Time reversal in random media and super resolution with shower curtain effects and backscattering enhancement

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ABSTRACT

Time reversal has attracted considerable attention in recent years, particularly because of its potential for communication and imaging through a complex environment. When a wave is emitted by a point source and is received by an array of receivers, and time-reversed and back-propagated in the same medium, the wave is refocused near the original source. The time-reversal array is also called the "time-reversed mirror" or the "conjugate mirror".

If the medium is free space, it is clear that the wave is refocused with the resolution determined by the aperture size of the array. What is unusual is that if the medium is random causing multiple scattering, the wave is refocused with the resolution better than that in free space, contrary to our intuition. This is called the "super resolution" which has been studied experimentally and numerically, and some theoretical explanations have been offered. This paper presents a detailed analytical theory of time reversal in random media. It includes several features which do not seem to have appeared in the literature. The relationship between the super resolution and the coherence length has been pointed out in the past, but this paper gives an analytical explanation of the role of the coherence in super resolution. The theory in this paper also shows clearly the shower-curtain effects and the backscattering enhancement in time reversal, which do not seem to have been discussed in the past.

The formulation is based on our previous studies on stochastic Green's functions. First, we consider the first moment of the refocused field, making use of the mutual coherence function and the Gaussian phase function for the random medium. The point source emits a Gaussian modulated pulse. The first moment consists of two terms. One is the coherent field which is attenuated due to the optical depth, and the other is the diffuse component. The coherent image is substantially the same as that in free space, except for attenuations. However, the diffuse component, which is dominant for large optical depth, has a much smaller spot size than that in free space. This super resolution is due to the coherence length which is smaller than the free space spot size. As the multiple scattering increases, the transverse coherence length decreases in proportion to the inverse of the square root of the scattering depth, resulting in a smaller spot size and super resolution. The longitudinal spot size along the propagation direction is substantially the same as the original pulse because this is the first moment. This formulation also gives the shower curtain effect giving higher resolution when the random medium is closer to the source.

Next, we consider the second moment. Because of the time-reversal back-propagation, this second moment requires the fourth-order Green's functions. We employ the circular complex Gaussian assumption to reduce the fourth moment to the second moment. Since we deal with the time-space Green's function, we developed two-frequency mutual coherence functions based on the extended Huygens-Fresnel formulations. The refocused second moment exhibits both the shower-curtain effects and backscattering enhancement, and the longitudinal spot size now reflects the temporal broadening due to the random medium. We conclude with potential applications to communication and imaging in complex and random environments.

FIRST AND SECOND MOMENTS OF THE TIME-REVERSED FIELD IN A RANDOM MEDIUM

Consider the problem shown in Fig. 1.

A point source at $\bar{r}_t(L,0)$ emits a Gaussian modulated pulse given by

$$f(t) = A_o \exp\left(-\frac{t^2}{T_o^2} - i\omega_o t\right) \tag{1}$$

Its spectrum is given by

$$F(\omega) = A_o \frac{2\sqrt{\pi}}{\Delta\omega} \exp\left(-\frac{(\omega-\omega_o)^2}{\Delta\omega^2}\right) \tag{2}$$

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where $\Delta\omega = 2/T_o$ is the bandwidth and ω_o is the carrier frequency. This is time-reversed and sent into the same medium. The spectrum of the time-reversed field is the complex conjugate of the spectrum of the original field. Thus, this is called the "time-reversal mirror" or the "conjugate mirror".

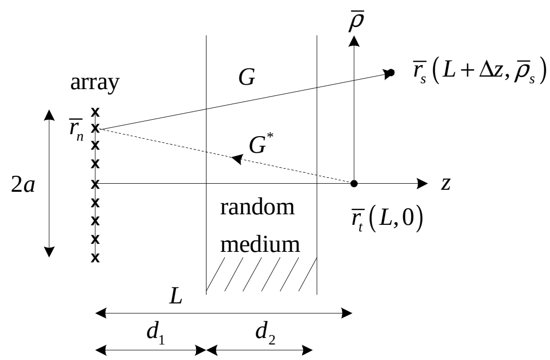

Figure 1: A point source at $\bar{r}_t$ emits a Gaussian pulse, which is received by $2N+1$ receivers, time-reversed, back propagated into the same medium, and observed at $\bar{r}_s$

The time-reversed field is then emitted into the same medium and the field at $\bar{r}_s$ is the sum of all contributions from $2N+1$ transducers and given by the spectrum:

$$\begin{aligned} \psi(\bar{r}_s, \omega) &= \sum_n \psi(\bar{r}_s, \bar{r}_n, \bar{r}_t, \omega) \\ \psi(\bar{r}_s, \bar{r}_n, \bar{r}_t, \omega) &= G(\bar{r}_s, \bar{r}_n, \omega) G^*(\bar{r}_n, \bar{r}_t, \omega) \mathrm{F}^*(\omega) \end{aligned} \tag{3}$$

The average field is therefore given by

$$\begin{aligned} \langle \psi(\bar{r}_s, t) \rangle &= \frac{1}{2\pi} \int \exp(-i\omega t)\, d\omega \sum_n \langle \psi(\bar{r}_s, \bar{r}_n, \bar{r}_t, \omega) \rangle \\ &= \frac{1}{2\pi} \int \exp(-i\omega t)\, d\omega \sum_n \Gamma_n(\bar{r}_s, \bar{r}_n, \bar{r}_t, \omega) F^*(\omega) \end{aligned} \tag{4}$$

where Γ is the mutual coherent function of the field at $\bar{r}_s$ and $\bar{r}_t$ with the source at $\bar{r}_n$.

$$\Gamma_n = \langle G(\bar{r}_s, \bar{r}_n, \omega) G^*(\bar{r}_n, \bar{r}_t, \omega) \rangle \tag{5}$$

The second moment is given by

$$\langle \psi(\bar{r}_s, t_1) \psi^*(\bar{r}_s, t_2) \rangle = \frac{1}{(2\pi)^2} \int d\omega_1\, d\omega_2\, \Gamma(\omega_1, \omega_2) F(\omega_1) F_2^*(\omega) \exp(-i\omega_1 t_1 + i\omega_2 t_2) \tag{6}$$

where

$$\begin{aligned} &\Gamma(\omega_1, \omega_2) = \sum_m \sum_n \langle G_1 G_2^* G_3 G_4^* \rangle \\ &G_1 = G(\bar{r}_s, \bar{r}_m, \omega_1), \quad G_2 = G(\bar{r}_m, \bar{r}_t, \omega_1), \\ &G_3 = G(\bar{r}_s, \bar{r}_n, \omega_2), \quad G_4 = G(\bar{r}_n, \bar{r}_t, \omega_2), \end{aligned} \tag{7}$$

The fourth order moments are expressed in terms of the second moments by using the following circular complex Gaussian assumption:

$$\langle G_1 G_2^* G_3 G_4^* \rangle = \langle G_1 G_2^* \rangle \langle G_3 G_4^* \rangle + \langle G_1 G_4^* \rangle \langle G_3 G_2^* \rangle - \langle G_1 \rangle \langle G_2^* \rangle \langle G_3 \rangle \langle G_4^* \rangle \tag{8}$$

Numerical calculations based on the above formulations are given in Fig. 2.

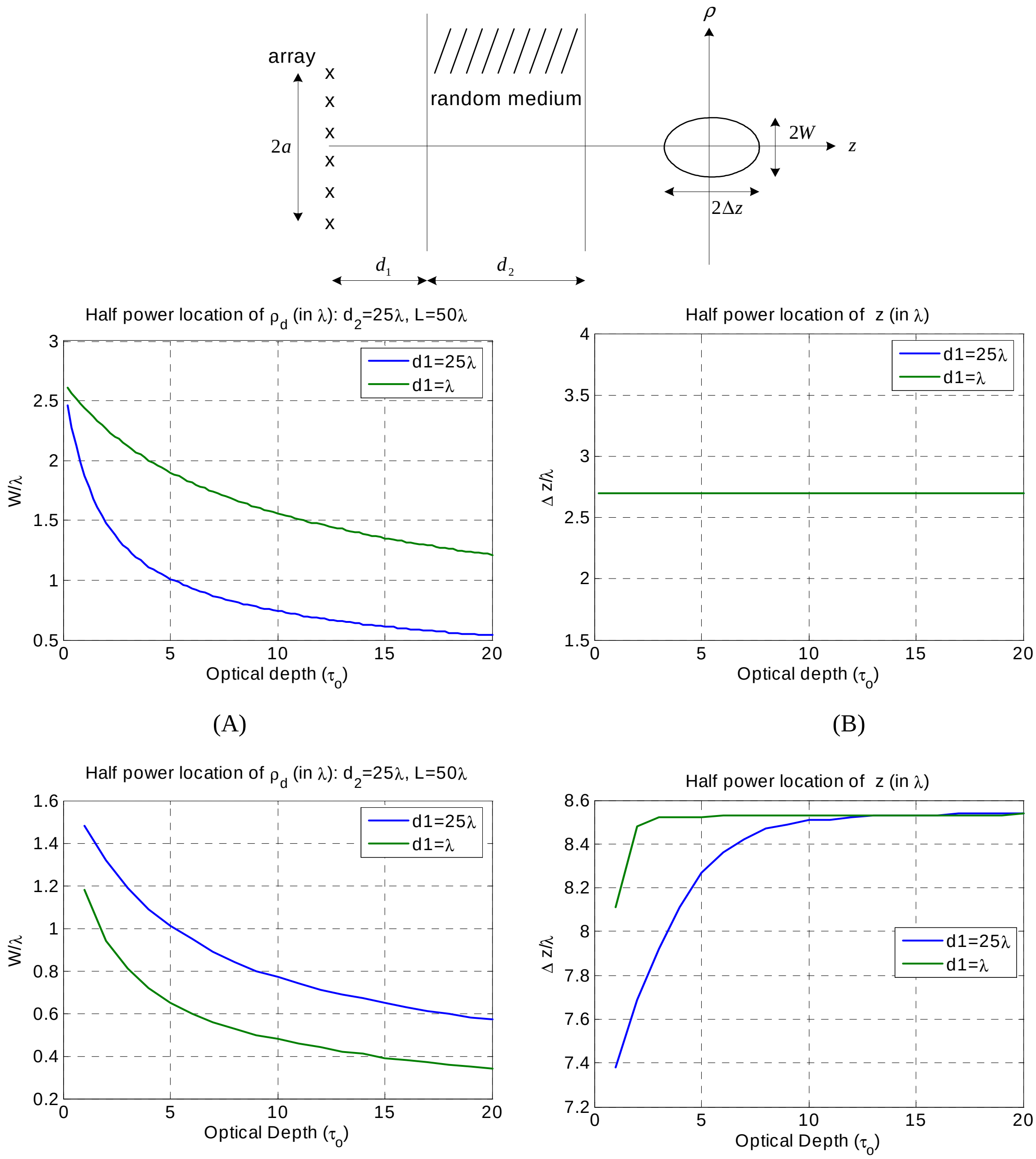

Figure 2. Time reversal in a random medium. Point source at $(\rho = 0,\ z = L)$ emits a Gaussian pulse which is received by an array of transducers and time-reversed and back propagated. This will be focused near the source with a resolution better than that of free space (super-resolution). This shows the super-resolution (W is smaller than that in free space) and the shower curtain effects (when the random medium is close to the source, $d_1 = 25\lambda$, the resolution is better). $\alpha_p = 44.578$ corresponding to anisotropy factor $g = 0.85$, $a = 5\lambda$, albedo $W_o = 0.9$, and $\Delta\omega / \omega_o = 0.1$. (A) and (B) are the first-moment solutions, (C) and (D) are the second-moment solutions.

We note that when the medium is far from the array ($d_2 = 25\lambda$), the transverse coherence length is smaller than when the medium is close to the array ($d_2 = \lambda$). For the first moment, the reduced coherence length gives a smaller spotsize (W/λ). For the second moment, the dispersion gives broader pulse duration ($\Delta z / c$) if the medium is closer to the array, and therefore, to conserve the power the transverse spot size (W/λ) decreases. The backscattering enhancement is included in (8). If we ignore the backscattering enhancement, we have

$$\left\langle G_1 G_2^* G_3 G_4^* \right\rangle = \left\langle G_1 G_2^* \right\rangle \left\langle G_3 G_4^* \right\rangle \tag{9}$$

The difference between (8) and (9) is the absence of the correlations between G_1 and G_4 and between G_3 and G_2 in (9).

CONCLUSIONS

This paper gives a theoretical explanation of several unusual time reversal phenomena including super-resolution, shower curtain effects, and backscattering enhancement.

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