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Short pulse detection and imaging of objects behind obscuring random layers

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This paper presents a theory of imaging objects behind layers of scattering media. The transmitter is a focused array or an aperture emitting a short pulse. The scattered pulse is received by a focused array or aperture. The received signal consists of two components: the **pulse scattered** from a random medium and from the target, and these two components can be distinguished by the use of ultra wide band (UWB) pulse. The second moment of the received signal includes the fourth order moments of stochastic Green's functions which are reduced to the second moments by the use of the circular complex Gaussian assumption, and of the generalized two-frequency mutual coherence function. This imaging theory is a generalization of optical coherence tomography (OCT), SAR and confocal imaging. It clarifies the relationships among resolution, coherence length, shower curtain effects, and backscattering enhancement.

1 Introduction

Detection of an object behind obscuring layers is one of the important problems today as it is applicable to numerous practical problems such as detection of a hidden object, mine detection, and imaging through scattering biological medium [1]. It has been noted that short pulse which penetrates through obscuring layers may be scattered by the layer and the object and the received signal scattered from the object and from the layer can be separated in time and therefore the detection of the object hidden behind the scattering layer may be possible. However, this requires the study of the interaction of a short pulse with random medium. Theoretically, this requires the study of pulse wave scattering and propagation in random medium, and the development of generalized two-frequency mutual coherence function [2]. In this paper, we investigate this detection and imaging problem and clarify several effects including resolutions, coherence length, shower curtain effects, and the backscattering enhancement. We make use of a focused aperture or an array of antennas, which emits a short pulse and receives it with focusing. This analysis is a generalization of OCT (Optical coherence tomography) [3], [4], [5], SAR

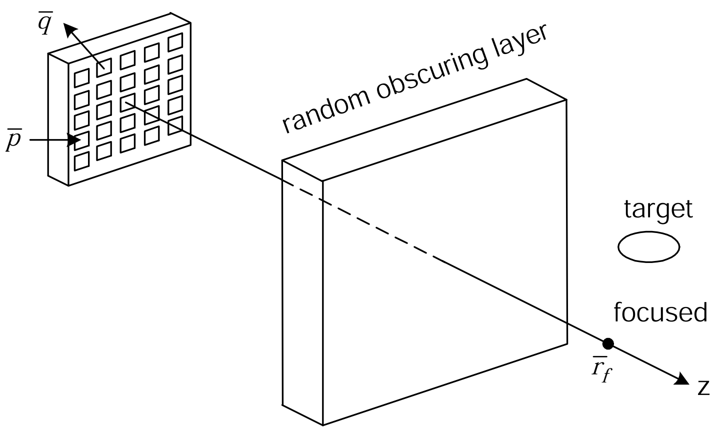

Figure 1. Array of transmitters and receivers to detect and image the target behind obscuring layers.

and confocal imaging [6], [7].

2 Formulation of the problem

Let us consider the detection and imaging of object shown in Fig. 1. A square array or aperture of transmitters and receivers are located at $z = 0$. The pulse wave is emitted by transmitters located at $\bar{p}$ focused at $\bar{r}_f(L, 0)$. The scattered wave is received at $\bar{q}$ and the phase is added to focus at $\bar{r}_f$. The waves are transmitted by all $\bar{p}$ transmitters and the received waves at all $\bar{q}$ receivers are summed. The temporal output wave ψ from a point scatterer is therefore given by

$$\psi(t) = \frac{1}{2\pi} \int \sum_p \sum_q g_1(\omega) g_2(\omega) F(\omega) \exp(-i\omega t)\, d\omega \tag{1}$$

where g_1 and g_2 are the forward and backward stochastic Green's function at ω. $F(\omega)$ is the transmitter spectrum. For a modulated Gaussian pulse, we have

$$\begin{aligned} f(t) &= \exp\left(-\frac{t^2}{T_o^2} - i\omega_o t\right) \\ F(\omega) &= \int f(t) \exp(i\omega t)\, dt = \sqrt{\pi} T_o \exp\left(-\frac{(\omega - \omega_o)^2}{\Delta\omega^2}\right) \end{aligned} \tag{2}$$

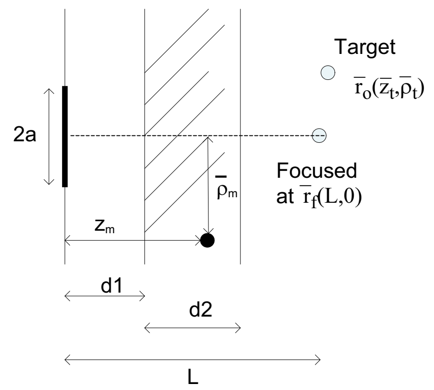

Figure 2. Target with **cross section** σ_t is located at $(z_t, \bar{\rho}_t)$. The medium with backscattering coefficient σ_b is located at $(z_m, \bar{\rho}_m)$ and the scattering from $d_1 < z_m < d_2$ are summed.

where $\Delta\omega = 2/T_o =$ bandwidth. The correlation function of the output is then given by

$$\Gamma(t,t') = \frac{1}{(2\pi)^2} \iint d\omega\, d\omega' \, \exp(-i\omega t + i\omega t')\Gamma(\omega,\omega')F(\omega)F^*(\omega') \tag{3}$$

The two-frequency mutual coherence function $\Gamma(\omega,\omega')$ consists of the component Γ_t from the target and Γ_m from the medium. (Figure 2)

$$\begin{aligned} \Gamma &= \sigma_t \Gamma_t + \Gamma_m \\ \Gamma_m &= \int_{d_1}^{d_2} dz_m \int d\bar{\rho}_m \sigma_b \Gamma\left(\bar{\rho}_m, z_m, \omega, \omega'\right) \end{aligned} \tag{4}$$

where σ_t is the backscattering crosssection of the target and σ_b is the backscattering coefficient of the medium. We will first examine Γ_t . The two-frequency mutual coherence function (MCF) is given by

$$\Gamma_t(\omega,\omega') = \sum_p \sum_q \sum_{p'} \sum_{q'} I \tag{5}$$

where $I = \left\langle g_1 g_2 g_1'^* g_2'^* \right\rangle$, $g_1 = G_1(\bar{p},\bar{r}_o,\omega)u(\bar{p},\bar{r}_f,\omega)$, G is the stochastic Green's function and u is the focusing function. $g_2 = G_2(\bar{q},\bar{r}_o,\omega)u(\bar{q},\bar{r}_f,\omega)$, $g_1' = G_1(\bar{p}',\bar{r}_o,\omega')u(\bar{p}',\bar{r}_f,\omega')$, $g_2' = G_2(\bar{q}',\bar{r}_o,\omega')u(\bar{q}',\bar{r}_f,\omega')$. $\bar{r}_o$ and $\bar{r}_f$ are the target location and the focus point respectively. Note that g_1 and g_2 are forward and backward waves respectively at ω and g_1' and g_2' are forward and backward waves at ω'. We also note that the focusing function is the conjugate of the free space Greens function G_o.

$$
\begin{aligned}
u(\bar{p}, \bar{r}_f, \omega) &= G_o^*(\bar{p}, \bar{r}_f, \omega) \\
u(\bar{q}, \bar{r}_f, \omega) &= G_o^*(\bar{q}, \bar{r}_f, \omega) \\
u(\bar{p}', \bar{r}_f, \omega') &= G_o^*(\bar{p}', \bar{r}_f, \omega') \\
u(\bar{q}', \bar{r}_f, \omega') &= G_o^*(\bar{q}', \bar{r}_f, \omega')
\end{aligned}
\tag{6}
$$

We can rewrite (3) for Γ_t using the center and difference time and frequency and obtain

$$
\Gamma(t, t') = \frac{1}{(2\pi)^2} \iint d\omega_c \, d\omega_d \; \Gamma_t(\omega_c, \omega_d) F(\omega) F^*(\omega') \exp(-i\omega_d t_c + i\omega_c t_d) \tag{7}
$$

where $\omega_d = \omega - \omega'$, $\omega_c = (\omega + \omega')/2$, $t_d = t - t'$, $t_c = (t + t')/2$. Furthermore, if $\Gamma_t(\omega_c, \omega_d)$ is a slowly varying function of ω_c , we can integrate (7) with respect to ω_c , and at $t = t'(t_d = 0)$, we get the pulse intensity

$$
\Gamma_t(t_c) = \frac{1}{\pi} \frac{1}{\Delta\omega} \sqrt{\frac{\pi}{2}} \int d\omega_d \exp(-i\omega_d t) \Gamma_t(\omega_d) \exp\left(-\frac{\omega_d^2}{2\Delta\omega^2}\right) \tag{8}
$$

where $\Gamma_t(\omega_d)$ is given in (5).

Let us now consider the two-frequency MCF.

$$
\begin{aligned}
\Gamma_t(\omega, \omega') &= \sum_p \sum_q \sum_{p'} \sum_{q'} I(\omega_d) \\
I(\omega_d) &= \left\langle g_1 g_2 {g'_1}^* {g'_2}^* \right\rangle
\end{aligned}
\tag{9}
$$

In order to express this fourth order moment in terms of the second order moment, we use the circular complex Gaussian assumption [8]

$$
\begin{aligned}
\langle u_1 u_2 u_3^* u_4^* \rangle &= \langle u_1 u_3^* \rangle \langle u_2 u_4^* \rangle + \langle u_1 u_4^* \rangle \langle u_2 u_3^* \rangle - \langle u_1 \rangle \langle u_2 \rangle \langle u_3^* \rangle \langle u_4^* \rangle \\
&= \langle u_1 \rangle \langle u_3^* \rangle \langle u_2 \rangle \langle u_4^* \rangle + \langle u_1 \rangle \langle u_3^* \rangle \langle v_2 v_4^* \rangle + \langle u_2 \rangle \langle u_4^* \rangle \langle v_1 v_3^* \rangle \\
&\quad + \langle v_1 v_3^* \rangle \langle v_2 v_4^* \rangle + \langle u_1 \rangle \langle u_4^* \rangle \langle v_2 v_3^* \rangle + \langle u_1 \rangle \langle u_3^* \rangle \langle v_1 v_4^* \rangle + \langle v_1 v_4^* \rangle \langle v_2 v_3^* \rangle
\end{aligned}
\tag{10}
$$

where $u = \langle u \rangle + v$. Making use of this, we write

$$\begin{aligned} I(\omega_d) &= \langle g_1 \rangle \langle g_2 \rangle \left\langle g_1'^* \right\rangle \left\langle g_2'^* \right\rangle + \langle g_1 \rangle \left\langle g_1'^* \right\rangle \left\langle g_{2f} g_{2f}'^* \right\rangle \\ &+ \langle g_2 \rangle \left\langle g_2'^* \right\rangle \left\langle g_{1f} g_{1f}'^* \right\rangle + \left\langle g_{1f} g_{1f}'^* \right\rangle \left\langle g_{2f} g_{2f}'^* \right\rangle \\ &+ \langle g_1 \rangle \left\langle g_2'^* \right\rangle \left\langle g_{2f} g_{1f}'^* \right\rangle + \langle g_2 \rangle \left\langle g_1'^* \right\rangle \left\langle g_{1f} g_{2f}'^* \right\rangle \\ &+ \left\langle g_{1f} g_{2f}'^* \right\rangle \left\langle g_{2f} g_{1f}'^* \right\rangle \end{aligned} \tag{11}$$

where $g_1 = \langle g_1 \rangle + g_{1f}$. Equation (8) with (9) and (11) gives the analytical form of the backscattered pulse from the target.

3 Coherent field and two-frequency MCF

It is now necessary to calculate the coherent field $\langle g_1(\omega) \rangle$ and the two-frequency MCF

$$\left\langle g_1(\omega) g_1'^*(\omega) \right\rangle = \langle g_1 \rangle \left\langle g_1'^* \right\rangle + \left\langle g_{1f} g_{1f}'^* \right\rangle \tag{12}$$

and similarly for all g_1, g_2 , g_1' , and g_2'. We make use of the parabolic approximation [1], [2] and write

$$\langle g_1 \rangle = \langle G_1(\bar{p}, \bar{r}_o, \omega) \rangle \, u(\bar{p}, \bar{r}_f, \omega) \tag{13}$$

where $\langle G_1 \rangle = G_o \exp\left(-\int \alpha \, dz\right)$, G_o is the free space Green's function $G_o = \frac{1}{4\pi z} \exp\left(ikz + i\frac{k}{2z} \left|\bar{p} - \bar{\rho}\right|\right)$, $\alpha = 2\pi^2 k^2 \int_0^\infty \Phi_n(\kappa, z) \, \kappa \, d\kappa = \frac{\tau_o}{2}$. The focusing function is given by (Fig. 3)

$$u = \frac{1}{4\pi L} \exp\left(-ikL - i\frac{kp^2}{2L}\right). \tag{14}$$

From (13), (14), we write $\langle g_1 \rangle = \langle G_1 \rangle u = I_p(\omega) \exp\left(-\int \alpha \, dz\right)$,

$$I_p(\omega) = \frac{1}{(4\pi z)} \frac{1}{(4\pi L)} \exp\left[ik_1(z - L) + \frac{ik_1}{2z}\rho^2 + \frac{ik_1 p^2}{2}\left(\frac{1}{z} - \frac{1}{L}\right) - i\frac{k_1}{z}\bar{p} \cdot \bar{\rho}\right] \tag{15}$$

where $z = z_t$ and $\bar{\rho} = \rho_t$ for target scattering. Rewriting (11) as $I = \sum_{i=1}^{7} I_i$, we get $I_1 = I_p(\omega) I_q(\omega) I_{p'}(\omega') I_{q'}(\omega') \exp\left[-2\tau_o\right]$. Let us next consider I_2.

$$I_2 = \langle g_1 \rangle \left\langle g_1'^* \right\rangle \left\langle g_{2f} g_{2f}'^* \right\rangle \tag{16}$$

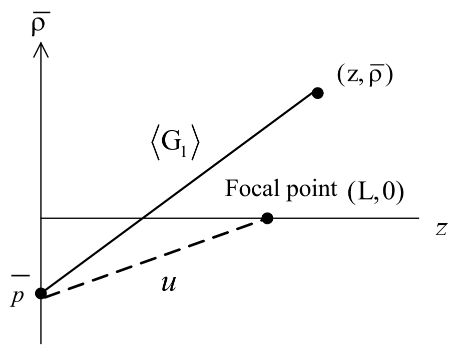

Figure 3. Coherent field $\langle g_1 \rangle$ and the focusing function $u = G_o^*$.

Here, we need to find the two-frequency MCF $\langle g_{2f} {g'_{2f}}^* \rangle$. From [2] for Gaussian phase function $p(s)$

$$p(s) = 4\alpha_p \exp\left(-\alpha_p s^2\right) \tag{17}$$

where $s = 2\sin(\theta/2)$, θ is the scattering angle, we have

$$\begin{aligned}
&\langle g_1 \rangle = I_p(k_1) \exp(-\tau_o/2), \\
&\left\langle {g'_1}^* \right\rangle = I_{p'}(k_2) \exp(-\tau_o/2), \\
&\left\langle g_{2f} {g'_{2f}}^* \right\rangle = I_q(k_1) I_q(k_2) F_s \exp\left[-E_f - D\left|\bar{q} - \bar{q}'\right|^2\right], \\
&F_s = 1 - \exp\left[\int dz\ b\ B_p\right], \qquad E_f = \int dz\ \ (1-b)\ \ B_p, \\
&D = \int dz\ b\ B_p \frac{k^2}{4\left(\alpha_p - A_p\right)} \frac{\left(z_m - z\right)}{z_m}, \\
&A_p = \frac{iz\left(z_m - z\right)}{2z_m} k_d, \qquad B_p = \left(1 - \frac{A_p}{\alpha_p}\right)^{-1},
\end{aligned} \tag{18}$$

where b is the scattering coefficient, and $z_m = z_t$, and $\bar{\rho} = \bar{\rho}_t$ for the target scattering. We use the same technique to calculate I_i for all $i = 3$ to 7.

4 Approximate analytical final expression

Now we go back to (9). Here, we need to calculate four summations. We can make use of Poison's sum formula and the following approximations to obtain analytical expressions for summation [8]. For square array, we use

$$\sum_p = \sum_{p_x}\sum_{p_y} = \left(\frac{(2N+1)}{2a}\right)^2 \iint \frac{4}{\pi}\exp\left(-\frac{x^2+y^2}{a^2}\right) f(x,y)\,dx\,dy. \tag{19}$$

We also use the following integral to obtain the analytical expression.

$$\begin{aligned} I\left(A,B,C,\alpha,\beta\right) &= \iint dx\;dy\;\;\exp\left(-Ax^2-By^2+Cxy+\alpha x+\beta y\right) \\ &= \frac{\pi}{\sqrt{AB-C^2/4}}\exp\left[\frac{\alpha^2 B+\beta^2 A+\alpha\beta C}{4(AB-C^2/4)}\right] \end{aligned} \tag{20}$$

We use similar procedure to calculate all $I_i, i=1,\ldots,7$. We get

$$\begin{aligned} &\sum_p\sum_{p'}\sum_q\sum_{q'}\left\langle g_1 g_2 {g_1'}^* {g_2'}^*\right\rangle = \sum\sum\sum\sum \left\langle g_1\right\rangle\left\langle g_2\right\rangle\left\langle {g_1'}^*\right\rangle\left\langle {g_2'}^*\right\rangle \\ &\quad + \left\langle g_1\right\rangle\left\langle {g_1'}^*\right\rangle\left\langle g_{2f}{g_{2f}'}^*\right\rangle + \left\langle g_2\right\rangle\left\langle {g_2'}^*\right\rangle\left\langle g_{1f}{g_{1f}'}^*\right\rangle + \left\langle g_{1f}{g_{1f}'}^*\right\rangle\left\langle g_{2f}{g_{2f}'}^*\right\rangle \\ &\quad + \left\langle g_1\right\rangle\left\langle {g_2'}^*\right\rangle\left\langle g_{2f}{g_{1f}'}^*\right\rangle + \left\langle g_2\right\rangle\left\langle {g_1'}^*\right\rangle\left\langle g_{1f}{g_{2f}'}^*\right\rangle + \left\langle g_{1f}{g_{2f}'}^*\right\rangle\left\langle g_{2f}{g_{1f}'}^*\right\rangle \\ &\quad \triangleq I_o^2 + I_o I_f + I_o I_f + I_f^2 + \left(I_o I_f\right)_b + \left(I_o I_f\right)_b + \left(I_f^2\right)_b \\ &\quad = I_o^2 + 4 I_o I_f + 2 I_f^2 \end{aligned} \tag{21}$$

where $()_b$ denotes backscattering enhancement (cross term) and represents the correlation between the forward and backward waves, giving rise to the backscattering enhancement. We now obtain the final expression for the received pulse , which consists of two components: medium and target, which are given by

$$\Gamma = \Gamma_{\text{target}} + \Gamma_{\text{medium}} \tag{22}$$

Note that $\Gamma_{\mathbf{target}}$ is the component of the received pulse for the transmitter-medium-target-medium-receiver and includes correlation between the forward (transmitter-medium-target) and the backward (target-medium-receiver). $\Gamma_{\mathbf{medium}}$ is the component for the transmitter-medium-reciever. The scattered intensity from the target located at $(z_m, \bar{\rho}_m)$ is given by

$$\Gamma_{\text{target}}\left(z_m,\bar{\rho}_m,t\right) = C\int \Gamma_o\left(z_m,\bar{\rho}_m,\omega_d\right)\exp\left(-2\tau_{am}\right)\exp\left(\Phi\right)\;\;d\omega_d \tag{23}$$

where z_m is the distance in the propagation direction, $\bar{\rho}_m = \rho_{mx}\hat{x} + \rho_{my}\hat{y}$ is the vector transverse to the propagation direction, $\tau_{am} = \int_{d_1}^{d_1+d_2} a_b \, dz = a_b(d_2)$ where a_b is the absorption coefficient, C is a constant, and

$$\Phi = i\frac{\omega_d}{2cz_m}2\left(\rho_{mx}^2 + \rho_{my}^2\right) + i\frac{\omega_d}{c}2\left(z_m - L\right) - \frac{\omega_d^2}{2\left(\Delta\omega\right)^2} - i\omega_d t \tag{24}$$

Here, L is the total distance to the focal point, c is the wave velocity in the medium, $\Delta\omega$ is the bandwidth of the signal, and

$$\Gamma_t(z_m, \rho_m, \omega_d) = \sigma_T\left(I_o^2 + 4I_oI_f + 2I_f^2\right). \tag{25}$$

The term I_o represents coherent component while the term I_f represents incoherent component. They are given by

$$I_o\left(A, B, C, \alpha, \beta\right) = \Theta_x\Theta_y \exp\left(-\tau_{sm}\right)\frac{1}{z_m^4} \tag{26}$$

where

$$\begin{aligned}
\Theta_x &= \frac{\pi}{\sqrt{AB - C^2/4}} \exp\left[\frac{\alpha_x^2 B + \beta_x^2 A + \alpha_x\beta_x C}{4\left(AB - C^2/4\right)}\right], \\
\Theta_y &= \frac{\pi}{\sqrt{AB - C^2/4}} \exp\left[\frac{\alpha_y^2 B + \beta_y^2 A + \alpha_y\beta_y C}{4\left(AB - C^2/4\right)}\right], \\
\tau_{sm} &= \int_{d_1}^{d_1+d_2} b\, dz, \quad a \text{ is the aperture size,}
\end{aligned} \tag{27}$$

$$A = \frac{1}{a^2} - i\frac{k_1}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right), \qquad B = \frac{1}{a^2} + i\frac{k_2}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right), \qquad C = 0,$$

$$\alpha_x = -i\frac{k_1\rho_{mx}}{z_m},\ \beta_x = i\frac{k_2\rho_{mx}}{z_m}, \qquad \alpha_y = -i\frac{k_1\rho_{my}}{z_m},\ \beta_y = i\frac{k_2\rho_{my}}{z_m}.$$

The incoherent intensity is given by

$$I_f\left(A_f, B_f, C_f, \alpha_f, \beta_f\right) = F_s\Upsilon_x\Upsilon_y \exp\left(-E_f\right)\frac{1}{z_m^4} \tag{28}$$

where

$$
\begin{aligned}
\Upsilon_x &= \frac{\pi}{\sqrt{A_f B_f - C_f^2/4}} \exp\left[\frac{\alpha_{fx}^2 B_f + \beta_{fx}^2 A_f + \alpha_{fx}\beta_{fx} C_f}{4\left(A_f B_f - C_f^2/4\right)}\right], \\
\Upsilon_y &= \frac{\pi}{\sqrt{A_f B_f - C_f^2/4}} \exp\left[\frac{\alpha_{fy}^2 B_f + \beta_{fy}^2 A_f + \alpha_{fy}\beta_{fy} C_f}{4\left(A_f B_f - C_f^2/4\right)}\right], \\
A_f &= \frac{1}{a^2} + \frac{D}{F_s} - M_1, \qquad B_f = \frac{1}{a^2} + \frac{D}{F_s} + M_2, \qquad C_f = 2\frac{D}{F_s}, \\
F_s &= 1 - \exp\left(-\int_{d_1}^{d_1+d_2} dz\ b\ B_p\right), \\
M_1 &= i\frac{k1}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right), \quad M_2 = i\frac{k2}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right), \\
D(z_m) &= \int_{d_1}^{d_1+d_2} dz\ b\ B_p \frac{k^2}{4\left(\alpha_p - A_p\right)} \frac{(z_m - z)}{z_m}
\end{aligned}
\tag{29}
$$

b is the scattering coefficient, α_p is the parameter related to the asymmetry factor g,

$$
\begin{aligned}
A_p &= \frac{iz\,(z_m - z)}{2z_m} k_d, \qquad B_p = \left(1 - \frac{A_p}{\alpha_p}\right)^{-1}, \\
E_f &= \int_{d_1}^{d_1+d_2} dz\ b\ \left(1 - B_p\right), \\
\alpha_{fx} &= -i\frac{k_1\rho_{mx}}{z_m}, \quad \beta_{fx} = i\frac{k_2\rho_{mx}}{z_m}, \quad \alpha_{fy} = -i\frac{k_1\rho_{my}}{z_m}, \quad \beta_{fy} = i\frac{k_2\rho_{my}}{z_m} \\
k_1 &= k + k_d/2, \quad k_2 = k - k_d/2.
\end{aligned}
$$

5 Mutual coherence function from the scattering medium

The mutual coherence function from the scattering medium is given by the above formula except that $\bar{\rho}_m$ is now inside the medium and the scattered intensity is obtained by integrating with respect to $\bar{\rho}_m$. The integral with respect to z_m is performed numerically.

$$\Gamma_{\text{medium}}(t) = \int d\omega_d \int_{d_1}^{d_1+d_2} dz_m \exp(\Phi_m)\Gamma_m \tag{30}$$

where

$$\Phi_m = i\frac{\omega_d}{c}2(z_m - L) - 2a_b(z_m - d_1) - \frac{\omega_d^2}{2(\Delta\omega)^2} - i\omega_d t$$

and

$$\Gamma_m = \sigma_b\left[I_{om}^2 + 4I_{om}I_{of} + 2I_{of}^2\right]$$

where σ_b is the backscattering coefficient given below, using Henyey-Greenstien phase function,

$$\sigma_b = \frac{1-g}{(1+g)^2}b \tag{31}$$

where g is the asymmetry factor. The I components are given by

$$I_{om}^2 = \left\{\left[\frac{\pi}{\sqrt{AB - C^2/4}}\right]^2\left[\frac{\pi}{-2Q_o - \phi_o}\right]^{1/2}\right\}^2 \exp(-2\tau_{sm})\frac{1}{z_m^4} \tag{32}$$

where

$$Q_o = \frac{\alpha_o^2 B + \beta_o^2 A + \alpha_o\beta_o C}{4\left[AB - C^2/4\right]},$$
$$\alpha_o = (-ik_1/z_m), \quad \beta_o = (-ik_2/z_m),$$
$$\tau_{sm} = b(z_m - d_1), \quad \phi_o = i\frac{\omega_d}{cz_m}.$$

$$4I_{om}I_{of} = 4\left\{\left[\frac{\pi}{\sqrt{AB - C^2/4}}\right]\left[\frac{\pi}{\sqrt{A_f B_f - C_f^2/4}}\right]\left[\frac{\pi}{-Q_o - Q_f - \phi_o}\right]^{1/2}\right\}^2 F_s \exp(-E_f - \tau_{sm})\frac{1}{z_m^4}, \tag{33}$$

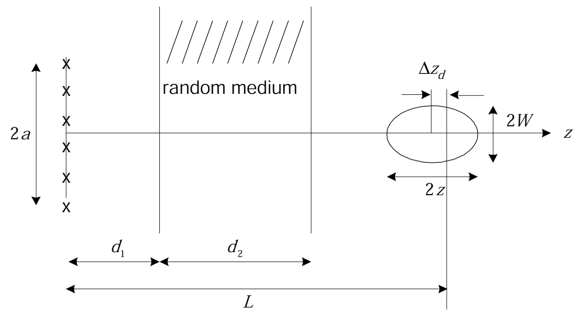

Figure 4. Geometry in numerical examples.

$$2I_{of}^2 = 2\left\{\left[\frac{\pi}{\sqrt{A_fB_f - C_f^2/4}}\right]^2\left[\frac{\pi}{-2Q_f - \phi_o}\right]^{1/2}\right\}^2 \left[F_s \exp\left(-E_f\right)\right]^2 \frac{1}{z_m^4}, \tag{34}$$

where E_f and F_s is now given by $E_f = \int_{d_1}^{z_m} dz\ b\ (1 - B_p)$, $F_s = 1 - \exp\left(-\int_{d_1}^{z_m} dz\ b\ B_p\right)$

$$Q_f = \frac{\alpha_o^2 B_f + \beta_o^2 A_f + \alpha_o\beta_o C_f}{4\left[A_fB_f - C_f^2/4\right]} \tag{35}$$

The terms A, B, C, A_f, B_f, C_f are given in (27) and (29).

6 Numerical examples

We now consider a numerical example. Fig. 4. We note several characteristics of the imaging problem. We use as the reference the following parameters: $a = 5\lambda$, $\Delta\omega/\omega_o = 0.2$, $g = 0.85$, $W_o = 0.9$, $L = 50\lambda$, $d_1 = d_2 = 25\lambda$, $\sigma_T = A\lambda^2, A = 1$ where Δz is the pulse spread, W is the lateral resolution, and Δz_d is the delay due to multiple scattering.

Figure 5 shows that the lateral spread increases with optical depth (OD), and the pulse delay. Figure 6 shows that for OD=1, the larger aperture gives

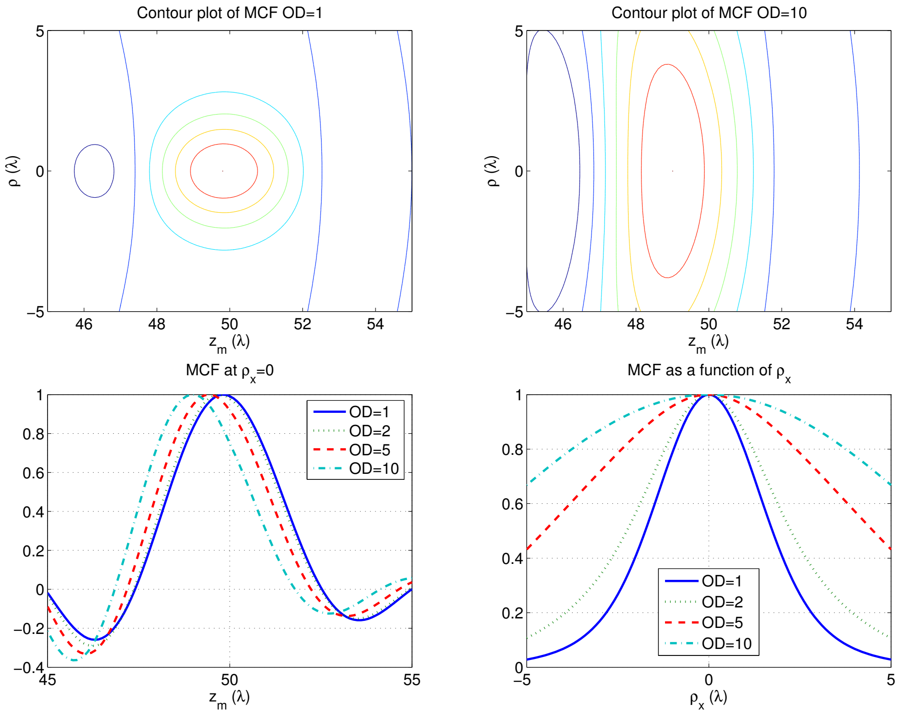

Figure 5. Imaging problem, pulse spread, delay, and lateral resolution.

smaller spot size as expected, but at OD =10, the aperture size does not affect the spot size significantly. Figure 7 shows the shower curtain effect. At OD=1, the location of the random medium ($d_1 = 0$ or $d_1 = 25\lambda$) does not make difference, but at OD=10, there is a significant difference. Figure 8 shows the effects of bandwidth. If BW=0.01 (normalized $\Delta\omega/\omega$), it is difficult to distinguish the target from the medium scattering, but if BW=0.1, it is possible to detect the target behind the medium even if OD=10.

7 Conclusion

We have presented a theory of imaging of objects behind a random medium using a short pulse and focused array or aperture antennas. General formulations are given and approximate analytical expression are obtained for the **scattering component $\Gamma_{\mathbf{target}}$ for the transmitter-medium-target-medium-receiver, and $\Gamma_{\mathbf{medium}}$ for the transmitter-medium-receiver**. Numerical examples are shown to illustrate pulse spread, pulse delay, lateral resolutions, the effects of aperture size, and the bandwidth. It is therefore possible to

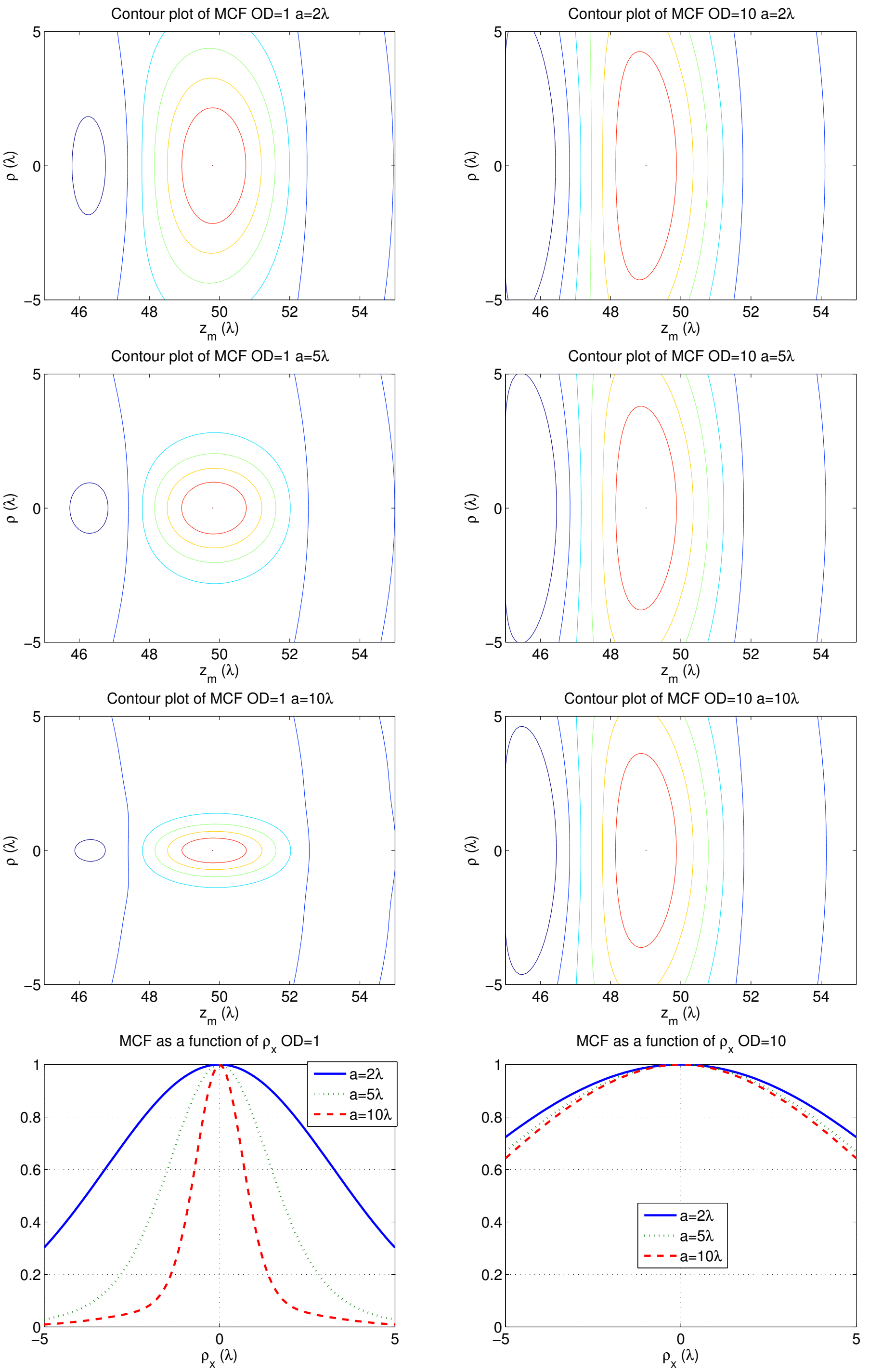

Figure 6. Effect of aperture size.

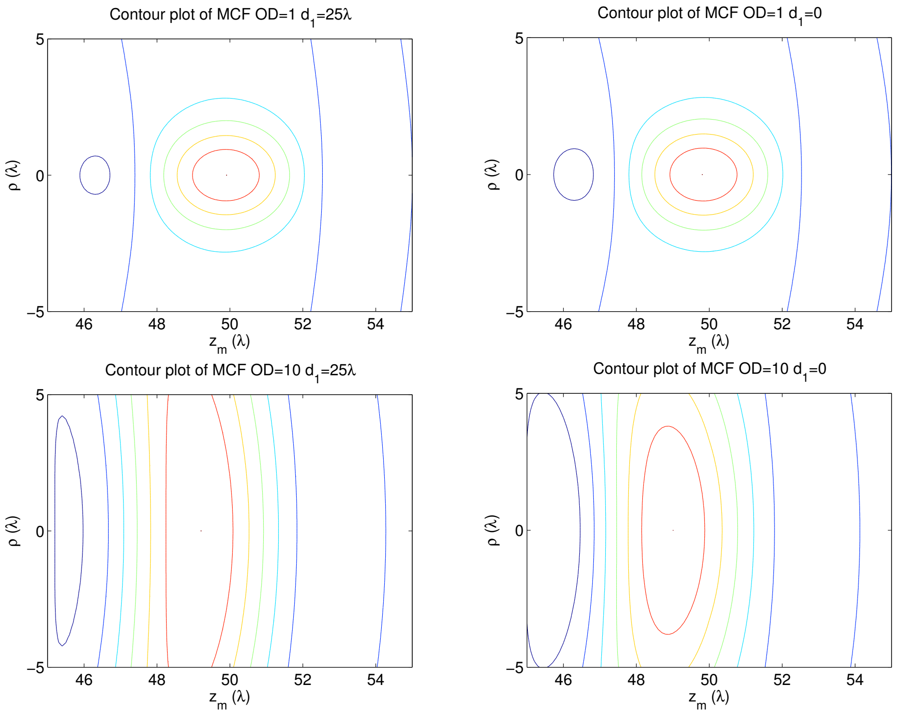

Figure 7. Shower curtain effects.

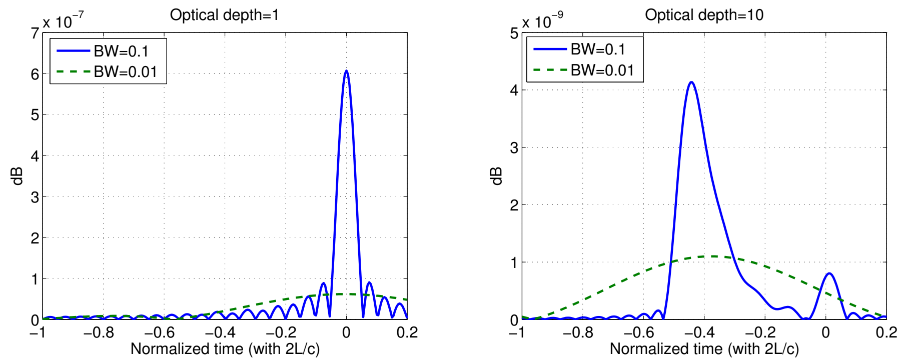

Figure 8. Effect of bandwidth. Target is located at the normalized time = 0. For broad band BW=0.1, the target can be detected even for Optical depth (OD) = 10.

distinguish the target scattering from the medium scattering.

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References

[1] Ishimaru, A., 1978, *Wave Propagation and Scattering in Random Media* (New York: Academic Press), 1997, (Piscataway, New Jersey: IEEE Press, Oxford, England: Oxford University Press).

[2] Andrews, L. C. and Phillips, R. L., *Laser Beam Propagation through Random Media* (Bellingham, Washington: SPIE), 1998.

[3] Fujimori, J., 2001, Optical coherence tomography *C. R. Acad. Paris* **2 IV**, 1099-1111.

[4] Schmitt, J. M. and Knuttle, A., 1997, Model of optical coherence tomography of hetrogeneous tissue *J. Optical Society of America A* **14:6**, 1231-42.

[5] Thrane, L., Yura, H., and Andersen, P. 2000, Analysis of optical coherence tomography systems based on the extended Huygens-Fresnel principle *J. Optical Society of America A* **17:3**, 484-90.

[6] Ishimaru, A., Chan, T.-K., and Kuga, Y. 1998, An imaging technique using confocal circular synthetic aperture radar *IEEE Trans. on Geoscience and Remote Sensing* **36:5**, 1524-30.

[7] Chan, T.-K., Kuga, Y., and Ishimaru, A. 1999, Experimental studies on circular SAR imaging in clutter using angular correlation function technique *IEEE Trans. on Geoscience and Remote Sensing* **37:5**, 2192-7.

[8] Ishimaru, A., Jaruwatanadilok, S., Kuga, Y., 2004, Multiple scattering effects on the radar cross section (RCS) of objects in a random medium including backscattering enhancement and shower curtain effects, *Waves in Random Media* **14**, 499–511.