Propagation and Scattering of Low Grazing Skimming Waves Over Conducting Rough Surfaces

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ABSTRACT

If both the transmitter and the observation points are located close to the rough conducting surface, the wave incident upon a point on the surface is a mixture of the coherent and incoherent waves and is no longer the incident plane or spherical wave in free space. If the surface is flat, this is the Sommerfeld problem which has been studied extensively. This paper considers the Sommerfeld problem for rough surfaces. First, we consider the coherent field over the one-dimensional rough surface which satisfies the Dyson equation. Using the flat surface Greens' function, the coherent field is expressed in a spatial Fourier transform which is equivalent to the Sommerfeld integral. From the complex reflection coefficient in the Fourier domain, we obtain the Sommerfeld pole and the final expressions are given for the attenuation function. Numerical examples are given for rough ocean and land surfaces showing the additional attenuation due to the scattering. The results are then compared with Monte-Carlo simulations showing good agreement. Next, the incoherent field is formulated based on the Bethe-Salpeter equation. The first-order solution indicated that the coherent wave propagates to a point on the surface where the incoherent wave is excited and is propagated to the observation point. The total incoherent field is a sum of contributions from all scattering points on the surface.

INTRODUCTION

There have been extensive studies made on the rough surface scattering problem. Most studies deal with plane wave incidence and the scattering characteristics are expressed in terms of the cross sections per unit area of the rough surface [1] – [3]. While this is appropriate for moderate angles of incidence (less then 75^{o}), the assumption of plane wave incidence is no longer appropriate when the transmitter and target are near the ocean surface or for LGA scattering. For larger angles of incidence, and scattering near the surface, careful examination of the plane wave assumption is required. For LGA scattering, it has already been pointed out by Barrick [4], [5] that "propagation and scatter become inextricably connected" and "the free-space plane wave description may not suffice". The wave incident at a point on a rough surface is not the direct plane or spherical wave from the transmitter. The incident wave is modified by the rough surface itself. The incident wave at a point on the rough surface is a sum of the free-space plane wave from the transmitter and the scattered wave from the surface. In this paper, we consider the radiation from a point source located at any point near the rough surface, and thus the field on the surface is the total field.

In recent years, several numerical Monte-Carlo techniques have been developed to obtain numerical solutions to the rough surface scattering problem [5]. While this is an excellent approach to the study of rough surface scattering, when the grazing angle becomes small, extremely large surface areas are required to properly take into account the large footprint area. Thus fast high performance computers are required for solutions. The rough surface Green's function is analytical, and the computer requirement is reduced. Fast analysis of the rough surface effects is possible. Therefore, it is important to consider problems in which the rough surface correction of scattering from near-surface objects must be included.

We present an analytical theory of rough surface Green's function for the one-dimensional rough surface. This provides a mathematically simple formulation including the effects of rough surfaces, but it does not include cross-polarization effects. We begin with Green's theorem, and using an equivalent boundary condition, we obtain Dyson's equation for the coherent field which is obtained by using a spatial Fourier transform. If the surface is Dirichlet, the equivalent impedance is zero for the flat surface. However, the impedance is not zero due to the presence of roughness. Also, corresponding to this impedance, there are surface wave poles which give rise to surface wave propagation along the surface. The coherent field is shown to be equivalent to Watson-Keller's results. Next we examine the Bethe-Salpeter equation and obtain the first-order iteration solution once again making use of the spatial Fourier transform. The cross section per unit length is calculated and is shown to be similar to Watson-Keller, but more

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importantly it is reciprocal. Discussions are also included on power conservation and the specific intensity. This paper discusses the first-order modified perturbation theory of the rough surface Green's function and the far-field approximations. We will discuss the surface wave contributions applicable to the low grazing angle case and the second-order modified perturbation techniques which extend the range of validity of this theory.

We next consider the electric and magnetic line source located above a finitely conducting rough surface. We spend some time here to analyze the TE and TM wave propagation over the surface. The Green's function for the rough surface is determined from

$$(\nabla^2 + k^2)G(r,r_o) = -\delta(r - r_o)$$

And satisfies the impedance boundary condition at the surface.

$$G + \beta_o \frac{\partial}{\partial n} G = 0 \qquad \text{for TE.}$$

$$\frac{\partial}{\partial n} G + \alpha_o G = 0 \qquad \text{for TM.}$$

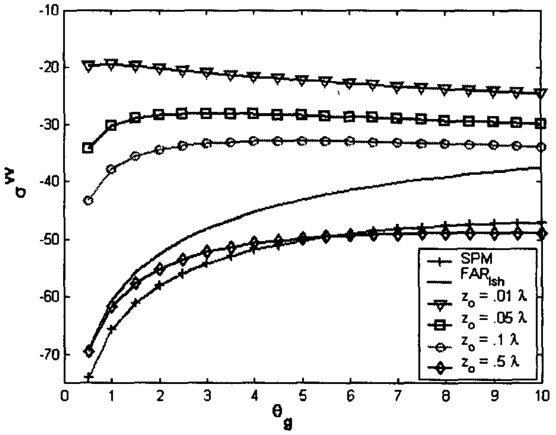
where $\alpha_o = ik_o Z_s / Z_o, \text{and } \beta_o = -iZ_s /(k_o Z_o)$. If the line source is an electric current, the TE electric field is given by

$$E_y(x,z) = iw\mu_o I_e G(r,r_o)$$

If the source is a magnetic line source, then the TM magnetic field is given by

$$H_y(x,z) = iw\varepsilon_o I_m G(r,r_o)$$

The coherent TM wave propagation over the impedance rough surface corresponds to the classic Sommerfeld problem. The modification of the Sommerfeld pole and Zenneck wave due to surface roughness can be obtained. Numerical examples are obtained for the Sommerfeld pole, the equivalent surface impedance and the propagation constant. We note that the Sommerfeld attenuation function shows increased attenuation due to surface roughness. The Zenneck wave propagation constant have been obtained and numerical examples are given for land and sea rough surfaces and compared to Monte Carlo simulations. We have also obtained the scattering cross-sections per unit length of the rough surface for Dirichlet, Neumann, TE and TM impedance surfaces showing the effects of roughness and surface conductivity. Examples include HH and VV cross-sections of finitely conducting rough surfaces. For moderate angles of incidence, the equivalent cross-sections are determined from the far-field approximation of the Bethe-Salpeter's equation. For low grazing angles, the TM cross-section is modified and the attenuation function for propagation along the surface is used. Application of the attenuation function results in a positioning of a source and receiver within the low grazing angle cross-section. The following figure is for the TM cross section, showing near grazing angle scattering with sources near the surface, and sources away from the source in the far field.



CONCLUSIONS

This paper presents an analytical theory of the coherent and the incoherent rough surface Green's function for one-dimensional smooth rough surface. The theory is applicable to surfaces with small rms height $k\sigma \leq 1.0$, but the range of validity is much greater than that of the conventional perturbation method. The coherent Green's function was determined from Dyson's equation, and its spatial Fourier transform representation is given. A saddle-point technique was used to evaluate this expression and is given. The mutual incoherent function was calculated based on the Bethe-Salpeter equation, and the general solution based on a spatial Fourier transform is given. This is also evaluated using a far-field asymptotic approximation and a surface wave approximation. The mutual coherence function was then used to calculate the specific intensity. Therefore, the theory should be useful for RCS signature related problems and for LGA scattering when both the transmitter and observation point are close to the surface.

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