Analysis of Electrically-Small Metamaterials with Lumped Tuning Elements Through a Time Domain Coupled EM-Circuit Solver

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Introduction

A rigorous time domain integral equation (TDIE) solver that couples the electric field integral equation (EFIE) and SPICE-like circuit simulation is developed to obtain the macroscopic properties of metamaterial elements with lumped tuning circuit elements. In this paper, as an example, the conducting split ring resonator (SRR) is investigated. In particular, the SRR is scaled to be much smaller than a wavelength without changing the resonance frequency drastically, by adding selected tuning elements. The method itself is sufficiently general to model any conducting structure with any arbitrary lumped passive or active circuit.

The induced surface current from the electromagnetic (EM) surface-current unknowns is used to compute microscopic quantities of interest, such as the element electric and magnetic dipole moment. By using a generalized constitutive relation [1] based on the quasi-static Lorentz theory, the macroscopic physical property such as permeability is obtained.

The advantage of the proposed TDIE solver over existing frequency-domain surface integral methods and over time domain volumetric methods is its efficiency in terms of surface-only broadband analysis and the ability to include nonlinear and linear, passive and active loads.

Formulation

For a conducting body illuminated by an incident wave, the following equation holds,

$$\left[\mathbf{E}^{s}(\mathbf{J})+\mathbf{E}^{inc}\right]_{\tan}=0 \tag{1}$$

where $\mathbf{E}^{inc}$ the incident field and $\mathbf{E}^{s}(\mathbf{J})$ is the scattered field produced by induced current $\mathbf{J}$. From potential theory, the scattered field can be expressed as electric scalar potential and magnetic vector potential,

$$\mathbf{E}^{S}(\mathbf{J})=-\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t}-\nabla\varphi(\mathbf{r},t) \tag{2}$$

$$\mathbf{A}(\mathbf{r},t)=\mu\int_{s}\frac{\mathbf{J}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c)}{4\pi|\mathbf{r}-\mathbf{r}'|}ds' \tag{3}$$

$$\varphi(\mathbf{r},t)=\frac{1}{\varepsilon}\int_{s}\frac{\rho_s(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c)}{4\pi|\mathbf{r}-\mathbf{r}'|}ds' \tag{4}$$

For triangular patch meshes used to model the surface of conducting objects, the sub-surfaces corresponding to areas where lumped circuits are connected are denoted by S_{CK}^{m}, where m is used to indicate the circuit node number. The remainder of the surface is denoted by S_{EM}. The current flow from the mth node onto S_{CK}^{m} is denoted by I_c^m. Consequently, the continuity equation linking the current density $\mathbf{J}$ and surface charge density ρ_s has the following form,

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$$\nabla \cdot \mathbf{J}(\mathbf{r},t) + \frac{\partial \rho_s(\mathbf{r},t)}{\partial t} = 0 \qquad \forall \mathbf{r} \in S_{EM} \tag{5}$$

$$\nabla \cdot \mathbf{J}(\mathbf{r},t) + \frac{\partial \rho_s(\mathbf{r},t)}{\partial t} = \frac{I_c^m}{A_c^m} \qquad \forall \mathbf{r} \in S_{CK}^m \tag{6}$$

where A_c^m denotes the total area associated with S_{CK}^m. The conditions in Eqns. (5 and 6) can be combined with Eqns. (2-4) to generate an expression for the scattered field,

$$\mathbf{E}^s(\mathbf{r},t) = -\frac{\mu}{4\pi}\frac{\partial}{\partial t}\sum_{i=1}^{N_e}\int_{T_{i^+}\cup T_{i^-}} \frac{\mathbf{J}(\mathbf{r}',|\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}'-\mathbf{r}|}ds' + \nabla\frac{1}{4\pi\varepsilon}\sum_{i=1}^{N_e}\int_{T_{i^+}\cup T_{i^-}}\int_0^{t-|\mathbf{r}-\mathbf{r}'|/c}\frac{\nabla_s\cdot\mathbf{J}(\mathbf{r}',\tau)}{|\mathbf{r}-\mathbf{r}'|}d\tau ds'$$
$$-\nabla\frac{1}{4\pi\varepsilon}\sum_{m=1}^{M}\sum_{k=1}^{K^m}\int_{T_k}\int_0^{t-|\mathbf{r}-\mathbf{r}'|}\frac{I_{c,k}^m(\tau)}{A_{c,k}}d\tau ds' \tag{7}$$

The well-known Rao-Wilton-Glisson spatial basis functions are used to approximate the surface current density $\mathbf{J}$ at a given time step.

$$\mathbf{J}(\mathbf{r},t) \cong \sum_{i=1}^{N_e}\sum_{n=1}^{N_t} h_{i,n} S_i(\mathbf{r}) T_m(t) \tag{8}$$

$h_{i,n}$ are the unknown weighting coefficients associated with the space-time basis functions $S_i(\mathbf{r})T_m(t)$, N_e is the total number of spatial basis functions, and N_t is the number of time steps. The scalar potential on each terminal n is treated the same as the node voltage V_n due to the electrically small nature of the terminals.

$$\frac{1}{4\pi\varepsilon}\sum_{i=1}^{N_e}\int_{T_{i^+}\cup T_{i^-}}\int_0^{t-|\mathbf{r}-\mathbf{r}'|/c}\frac{-\nabla_s\cdot\mathbf{J}(\mathbf{r}',\tau)}{|\mathbf{r}-\mathbf{r}'|}d\tau ds' + \frac{1}{4\pi\varepsilon}\sum_{m=1}^{M}\sum_{k=1}^{K^m}\int_{T_k}\int_0^{t-|\mathbf{r}-\mathbf{r}'|/c}\frac{I_{c,k}^m(\tau)}{A_{c,k}}d\tau ds' = V_n \quad \forall \mathbf{r}\in S_{CK}^n \tag{9}$$

After basis expansion and Galerkin testing on (1), a time-domain circuit-EM coupled system is formed as following,

$$\begin{bmatrix} \overline{\mathbf{Z}_0^{JJ}} & \overline{\mathbf{Q}_0^{JI}} & \bar{\mathbf{0}} \\ \overline{\mathbf{Z}_0^{IJ}} & \overline{\mathbf{Q}_0^{II}} & \overline{\mathbf{C}} \\ \bar{\mathbf{0}} & \overline{\mathbf{C}^T} & \overline{\mathbf{MNA}_0} \end{bmatrix}\begin{bmatrix} \mathbf{J}(\mathbf{t}_j) \\ \mathbf{I}(\mathbf{t}_j) \\ \mathbf{ckt}(\mathbf{t}_j) \end{bmatrix} = \sum_{l=1}^{j}\begin{bmatrix} \overline{\mathbf{Z}_l^{JJ}} & \overline{\mathbf{Q}_l^{JI}} & \bar{\mathbf{0}} \\ \overline{\mathbf{Z}_l^{IJ}} & \overline{\mathbf{Q}_l^{II}} & \bar{\mathbf{0}} \\ \bar{\mathbf{0}} & \bar{\mathbf{0}} & \overline{\mathbf{MNA}_l} \end{bmatrix}\begin{bmatrix} \mathbf{J}(\mathbf{t}_{j-l}) \\ \mathbf{I}(\mathbf{t}_{j-l}) \\ \mathbf{ckt}(\mathbf{t}_{j-l}) \end{bmatrix} + \begin{bmatrix} \mathbf{src}_{EM}(\mathbf{t}_j) \\ \mathbf{0} \\ \mathbf{src}_{CK}(\mathbf{t}_j) \end{bmatrix} \tag{10}$$

$\mathbf{J}(t_j)$, $\mathbf{I}(t_j)$, $\mathbf{ckt}(t_j)$ are the EM current unknowns, branch current unknowns and circuit unknowns at time t_j, respectively; $\mathbf{src}_{EM}(t_j)$, $\mathbf{src}_{CK}(t_j)$ are the EM and circuit excitations, respectively. The matrix $\overline{\mathbf{C}}$ is a sparse bipolar adjacency matrix that is used for enforcing Kirchoff's Voltage and Current Laws at the terminals. Once the surface induced current is obtained, an FFT of the current distribution on surface yields the frequency domain current; the electric dipole moment $\mathbf{p}_e$ and magnetic dipole $\mathbf{p}_m$ can be calculated by the following formulae

$$\mathbf{p}_e = \frac{1}{j\varpi}\int \mathbf{J}(\mathbf{r},\varpi)ds \quad (11a)$$

$$\mathbf{p}_m = \frac{1}{2}\int \mathbf{r}\times\mathbf{J}(\mathbf{r},\varpi)ds \quad (11b)$$

And the general constitutive relation of bianisotropic media [1] is used,

$$\begin{bmatrix} \boldsymbol{\varepsilon}_p & \boldsymbol{\alpha} \\ \boldsymbol{\beta} & \boldsymbol{\mu}_p^{-1} \end{bmatrix} = \begin{bmatrix} \varepsilon_0\varepsilon_b\mathbf{U} & \mathbf{O} \\ \mathbf{O} & \frac{1}{\mu_0}\mathbf{U} \end{bmatrix} + N\begin{bmatrix} \mathbf{U} & \mathbf{O} \\ \mathbf{O} & -\mathbf{U} \end{bmatrix}\bar{\alpha}\left(\begin{bmatrix} \mathbf{U} & \mathbf{O} \\ \mathbf{O} & \mathbf{U} \end{bmatrix} - N\begin{bmatrix} \frac{1}{\varepsilon_0\varepsilon_b}\mathbf{C} & \mathbf{O} \\ \mathbf{O} & \mu_0\mathbf{C} \end{bmatrix}\bar{\alpha}\right)^{-1} \quad (12)$$

where $\bar{\boldsymbol{\alpha}}$ is a 6×6 polarizability matrix which is obtained directly from $\mathbf{p}_e$ and $\mathbf{p}_m$. **C** is the interaction matrix, which is related to the spacing in *x, y, z* directions. N is the concentration of the material. $\boldsymbol{\varepsilon}_p$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\mu}_p$ are defined through constitutive relation for bianisotropic media,

$$\begin{bmatrix} \mathbf{D} \\ \mathbf{H} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varepsilon}_p & \boldsymbol{\alpha} \\ \boldsymbol{\beta} & \boldsymbol{\mu}_p^{-1} \end{bmatrix}\begin{bmatrix} \mathbf{E} \\ \mathbf{B} \end{bmatrix} \quad (13)$$

The effective permittivity, permeability and chirality of the material can then be obtained.

Numerical results

3D periodic SRRs in an open and circuit-coupled configuration are simulated. The spacing between elements is less than 0.1 λ, which ensures the suitability of the quasi-static Lorenz condition. Figure 1 shows a validation example, for an SRR (inset) in the x-y plane, with parameters from [1], with r = 1.5 mm, w = 0.8 mm, d = 0.2 mm. For bulk permeability, the spacing between SRRs in the x,y,z axes is a = b = 8 mm, c = 3.7 mm. The resonance frequency is at about 4.8GHz and negative permeability is demonstrated, with agreement with the simulation result from [1] and the experimental result [2]. Figure 2 shows the combined tunability and scaling effect obtained by compressing the SRR and adding tuning elements (inset). The size of SRR is reduced to one tenth of the original size (r = 0.15 mm, w = 0.08 mm, d = 0.2 mm) and the spacing is scaled by the same factor. The simultaneous scaling of size and spacing ensures that the Lorentz condition continues to be satisfied. The capacitor is added in the gap of the outer ring of SRR, and two curves in Fig. 2 correspond to a 1pF and 2pF capacitor respectively. The following desirable features are seen: the resonance shift is tunable via the capacitor value, negative permeability is produced around the resonance region, and the resonance frequency region is in the same range as before scaling (not scaled by ten), which indicates the possibility of electrically small tunable resonant antenna arrays.

Conclusion

In this paper, a time domain integral equation based EM-circuit coupled solver is used to simulate metamaterials with lumped circuit elements, and physical properties are then extracted through a generalized constitutive relation. The simulation results point to the possible flexibility of tuning and scaling of meta-material elements with a consequent effect on the required size of radiating structures and antennas. The proposed method is sufficiently general to model arbitrary active and passive loads.

References

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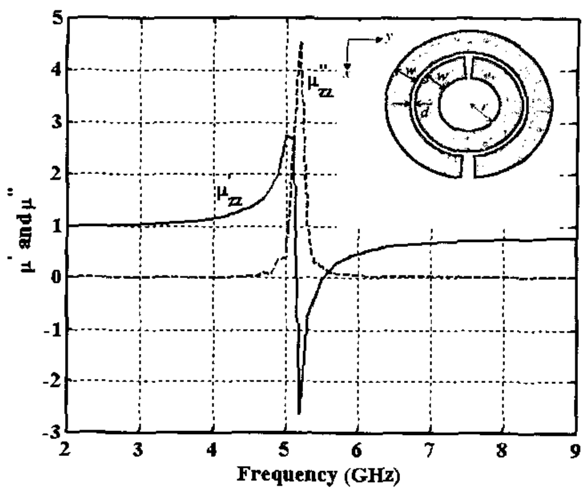

Fig.1 Negative permeability of SRR (inset) layout in 3D

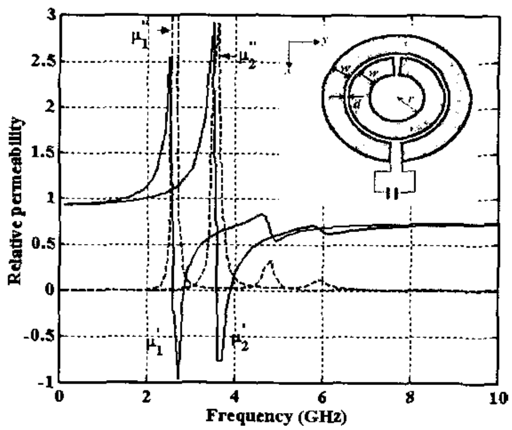

Fig.2 Tunable scaled SRR (1/10th size) with capacitor; C =2pF (left curve) and C = 1pF (right curve)