-hour ramps, depicted with solid lines in Fig. 1b, are enforced using additional constraints. The objective function of the interval stochastic UC minimizes the operating cost for the central forecast, while only the feasibility of the solution is guaranteed for the upper and lower bounds. In contrast with the scenario-based formulation, the interval stochastic UC does not account for the likelihood of the upper and lower bounds. Therefore, it produces a schedule that is more expensive, but also more robust than the scenario-based UC [13]. In addition, the interval stochastic UC is computationally less demanding than the scenario-based formulation since it considers fewer scenarios.

C. Contributions

This paper compares solutions of the scenario-based stochastic UC obtained with four scenario reduction techniques (k-means [8], fast forward scenario selection [11], backward scenario reduction [10], and importance-sampling scenario reduction [12]) and the interval stochastic UC for different magnitudes of the upper and lower bounds. The comparison is performed in terms of both the operational cost and computational time. The cost comparison is based not only on the cost of the computed schedules (as in [13]) but also on a Monte Carlo simulation of the implementation of these schedules. The Monte Carlo simulation highlights differences in robustness between the schedules produced using different scenario reduction techniques.

II. Stochastic UC Formulation

This section summarizes the scenario-based and interval stochastic formulations used in the case study. Complete formulations can be found in [6], [7]. In the following equations indices i, s, b and t refer to the sets of controllable generators I, scenario S, buses B, and time intervals T. The objective function of the scenario-based UC is:

$$\min \sum_{t\in T}\sum_{i\in I}\left[SC_{t,i}\cdot x_{t,i}+\sum_{s\in S}\pi_s\cdot F_i\left(p_{t,i,s}\right)\right]+ \\ +\sum_{t\in T}\sum_{s\in S}\pi_s\cdot\left(ENS_{t,s}\cdot VoLL+WS_{t,s}\cdot VoWS\right) \quad (1)$$

The first term of this objective function represents the start-up cost, $SC_{i,t}$, which is incurred if generator i is started at hour t, i.e. when binary variable $x_{t,i}$ is equal to 1. The second term accounts for the dispatch cost for each scenario s with probability π_s. Function $F_i\left(p_{t,i,s}\right)$ accounts for the fuel cost of each on-line generator with output $p_{t,i,s}$. The Energy Not Served ($ENS_{t,s}$) and Wind Spillage ($WS_{t,s}$) that a particular scenario s may cause at hour t are penalized by the Value of Lost Load ($VoLL$) and the Value of Wind Spillage ($VoWS$). The objective function of the interval stochastic UC can be written in a similar fashion:

$$\min \sum_{t\in T}\sum_{i\in I}\left[SC_{t,i}\cdot x_{t,i}+F_i\left(p_{t,i,bc}\right)+WS_{t,bc}\cdot VoWS\right] \quad (2)$$

This objective function minimizes the start-up cost of the generators and the dispatch cost under the base case (*bc*), which is assumed to be the central forecast. Since it accounts for a single forecast, load shedding is not allowed. However, wind spillage can occur at a penalty cost of $VoWS$.

Both the scenario-based and the interval stochastic UC formulations enforce the minimum up- and down-time constraints on the generators. In the scenario-based formulation, dispatch constraints include minimum and maximum limits on the output of the generators, the maximum upward and downward ramp of the generators, and the load/generation balance. In the interval formulation, the same dispatch constraints are enforced for the base case and bounds as well as the additional ramping constraints as illustrated in Fig. 1b. Network constraints in both formulations are enforced using a dc power flow approximation.

III. Case Study

Four scenario reduction techniques using the UC formulations described above have been tested on a modified version of the 24-bus IEEE RTS; details can be found in [14]. The wind penetration is assumed to be 20% in terms of the energy consumed daily system-wide. The $VoWS$ is set to \$35/MWh [15] and the $VoLL$ is set to \$5000/MWh [16].

The wind and load forecasts are based on the BPA data [17] and are obtained using an ensemble approach, which implements the random feature selection and bootstrap sampling methods [18] to generate training samples for the NN and SVR scenario generators. A cross-validation set is used to select the parameters of the model [3]. Two sets of 1000 wind generation scenarios were generated with positive and negative correlation between the central forecast and the load. The central forecast for each scenario set was calculated as the average of 1000 scenarios. Fig. 2 illustrates these central wind forecasts. For

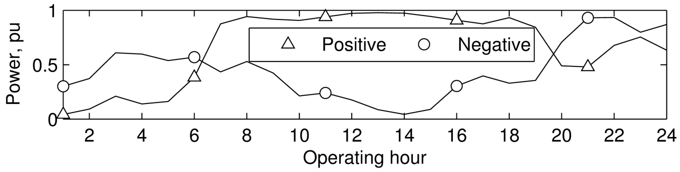

Fig. 2. Wind forecast profiles with positive and negative correlations with load. Data: BPA [17]

positively correlated wind and load forecasts, the peak wind production occurs during daytime hours, and during the night for negatively correlated wind and load forecasts.

To reflect standard practice in power system operation, the stochastic UC is solved based on the day-ahead load and wind scenarios to produce the optimal day-ahead schedule. The value of the objective function for this schedule gives the *day-ahead cost* (DAC) of operating the power system for the set of scenarios considered. Since these day-ahead scenarios are by definition uncertain, the DAC is a projected or expected cost, rather than an actual cost. A Monte Carlo simulation is required to estimate the *actual operating cost* (AOC) i.e. the cost that would be incurred on the day, when the realization of load and wind uncertainty is known and the day-ahead schedule must be adjusted to keep the system in balance. The difference between the DAC and the AOC is the *cost of corrective dispatch*, i.e. the cost of the adjustments to the day-ahead schedules that are required in real time to meet the load and wind realization. Changes in the commitment decisions made in the day-ahead schedule are allowed in the Monte Carlo simulation if constraints on the minimum up- and down-time limits are not violated.

At each Monte Carlo trial, the day-ahead schedule, produced with the scenario-based and interval stochastic UC, is dispatched to meet a particular realization of wind and load uncertainty. Realizations of load and wind uncertainty are modeled using the normal [19] and skew-Laplace distributions [20]. The cost of this dispatch represents the AOC for a particular realization of uncertainties. This cost includes the start-up cost of the day-ahead and additional real-time commitments, the cost of dispatch of generators under known realizations of wind and load uncertainties, as well as the wind spillage and load shedding costs. The number of Monte Carlo trials required for each schedule is $N_{MC} = \max\left[1000, N_{MC}\right]$, where N_{MC} is the number of trials required to ensure 95% confidence that the AOC estimate has an error of 0.1% or less [21].

A. Day-Ahead Schedules and Costs

At the day-ahead stage, the scenario-based stochastic UC is solved for 5, 10, 20, and 40 scenarios, obtained using the k-means, Backward Scenario Reduction (BSR), Forward Scenario Selection (FSS), and Importance Sampling (IS) scenario reduction techniques. The interval stochastic UC is solved for the range of uncertainty between bounds that discard 30, 20, 10, and 1% of the extreme values at each time period. Fig. 3 shows the range of uncertainty obtained with different numbers of scenarios produced by the forward scenario selection technique and compares them to the range of uncertainty obtained when 10% largest and smallest values are discarded. The range of uncertainty increases with the number of scenarios considered. Note that during some operating hours the ranges of uncertainty of the scenario-based stochastic UC with 40 scenarios and of the interval UC are equal.

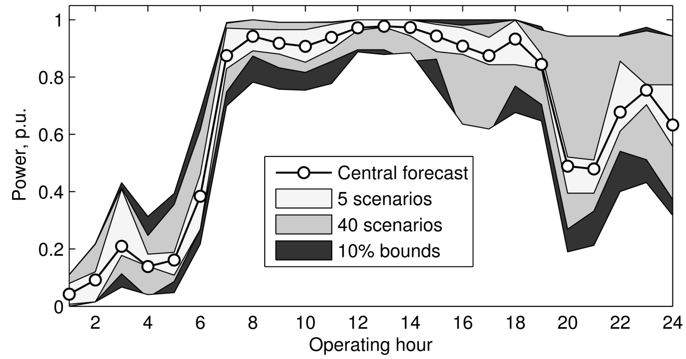

Fig. 3. Comparison of the range of uncertainty obtained for different number of scenarios and the 10% bounds.

Table I shows that the DAC for the scenario-based UC depends on the number of scenarios considered and increases with the number of scenarios. This increase is explained by the larger range of uncertainty captured by a greater number of scenarios. For a given number of scenarios, the DAC also depends on the scenario reduction techniques. However, no scenario reduction technique appears to be significantly better than the others. These variations in DAC remain in the range $[0.37, 0.93]\,\%$ and $[0.44, 0.79]\,\%$ for positively and negatively correlated wind and load forecasts, respectively. Table II shows that the DAC of the interval UC increases as the range of uncertainty increases. If the range of uncertainty is relatively small, i.e. 30% of extreme scenarios are discarded, the interval UC has a smaller DAC than the scenario-based UC with any number of scenarios and any scenario reduction technique. As the range of uncertainty increases, the DAC of the interval formulation also increases. The difference between the DAC of the scenario-based formulation with 40 scenarios and the interval formulation is in the range $[0.15, 0.53]\,\%$ and $[0.73, 1.1]\,\%$ for positively and negatively correlated wind and load forecasts, respectively.

Table III presents the computation time for the interval and scenario-based stochastic UC. As long as the number of scenarios in the scenario-based approach remains relatively low, there are no significant differences in the scenarios

TABLE I
DAY-AHEAD COST (IN 10^3\$) OF THE SCENARIO-BASED SCHEDULE

Positively Correlated Wind and Load

Scenarios	FSS	BSR	k-means	IS
5	566.7*	568.8	567.0	567.7
10	570.6	571.2	570.5*	575.2
20	573.0	572.4*	577.5	577.8
40	576.0*	576.6	578.1	578.2

Negatively Correlated Wind and Load

Scenarios	FSS	BSR	k-means	IS
5	657.1*	657.6	656.7	652.4
10	657.1	659.1	656.9*	660.3
20	658.9	658.5*	659.7	662.1
40	660.6*	662.6	663.5	663.2

* – the minimum DAC for a given number of scenarios.

TABLE II
KEY STATISTICS FOR THE INTERVAL UC SCHEDULES

Positively Correlated Wind and Load

Bound	30%	20%	10%	1%
CPU time, sec	61.8	42.4	94.3	49.3
DAC, $\cdot 10^3$\$	565.2	568.2	572.4	579.3

Negatively Correlated Wind and Load

Bound	30%	20%	10%	1%
CPU time, sec	48.7	41.9	37.3	31.4
DAC, $\cdot 10^3$\$	658.1	658.5	659.3	668.1

TABLE III
COMPUTATION TIME (SEC) FOR THE SCENARIO-BASED UC SCHEDULES

Positively Correlated Wind and Load

Scenarios	FSS	BSR	k-means	IS
5	53.8	54.1	58.9	59.0
10	168.0	156.6	290.8	271.6
20	864.3	1027	1143	1943
40	2208	3516	4097	4801

Negatively Correlated Wind and Load

Scenarios	FSS	BSR	k-means	IS
5	45.5	42.7	52.8	55.8
10	108.0	115.7	101.9	111.9
20	433.2	407.1	384.8	554.0
40	2635	1645	3797	7890

generated by different scenario reduction techniques and, therefore, the computation time is weakly dependent on the scenario reduction technique used. As the number of scenarios increases, scenarios generated by different scenario reduction techniques become more varied and require different amounts of computation time. The forward scenario selection technique produces scenarios that result in the fastest solution of the scenario-based stochastic UC. On the other hand, the IS scenario reduction technique produces scenarios that require the largest computation time. Table II shows that the interval UC usually requires less computing time than the scenario-based stochastic UC and does not depend on the range of uncertainty considered.

The day-ahead results obtained for the scenario-based and the interval UC indicate that the scenario reduction technique is a factor that significantly affects the computation time of the stochastic UC. On the other hand, if the scenario-based UC is solved with different scenario reduction techniques, the DAC variations would be of the same order of magnitude as the DAC difference between the scenario-based and interval formulations.

B. Real-Time Re-dispatch and Actual Operating Cost

Table IV shows results obtained via Monte Carlo simulations: the expected value of the actual operating cost, $E\left(AOC\right)$, its standard deviation, $\sigma\left(AOC\right)$, the expected cost of corrective dispatch, $E(\Delta)$, the expected start-up cost, $E\left(SC\right)$, and its standard deviation, $\sigma\left(SC\right)$, for the scenario-based schedules obtained with different scenario reduction techniques and for different numbers of scenarios. The expected value of the start-up cost includes the day-ahead start-up cost and the expected start-up cost of real-time changes to the commitment decisions. The forward scenario selection techniques technique results in the least expensive AOC and, therefore, produces the most cost-effective day-ahead schedule. On the other hand, the schedules produced by this technique also result in the largest standard deviation of the AOC. This indicates that the least expensive solution is also less adaptable to worst-case realizations of uncertainty than more expensive schedules obtained with the other scenario-reduction techniques. The AOC also depends on the number of scenarios considered in the scenario-based stochastic UC. If only five scenarios are used, the schedule is based on an inaccurate representation of the uncertainty and, therefore, this schedule requires expensive real-time adjustments. On the other hand,

TABLE IV
ACTUAL OPERATING COST (IN 10^3\$) OF THE SCENARIO-BASED UC SCHEDULES

Positively Correlated Wind and Load

Scenarios	Parameter	FSS	BSR	k-means	IS
5	$E\left(AOC\right)$	593.0*	593.1	593.1	596.2
	$\sigma\left(AOC\right)$	17.7	17.7	16.2	16.1
	$E\left(\Delta\right)$	26.3	24.3	26.1	28.5
	$E\left(SC\right)40$	21.3	21.1	21.3	22.9
	$\sigma\left(SC\right)$	0.719	0.728	0.715	0.708
10	$E\left(AOC\right)$	590.4*	591.0	590.8	592.2
	$\sigma\left(AOC\right)$	17.3	17.3	17.3	15.8
	$E\left(\Delta\right)$	19.8	19.8	20.3	17
	$E\left(SC\right)40$	19.5	21.1	21.3	10.8
	$\sigma\left(SC\right)$	0.188	0.186	0.182	0.204
20	$E\left(AOC\right)$	590.5*	591.5	590.9	593.1
	$\sigma\left(AOC\right)$	16.1	16.0	15.7	15.7
	$E\left(\Delta\right)$	17.5	19.1	13.4	15.3
	$E\left(SC\right)40$	21.4	21.1	23.1	23.1
	$\sigma\left(SC\right)$	0.167	0.157	0.193	0.150
40	$E\left(AOC\right)$	593.3*	593.5	594.2	596.7
	$\sigma\left(AOC\right)$	15.7	15.8	15.5	15.7
	$E\left(\Delta\right)$	17.3	16.9	16.1	18.5
	$E\left(SC\right)40$	22.9	22.9	22.8	22.8
	$\sigma\left(SC\right)$	0.155	0.191	0.138	0.138

Negatively Correlated Wind and Load

Scenarios	Parameter	FSS	BSR	k-means	IS
5	$E\left(AOC\right)$	666.6*	669.0	668.4	667.6
	$\sigma\left(AOC\right)$	13.9	13.8	13.9	13.0
	$E\left(\Delta\right)$	9.5	11.4	11.7	15.2
	$E\left(SC\right)40$	12.0	12.2	12.1	12.5
	$\sigma\left(SC\right)$	0.693	0.754	0.605	0.229
10	$E\left(AOC\right)$	667.4*	669.1	668.7	668.1
	$\sigma\left(AOC\right)$	13.2	13.0	13.1	12.6
	$E\left(\Delta\right)$	10.7	10.0	10.6	7.8
	$E\left(SC\right)40$	12.1	12.3	12.2	12.1
	$\sigma\left(SC\right)$	0.656	0.646	0.584	0.161
20	$E\left(AOC\right)$	665.5*	665.7	665.7	670.6
	$\sigma\left(AOC\right)$	12.9	12.9	13.0	12.7
	$E\left(\Delta\right)$	6.6	6.2	6.0	8.5
	$E\left(SC\right)40$	12.3	12.1	12.3	12.2
	$\sigma\left(SC\right)$	0.596	0.619	0.214	0.129
40	$E\left(AOC\right)$	669.1*	671.9	671.7	671.0
	$\sigma\left(AOC\right)$	12.6	12.4	12.3	12.4
	$E\left(\Delta\right)$	8.5	9.3	8.2	7.8
	$E\left(SC\right)40$	12.3	12.1	12.2	12.6
	$\sigma\left(SC\right)$	0.133	0.119	0.136	0.111

* – the minimum AOC for a given number of scenarios.

TABLE V
ACTUAL OPERATING COST (IN 10^3\$) OF THE INTERVAL UC SCHEDULES

Positively Correlated Wind and Load

Parameter	30%	20%	10%	1%
$E(AOC)$	587.8	590.1	591.2	596.3
$\sigma(AOC)$	18.1	16.8	15.6	1.5
$E(\Delta)$	22.3	21.9	18.8	17.0
$E(SC)$	12.3	12.5	12.2	12.2
$\sigma(SC)$	0.201	0.185	0.207	0.123

Negatively Correlated Wind and Load

Parameter	30%	20%	10%	1%
$E(AOC)$	685.3	668.9	665.9	669.7
$\sigma(AOC)$	21.3	20.9	1.30	0.891
$E(\Delta)$	27.2	10.4	6.6	1.6
$E(SC)$	21.2	21.1	21.2	22.7
$\sigma(SC)$	0.825	0.697	0.570	0.113

if the number of scenarios is 40, the schedule accommodates scenarios with low probabilities and is unnecessarily robust and expensive. The minimum AOC occurs for 10 or 20 scenarios for all the scenario reduction technique considered. The standard deviation of the AOC monotonically decreases as the number of scenarios increases. This trend indicates that a larger number of scenarios improves the adaptability of the schedule. Although the expected start-up cost does not vary significantly for different numbers of scenarios or scenario reduction techniques, its standard deviation decreases as the number of scenarios increases. Therefore, a more robust representation of uncertainty causes less cycling of generators.

Table V summarizes key statistics of the Monte Carlo trials for the interval UC using the same notations as in Table IV. The AOC increases as the range of uncertainty increases and is sensitive to the bounds. If the 30% bounds are used, the schedule is not robust enough and results in the largest cost of corrective actions. On the other hand, the schedule with the 1% bounds is more expensive than any scenario-based stochastic UC in Table IV. On the other hand, it requires the least costly corrective actions. The AOC of the interval solution is larger than the cost of the scenario-based solution for the positively correlated wind and load profiles and the difference is in the range $[0.13, 0.35]\,\%$ for different numbers of scenarios. On the other hand, for the negatively correlated wind and load profiles, this range is $[-0.48, 0.06]\,\%$. This indicates that the cost of the interval approach can be lower than the cost of the scenario-based approach (e.g. the cost for 5 scenarios) if scenarios do not represent uncertainty accurately and require expensive corrective dispatch.

IV. CONCLUSION

This paper compared the scenario-based stochastic UC with different scenario reduction techniques and the interval UC. Results demonstrate that different scenario reduction techniques affect the operating cost and the computation time of the stochastic UC. The forward scenario selection technique generates scenarios, which result in the least expensive actual operating cost for the scenario-based stochastic UC formulation. This scenario selection technique is also shown to produce scenarios that cause the lowest computation burden. Although this computation time is relatively small, as compared to the other scenario reduction techniques, it is an order of magnitude larger than the computation time of the interval formulation. The results show that the difference between the cost of the interval and stochastic-based UC can be kept at a minimum if the bounds of the interval method are chosen carefully. Further studies are needed to optimally determine the bounds of the interval formulation and thereby to minimize the cost difference between the interval and scenario-based stochastic UC.

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