Time reversal in random media and super-resolution

Akira Ishimaru*, Sermsak Jaruwatanadilok, and Yasuo Kuga
Department of Electrical Engineering, University of Washington
Box 352500, Seattle, WA 98195, email: ishimaru@ee.washington.edu

Time reversal is the phenomenon that a wave emitted by a point source is received by an array of transducers, time-reversed, and back-propagated in the same medium. If the medium is free space, the back propagated wave is refocused near the original source with the resolution determined by the aperture size of the array. If the medium is a random medium, multiple scattering takes place and the refocused image is different from that in free space. It may be conjectured that the multiple scattering worsens the resolution in random media. Instead, it has been observed that the resolution is better in a random medium than in free space. This is called *super resolution* and has been studied experimentally and numerically, and some theoretical explanations have been offered. This paper presents an analytical theory to explain super resolution in a random medium making use of our previous studies on stochastic Green's functions in random media.

A point source emits a modulated pulse which propagates through a random medium and is received by an array and then time-reversed and reradiated. The time-reversed array is also called the "time-reversed mirror" and is equivalent to the "conjugate mirror" used in adaptive optics and retro directive arrays. The formulation is given in analytical signal representations, and the time-reversal is expressed by the conjugate of the received signal. We consider the first moment of the refocused field, making use of the mutual coherence function and the Gaussian phase function for the random medium. The first moment consists of two terms. One is the coherent field which is attenuated due to the optical depth, and the other is the diffuse component. The coherent image is substantially the same as that in free space, except for attenuations. However, the diffuse component, which is dominant for large optical depth, has a much smaller spot size than that in free space. This super resolution is due to the coherence length which is smaller than the free space spot size. As the multiple scattering increases, the transverse coherence length decreases in proportion to the inverse of the square root of the scattering depth, resulting in smaller spot size and super resolution. The longitudinal spot size along the propagation direction is substantially the same as the original pulse because this is the first moment. This formulation also gives the shower curtain effect giving higher resolution when the random medium is closer to the source.