

**ON THE FEATURES OF SPATIAL SPECTRUM OF SCATTERED  
ELECTROMAGNETIC WAVES IN A RANDOM ABSORBING SLAB**

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**Introduction**

It is known, that absorption of energy of light waves exerts essential influence on their statistical characteristics at scattering in randomly media [1,2]. Absorption can lead to significant distortions of an angular power spectrum (APS) of scattered radiation at asymmetric statement of a problem [3,4]. The case when a point source and the receiver are located on the opposite sides with respect to absorptive chaotically random layer is of practical interest. In this paper the problem of passing of radiation from a point source through a plane absorbing layer with randomly smooth inhomogeneities of dielectric permittivity in the case when a source and the receiver are located at various distances from layer's boundaries is considered. APS of scattered radiation is statistically simulated for different thickness of a layer, positions of a source and the receiver concerning it and regular absorption in a slab.

**Statement of the problem and its solution**

Let's assume that a point source (Fig. 1) is located in the homogeneous nonabsorbing medium with dielectric permittivity  $\varepsilon = \varepsilon_0$  at a distance  $L_1$  above randomly inhomogeneous absorptive plane layer with thickness  $Z$ . Dielectric permittivity of this layer is  $\varepsilon = \varepsilon_0 + i\varepsilon'' + \varepsilon_1(\mathbf{r})$ , where  $\varepsilon_1(\mathbf{r})$  is random variable with zero mean value describing fluctuations of the real part of dielectric permittivity inside a layer, and  $\varepsilon''$  describes regular absorption of wave in a slab. The source has directivity diagram as a cone. Apex angle of the cone is supposed big enough that at  $L_1 \approx L_2 \approx Z$  lighted area on the upper boundary of a layer allowed scattered radiation propagate along three directions designated in Fig. 1 by dashed line. The receiver is located in the homogeneous nonabsorbing medium with  $\varepsilon = \varepsilon_0$  in plane XZ at a distance  $L_2$  below slab. Further without restriction of generality  $\varepsilon_0$  is considered equal to unit. The line-of-sight connecting the source and the receiver makes angle  $\theta$  with z-axis. This angle and the apex angle of a cone further are considered fixed at motion of a source and the receiver concerning layer. Let the characteristic size of inhomogeneities in a layer substantially exceed the wavelength of radiation, and  $\varepsilon_1 \ll \varepsilon_0$ . This problem for the case of small-angle scattering has been solved in [5]. In this paper the method of

complex geometrical optics was used, allowing solve the transfer equation for a phase by perturbation method and find its correlation function. APS is the Fourier transform of this correlation function. Trustworthy information about behaviour of the APS is possible to obtain using the most general representations of propagation and scattering processes in chaotically inhomogeneous medium, in particular, statistical simulation – Monte Carlo method. In the given paper we have used its modification, so-called "weight" algorithm.

#### Numerical analyses of the APS

For receiving the most reliable representation about character of dependence of an APS versus location both a source and the receiver, thickness of a slab and absorption in the layer, we shall choose the following model of spatial spectrum of dielectric permittivity fluctuations presenting in [5]. Further we shall use the scattered indicatrix

$$\sigma(\alpha) = \frac{1}{2} \pi k_0^4 \Phi\left(\frac{1}{2} k_0 \sin \frac{\alpha}{2}\right), \quad (1)$$

which quite good describes single scattered processes of light in sea water, infra-red radiation on water drops in clouds and in living tissues;  $k_0$  is the module of a wave vector of electromagnetic radiation in vacuum. In all numerical experiments  $\theta$  was equaled  $36.89^\circ$ , and absorption was determined so-called by "probability of a survival"  $\Lambda = \sigma_s / (\sigma_s + \sigma_a)$  [2], where  $\sigma_s$  and  $\sigma_a$  are extinction coefficients of dispersion and absorption respectively. The value of  $\sigma_s$  is integral from scattering indicatrix over all possible scattered directions:

$$\sigma_s = \oint \sigma(\alpha, \varphi) d\Omega = (1/2) \pi k_0^4 \oint \Phi(k) d\Omega, \quad (2)$$

where  $d\Omega$  is the infinitesimal solid angle of scattering. In the transfer theory inverse value of extinction coefficient of radiation corresponds to mean straightforward length of radiation between two acts of scattering on random inhomogeneities. The magnitude of  $\sigma_a = k_0 \varepsilon''$  is a reverse value of path length of radiation, over which its amplitude decreases in  $e$  (2.71 ...) time due to regular absorption in a slab. Comparison with the results obtained in [5] has been carried out for thickness of a slab  $\sigma_s Z = 20$  and absorption in the layer corresponding probability of survival  $\Lambda = 0.5$ . Statistical simulation has shown that even, despite of strong qualitative distinction of spectra of dielectric permittivity fluctuations, in this paper and in [5] the APS of scattered radiation approximately has the Gaussian form. The results of numerical simulation of dependence of position of the centre of gravity  $M[s_x]$  versus  $L_1$  for thickness of a layer  $\sigma_s Z = 40$  and different values  $L_2$  are presented in Fig. 2. The centre of gravity is considered as the statistical moment of the first order with respect to the argument  $s_x$  from a resulting APS  $I(s_x, s_y)$ :

$$M[s_x] = \frac{\int_{-1}^1 ds_x \int_{-1}^1 ds_y s_x I(s_x, s_y)}{\int_{-1}^1 ds_x \int_{-1}^1 ds_y I(s_x, s_y)}. \quad (3)$$

Curve 1 corresponds to the case  $\sigma_z L_2 = 80$ , curve 2 to the case  $L_2 = 0$  at  $\Lambda = 0.5$ . For comparison similar dependence without absorption (a curve 3  $\sigma_z L_2 = 80$ ,  $\Lambda = 1$ ) is shown in the same figure. From these diagrams follows that the deep regime of scattered waves is not realized yet in slab of such thickness. Strong scattering of radiation plays important role in formation of exiting power spectrum due to significant thickness of layer and power-law model of the spatial spectrum of dielectric permittivity fluctuations. The dotted line shows values of the centre of gravity  $M[s_x]$  for limiting case of an incident plane wave (i.e. at  $L_1 \rightarrow \infty$ ) on a layer having thickness  $\sigma_z Z = 40$  at angle  $36.89^\circ$  and probability of survival  $\Lambda = 0.5$ .

#### Conclusion

1. APS of scattered radiation at small angle approximation in the case of multiple scattering approximately has the Gaussian form for a slab having small thickness despite of essentially non-Gaussian character of the spectrum of dielectric permittivity fluctuations. 2. Numerical simulation has shown that strong distortion of spectrum of received radiation is possible at a certain location both source and the receiver concerning thick layer, even without absorption. 3. It is found out that the APS has non-Gaussian form and some maxima which correspond to some separate directions of wave propagation for the case of an absorptive layer with significant thickness. Depending on location of a source or the receiver with respect to the layer, propagation of radiation along these directions can be practically impossible and it determines number of maxima of the APS.

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