Generation Scheduling, Pricing Mechanisms and Bidding Strategies in Competitive Electricity Markets

A thesis submitted by

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to

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to my beloved mama & papa

Dalva & Edimar

to my dearest wife

Rogéria

to my adorable son & daughter

Marcus Vinícius & Julia

Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification in this or any other university, or other institution of learning.

About the Author

Dilcemar de Paiva Mendes was born in Rio de Janeiro, Brazil in 1965. He received his B.Sc. and M.Sc. Degrees in electrical engineering from *Universidade Federal do Rio de Janeiro* - UFRJ in Brazil in 1988 and 1992 respectively. From 1989 to 1992 he worked at *Centro de Pesquisas de Energia Elétrica* - CEPEL, in Brazil, where his main activities were related to the reliability evaluation of power systems. In 1993, he became a lecturer of *Departamento de Engenharia Elétrica* - DEE in *Universidade Federal do Ceará* - UFC, also in Brazil. His current research interests lie in the area of power system operation and economics. His *E-mail* address is dilcemar@dee.ufc.br.

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Abstract

This thesis addresses some fundamental issues regarding the formulation and solution of the generation scheduling problem, the implementation of mechanisms for pricing the electricity and the assessment of generators' bidding strategies in competitive electricity markets. The core of the investigation performed in this research project is related to the assessment of the equity and efficiency of such electricity markets.

The investigation of those issues was carried out through the implementation of mathematical models and the development of computational algorithms designed to simulate the operation of an electricity market. Amongst these algorithms there are (*i*) a hybrid Lagrangian relaxation-Dynamic Programming (LR-DP) algorithm for scheduling thermal generating units; (*ii*) alternative schemes to allocate the generators' fixed costs in the pricing mechanisms; (*iii*) a re-dispatching algorithm that aims to reduce the electricity prices and hence the customers' payments in an electricity model of which the generation scheduling and price computation are two stand-alone modules; (*iv*) a generation scheduling algorithm in which the objective is to minimise the customers' payments rather than the generators' production costs; and (*v*) an alternative economic dispatch (ED) that attempts to minimise the customers' payments and that is merged with the alternative generation scheduling model.

The issues discussed in this thesis are illustrated using 4-, 10-, 26- and 110-generating unit systems.

Changes in the market rules have been investigated and the results have shown that they can affect the efficiency and equity of the market and the profitability of the market participants in different ways. For example, the hybrid LR-DP algorithm can produce a high quality solution for the generation scheduling problem. Negligible variations in the UC schedule can introduce considerable changes in the total customers' payments. In addition, the customers' payments can be further reduced by alternative fixed cost allocation schemes in the pricing mechanisms. An alternative ED can also contribute to lowering the customers payments, however at the expenses of the equity of the market as a whole. Formulating a generation scheduling which optimises the customers' payments rather than the generators' production costs is feasible. A sub-optimal algorithm to solve this problem reduces the total payments by consumers while ensuring cost recovery to generators. Great care should be taken in formulating the problem to ensure the efficiency and equity of the electricity market. Furthermore, the assessment of generators' bidding strategies is a very complex task due to the large number of parameters in the generators bidding files and due to the complexities of the market rules. During simple "games", generators had no incentive to bid beyond their true costs in an equitable and efficient market.

Publications

The algorithms and models developed throughout the research project and the conceptual ideas reported in this thesis were useful in the preparation of the following conferences and journals articles:

i. D. P. Mendes and D. S. Kirschen, "Modelling of a Competitive Electricity Power Pool," *in Proceedings of the 32nd Universities Power Engineering Conference - UPEC '97*, Manchester - England, September 10-12, 1997.

ii. D. P. Mendes and D. S. Kirschen, "Cost and Price Optimisation in a Competitive Electricity Pool," *in Proceedings of the VI Symposium of Specialists in Electric Operational and Expansion Planning - VI SEPOPE*, Salvador-Brazil, May 23-28, 1998.

iii. H. B. Gooi, D. P. Mendes, K. R. W. Bell, and D. S. Kirschen, "Optimal Scheduling of Spinning Reserve," *IEEE Transactions on Power Systems*, N° PE-302-PWRS-0-06-1998, 1998.

iv. D. S. Kirschen, G. Strbac, P. Cumperayot, and D. P. Mendes, "Factoring the Elasticity of Demand in Electricity Prices," *IEEE Transactions on Power Systems*, paper N° TR8 109, 1999.

v. D. P. Mendes and D. S. Kirschen, "Generation Scheduling and Pricing Mechanism in Competitive Electricity Markets," to be published *in Proceedings of the VII Symposium of Specialists in Electric Operational and Expansion Planning - VII SEPOPE*, Curitiba-PA, Brazil, May 21-26, 2000.

vi. D. P. Mendes and D. S. Kirschen, "Assessing Pool-Based Pricing Mechanisms in Competitive Electricity Markets," submitted to the *2000 IEEE PES Summer Meeting*, Seattle, Washington, USA, July 16-20, 2000.

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List of Symbols

The main symbols and acronyms used in this thesis are as follows:

Symbol	***Description***	***Unit***	***Page of first appearance***
inc_i^1	incremental cost of the first segment of the piece-wise linear cost function of unit *i*	\$/MWh	35
inc_i^2	incremental cost of the second segment of the piece-wise linear cost function of unit *i*	\$/MWh	35
inc_i^3	incremental cost of the third segment of the piece-wise linear cost function of unit *i*	\$/MWh	35
nl_i^1	no-load cost 1 of the piece-wise linear cost function of unit *i*	\$/h	35
nl_i^2	no-load cost 2 of the piece-wise linear cost function of unit *i*	\$/h	35
nl_i^3	no-load cost 3 of the piece-wise linear cost function of unit *i*	\$/h	35
e_i^1	first elbow point of the piece-wise linear cost function of unit *i*	MW	35
e_i^2	second elbow point of the piece-wise linear cost function of unit *i*	MW	35
$P_i^{\min}$	lower generation limit of unit *i*	MW	36
$P_i^{\max}$	upper generation limit of unit *i*	MW	36
t_{on}	period at which a unit is started up	h	38
t_{off}	last period before the one at which a unit is shut down	h	38
S_i^t	start-up price of unit *i* at period *t*	\$	38

Symbol	***Description***		***Page of first***
pl	length of the scheduling period. (Half an hour in the EPEW)	h	38
inc_i^t	incremental cost of unit *i* at period *t*	\$/MWh	38
nl_i^t	no-load cost of unit *i* at period *t*	\$/MWh	38
gp_i^t	price of unit *i* at period *t*	\$/MWh	38
Ω_A	set of Table A periods from continuous running periods	_	38
Ω_B	set of Table B periods from continuous running periods	_	38
smp^t	system marginal price at period *t*	\$/MWh	39
ppp^t	pool purchase price at period *t*	\$/MWh	39
$lolp$	loss of load probability		39
$voll$	value of loss of load	\$/MWh	39
psp^t	pool selling price at period *t*	\$/MWh	40
W_i^t	revised unconstrained generation of unit *i* at period *t*	\$/MWh	41
EP_i	energy payments of unit *i* for the entire time horizon	\$	41
P_i^t	output power of unit *i* at period *t*	MW	49
$F_i(P_i^t)$	fuel cost of unit *i* when its output power is P_i^t	\$	49
a_i	combined crew start-up costs and the equipment maintenance costs of unit *i*	\$	49
b_i	cold start-up fuel cost of unit *i*	\$	49
t_i	cooling rate of unit *i*	h	49
N	total number of generating units	_	49
T	total number of scheduling periods	_	49

Symbol	***Description***	***Unit***	***Page of first appearance***
$\overline{P} = \left[P_i^t \right]$	(N ´ T) power matrix, in which the element in row *i* column *t* represents the output power of unit *i* at period *t*	MW	49
$\overline{U} = \left[u_i^t \right]$	(N ´ T) commitment matrix, in which the element in row *i* column *t* represents the commitment state of unit *i* at period *t*	_	49
$\mathrm{F}(\overline{P}, \overline{U})$	total operation cost of the system	$	49
$X_{off,i}^{t}$	continuous off-line time of unit *i* at period *t*	h	50
$a_i,\ b_i,\ c_i$	parameters of polynomial cost function of unit *i*	$/MWh^2; $/MWh; $	50
D^t	system load at period *t*	MW	51
R^t	system spinning reserve requirement at period *t*	MW	51
T_i^{up}	minimum up-time of unit *i*	h	53
T_i^{down}	minimum down-time of unit *i*	h	53
$X_{on,i}^{t}$	continuous running time of unit *i* at period *t*	h	53
l^t	Lagrangian multiplier for the power balance constraint at period *t*	$/MWh	65
m^t	Lagrangian multiplier for the system spinning reserve constraint at period *t*	$/MWh	65
$g(P_i^t, u_i^t)$	Generators' production cost at period *t*	$	69
f	constant parameter to initialise the Lagrangian multiplier for the spinning reserve constraints	_	72
s	relaxation coefficient to adjust the Lagrangian multipliers for the demand constraint	_	73
d	scaling factor to adjust the Lagrangian multiplier for the demand constraint	_	73

Symbol	***Description***	***Unit***	***Page of first appearance***
q	tuning constant to adjust the Lagrangian multiplier for the demand constraint	_	73
K	iteration counter the LR-based UC algorithm	_	73
MC_k^t	marginal cost of period t iteration k, i.e., the incremental cost of last unit loaded by the economic dispatch algorithm	$/MWh	73
e_k^t	slack term to adjust the Lagrange multipliers for the reserve constraint	MW	74
d'	scaling factor to adjust the Lagrangian multiplier for the spinning reserve constraint	_	74
q'	tuning constant to adjust the Lagrangian multiplier for the spinning reserve constraint	_	74
ε	constant equal to the maximum capacity of the smallest unit of the system	MW	75
N_c^t	number of candidate units at period t for the DP-based post-processor	_	84
GC	generators' production cost over the entire time horizon	$	85
u_i^t	commitment state of unit i at period t (u_i^t =1: unit is on-line and u_i^t = 0 unit is off-line)	_	91
PC	pool cost (total customers' payments) over the entire time horizon	$	91
$PC_1^{t_a}$	accumulated pool cost (customers' payments) from period *1* to period t_a	$	126
Sd_i	shut-down price of unit i	$	193
q	dual solution of the LR-based UC algorithm	$	65
$\boldsymbol{J}$	primal solution of the LR-based UC algorithm	$	63

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CHAPTER 1

Introduction

1.1 Objective and Motivation

The main objective of the introduction of competition in the electricity sector is to increase the efficiency of electricity production and distribution, offering lower prices, higher quality and a more secure and reliable product. The ideas of a genuinely competitive electricity market were first introduced by Schweppe et al. [1], whose theoretical development established the foundation for the liberalisation of the electricity trade and launched the concepts of spot prices of electricity, based upon a model of price driven economic competition. One of the essential components required for the establishment of a competitive electricity market is the existence of a fair and open mechanism for setting the price of electricity. Moreover, an electricity market should be efficient and equitable.

By efficiency, this author means that the market should operate either at its optimal operating point or very close to it. A market is considered equitable if changes in generators' bids result in changes in market share, spot prices and customers' payments that are in accordance with the theoretical economic concepts (e.g., a generating unit should not increase its market share by increasing its bidding price).

The precise framework of an electricity market is determined by a number of key design features that depend on various factors such as the structure of the power system before the implementation of the new market environment, the utilities financial situation, the power system investment needs, and even the government's political structure. The electricity markets in different countries have applied different solutions to a broadly common set of issues.

The general structure of a competitive electricity market is such that electricity can be traded through a centralised market, bilateral contracts or a combination of both. In a pool-based market, participants make offers for the amount of electricity they are willing to trade. These prices are ranked and taken in an increasing order until the balance of supply and demand is achieved. In the remainder of this thesis this problem will be called generation scheduling. The last price taken sets the market-clearing price. The details of the method used to set this price will be called the pricing mechanisms. Consumers pay this market-clearing price whereas generators receive this price for every unit of energy traded. Bilateral contracts can comprise terms and conditions for physical delivery or can be pure financial agreements to hedge participants against oscillations of the spot prices of electricity.

In traditional vertically integrated power systems, which are being replaced by new wholesale electricity markets around the world, each utility optimises its generation schedule pretty much independently of what the other utilities do. There may be interchanges between utilities but these are relatively straightforward to model in unit commitment, optimal power flow and other programs. On the other hand, in a pool-based system, generators bid into the pool in such a way as to maximise their total profit. However, the results are affected by the ways in which the other generators bid into the pool. The identification of the generators' bidding strategies is an interesting issue that will be addressed in this thesis.

The implementation of electricity markets around the world has raised some practical problems that have made it difficult to achieve the ideal of a free market with perfect competition. While the principles at the basis of an electricity market are relatively simple, its implementation is very complex and there have been numerous complaints about the lack of transparency and the ability of generating companies to manipulate prices in some markets such as the Electricity Pool of England and Wales (EPEW)[2-4].

The objective of this research project is to investigate some fundamental issues related to generation scheduling, pricing mechanisms and bidding strategies in competitive electricity markets. To perform this investigation, a mathematical model of a competitive electricity market is developed because it is difficult, if not impossible, to predict the effects of some of the market rules through analytical speculation. This model can be used to study alternative methods for selecting generating units to supply the demand, new mechanisms for electricity pricing and strategies for bidding generation. This model can also be used to assess the effect of an increase in the number of market participants, and the effects of changes in the rules from the perspective of the market participants or from the market as a whole. Based on these results, recommendations can be made for the implementation of similar mechanisms in other parts of the world. The rules of the EPEW, operating since March 1990, are the basis of this research project.

1.2 Aims of the Research

In some electricity markets, such as the EPEW, the generators' bids are composed of the various elements of the cost of running a generating unit, including incremental costs and fixed costs (start-up, shut down and no-load costs). In such a complex bidding structure, the problem of scheduling the generating units is normally formulated and solved by a centralised independent system operator (ISO), as a unit commitment (UC) problem, in which the objective is to minimise the generators' production costs. The UC problem is a large-scale mixed-integer non-linear optimisation problem [5] whose optimum solution may not always be found. Indeed, for large-scale power systems, the current techniques can only achieve sub-optimum solutions. The quality of this sub-optimum solution is crucial to the efficiency and equity of a competitive electricity market.

Research questions: are the solutions of the UC problem generated by the available optimisation techniques of satisfactory quality for the implementation of a competitive electricity market? How can the quality of the UC schedule be improved and how does this solution affect the electricity markets? Is the traditional UC problem suitable for simulating the operation of a competitive electricity market?

In this research project, a hybrid algorithm that combines the Lagrangian relaxation and the Dynamic Programming technique has been developed for scheduling generating units in a competitive electricity market. A high quality solution of the scheduling problem is achieved in two steps. Firstly, an LR-based UC program determines a preliminary schedule. The behaviour of the units during the iterative search of the LR-based solution and the preliminary schedule itself are assessed to define a variable window size for each interval. The Dynamic Programming (DP) technique is then used as a post-processor to improve the preliminary solution.

In electricity markets with a complex bidding structure the allocation of fixed costs has a significant effect on the electricity prices. In the pricing mechanism of the EPEW the fixed costs are allocated according to the Table A/B scheme, which attempts to produce lower prices of electricity during hours of low demand and higher prices during peak hours, therefore encouraging voluntary demand side management. Nevertheless, due to the nature of the piece-wise linear price functions (*Willans line*), prices during peak times may end up lower than prices during low demand. One of the basic rules of an electricity market defines the methods to calculate the electricity prices. The allocation of fixed costs plays an important role in the pricing mechanisms.

Research questions: What are the consequences of changes in the fixed cost allocation scheme to the prices of electricity? Would one scheme benefit generators and another benefit customers?

Alternative schemes to allocate the generators' fixed costs in the price mechanism have been implemented in this project and their application is discussed in this thesis.

The idea of allocating fixed cost using different approaches can be further explored as an attempt to reduce electricity prices. From the price formulation of the four cost allocation schemes, it can be seen that a reduction in prices can sometimes be achieved by amortising the fixed costs over a larger amount of energy. Hence, by increasing the production power of generating units, their unit prices can sometimes be reduced. The economic dispatch (ED) problem is part of the generation scheduling problem. The standard ED is formulated as an optimisation problem of which the objective function is the total generators' production costs.

Research questions: Is the standard ED problem in the interest of the customers? Is it possible to formulate and solve an alternative ED to minimise the total customers' payments? Would this alternative ED algorithm be equitable in the context of an electricity market?

A re-dispatching algorithm that aims to reduce the prices and hence the customers' payments in an electricity model, of which the generation scheduling and price computation are two stand-alone modules has been developed in this project and is presented in this thesis. The loading of the units is performed in a step-wise procedure, following the four "capital" points of the *Willans line*. The calculation of the unit prices depends on the choice of the cost allocation scheme.

It has been argued that centralised scheduling of multi-owned resources, which has to rely upon imperfect information may face difficulties that do not arise when resources are centrally owned [6]. The structure of the UC problem may produce several near optimum solutions, which are of equal quality in terms of total production costs, but may vary significantly in terms of individual costs, profits and commitments. These effects are inherent when attempting to optimise UC from the perspective of a central operator, because of the near-degeneracy of the UC problem and of the presence of many near-optimal solutions. Moreover, some examples that point out the failures of administrative mechanisms to mimic the operation of a power market have also been presented by Jacobs [7]. He argues that the "least-cost" dispatch of a central authority combined with a payment rule based on a market-clearing price does not necessarily minimise the cost to consumers of electricity. This happens because the scoring rule, which is used to select resources based upon a production cost minimisation, differs from the payment rule. He suggests that the UC problem should incorporate in its objective the consumer cost instead of the production cost. Furthermore, Hao et al. presented a new methodology for calculating optimal generating schedules that minimises energy payments by power pool consumers instead of minimising generators' production costs [8]. Payment adequacy constraints are introduced in the problem formulation to ensure that all units winning the auction recover their fixed costs as well their incremental costs. The total fixed costs for each selected unit appears in the payment adequacy constraint and is computed in the solution process. The portions of start-up and no-load costs allocated to each hour of the scheduling period are decision variables in the optimisation process. The authors claim that the start-up and no-load costs are optimally allocated throughout the scheduling period, and hence the presented methodology can produce the lowest possible consumer payment under a uniform pricing rule and payment adequacy requirements. Unfortunately, their paper does not provide enough information for the reader to reproduce their results or to study the effectiveness of their methodology when applied to real-size systems.

Research questions: What are the effects of incorporating the customers' interests in the scheduling problem? Is it possible to formulate such a scheduling problem and implement an efficient algorithm to solve it? Would this problem ensure the equity of the electricity market?

This thesis presents an alternative generation scheduling algorithm in which the objective is to minimise the total customers' payments. The generation scheduling and pricing algorithms, normally two stand-alone modules, are then merged in a unique algorithm. The search for the optimum solution of the UC problem is done by a forward DP algorithm.

Several authors have worked on the problem of assessing the generators' strategic bidding behaviour in competitive electricity markets [9-19]. The complexity of the problem however has stimulated the implementation of a series of simplifications to make it manageable. Some of those works have considered a market with simple quantity-price bids. The generators' operational characteristics, such as minimum up and down times, start-up prices and minimum stable generation are sometimes neglected.

Research questions: Is it possible to predict the generators' bidding strategies in an electricity market of great complexity such as the EPEW? Can a model capable of simulating different bidding strategies be developed? Is it possible to assess their effects on market participants? Can the scope for "gaming" through the complex rules of the market be identified?

The mathematical model of a competitive electricity market developed in this research project has been used to investigate the generators' bidding aspects. Initially, each generator is modelled as an independent agent trying to maximise its profit. In preparing its bid, each agent assumes that the other agents may be using the same bidding strategy as in the previous iteration. Once each agent has prepared its bid, they are passed on to the scheduling program which establishes the global schedule. Each generator then analyses this schedule and tries to improve its bidding prices for the next round. It is assumed that no market participant exercises market power. The results are presented and discussed.

The issues presented and discussed in this thesis are illustrated using 4-, 10-, 26- and 110-generating unit systems.

1.3 Outline of the Thesis

The first chapter of this thesis intends to present the main objectives of this research project in the context of the introduction of competition in power systems.

In Chapter 2, some key design features of electricity markets are discussed and illustrated with examples from electricity markets from around the world. Special attention is given to the EPEW, which constitutes the basis of this research. The aim of this chapter is to show how the market operator schedules the generating units based on their offer files and how the payments to generating units are calculated. This information is needed to understand the optimisation method proposed in Chapter 5.

Chapter 3 focuses on the problem of scheduling thermal generating units in electricity markets with a complex bidding structure. It reviews the basic features of some optimisation techniques currently available to solve the UC problem, and presents a hybrid algorithm that combines the Lagrangian Relaxation and the Dynamic Programming technique.

Chapter 4 discusses the issues regarding the pricing mechanism in electricity markets. It presents different ways of allocating the generating units' fixed costs in the computation of the electricity prices. A re-dispatching algorithm that can reduce the prices of scheduled units and consequently the market-clearing prices is also presented in this chapter.

Chapter 5 presents an alternative generation scheduling algorithm in which the objective is to minimise the total customers' payments, rather than the total generators' production costs as in the traditional UC problem. A forward Dynamic Programming algorithm is used to search for the optimal UC schedule. The alternative fixed cost allocation schemes presented in Chapter 4 are used in this algorithm. The proposed methodology attempts to ensure the equity and efficiency of a pool-based competitive electricity market.

The knowledge gained in the previous chapters on generation scheduling and auction pricing mechanisms is used to further discuss the generators' bidding strategies in a competitive electricity market in Chapter 6. The simulation of simple "games", in which some bidding parameters were kept constant and others were multiplied by an adjustable factor, is presented in this chapter.

Chapter 7 contains the conclusions of this work and proposes some topics for further investigation.

Appendix A presents the technique used to convert the original polynomial price functions into piece-wise linear functions, Appendix B shows the complete data for the case studies, Appendix C describes the *Wollenberg's Paradox*, and Appendix D shows additional results regarding the 110-unit system.

CHAPTER 2

Modelling of Competitive Electricity Markets

2.1 Introduction

A competitive electricity market can rely on a centralised market, on bilateral contracts or a combination of both. In a pool-based market, participants offer prices for the amount of electricity they want to buy or sell. These prices are ranked and taken in an increasing order until the balance of supply and demand is achieved. The last price taken sets the market-clearing price. Consumers pay this market-clearing price whereas generators receive this price for every MWh traded. Bilateral contracts can comprise terms and conditions for physical delivery or can be pure financial agreements to hedge participants against oscillations of the spot prices of electricity.

The electricity markets in different countries have applied different solutions to a broadly common set of issues and problems related to some key design features. In this chapter, some of those features will be discussed and illustrated by using examples of electricity markets from around the world. Special attention will be given to the Electricity Pool of England and Wales (EPEW), which constitutes the basis of this research.

2.2 Key Design Features of a Competitive Electricity Market

The precise framework of an electricity market is determined by a number of key design features that depend on several factors such as the structure of the power system before the implementation of the new market environment, the utilities financial situation, the power system investment needs, and even the government political structure.

2.2.1 Market and System Operation

The implementation of an electricity trading arrangement requires a clear and precise definition of the roles of two key entities: the system operator (SO) and market operator (MO). In general, a system operator is responsible for ensuring the technical security of the electrical network by co-ordinating the actions of the market participants. The primary role of a market operator is to determine a market-clearing price for each trading period by providing a forum for market participants to submit their bids and offers. In some electricity markets, the roles of system and market operators are carried out by the same organisation whereas in others they are undertaken independently. The system operator is the owner of the transmission network assets in some markets, while the control and ownership of these assets are unbundled in some others.

2.2.2 Trading Options

In some electricity markets (e.g., the Pool in England and Wales) trading through an independent entity that manages the bids and sets the market-clearing prices is compulsory. Market participants are eligible to enter into pure financial bilateral contracts to insulate themselves from the volatility of the spot price. In others, market participants are entitled to enter into bilateral contracts for physical delivery of all or part of their selling and purchasing requirements. The key issues in trading outside the Pool have been to ensure that bilateral arrangements do not prevent maintaining system security and that the system costs be appropriately shared amongst the market participants whether or not trading through the Pool.

Contracts are trading agreements in which one trader (seller) agrees to deliver a product to another trader (buyer), on certain conditions and in return for a certain payment. Electricity can be traded in different ways using standard forms of contracts. Contracts can be used to allow market participants to deal with one another and to overcome particular problems, such as managing transaction costs in a decentralised system and providing incentives for efficient behaviour. They are also useful to handle uncertainties as they provide mechanisms for transferring financial risks from one party to another, whenever there is a benefit for doing so. The main features used to classify a contract are the time and conditions of delivery, and the method of settlement [1, 20].

Spot transactions are sales of electricity for immediate delivery but often do not involve the establishment of a formal contract. Their key characteristics are immediate and unconditional delivery, and like any transaction, their financial terms include not only the price per unit of the asset but also the method of settlement. Forward and futures contracts are agreements for the delivery of electricity at a certain price in a defined location at a specified time in the future. The settlement is made only at the time of delivery, when the energy is delivered, and the difference between the contract price (strike price) and the market value of the electricity at the time of delivery (spot price) represents the profit or loss of the contract holder. Forward and futures contracts are hedging instruments to hedge market participants against oscillations in the spot prices of electricity. Futures may not necessarily result in physical delivery of the asset.

Regarding the conditions of delivery, options contracts allow traders to decide whether or not the asset should be delivered. A *call* option gives the holder the *right to buy* whereas a *put* option provides the holder the *right to sell* the asset at the strike price, at some time in the future. Combinations of call and put options can be used by traders to limit any risk. The combination of a call and put option at the same strike price is called a two-way option, and is equivalent to a fixed price contract.

In a conventional contract, the seller must deliver the electricity at a specified time and place. However, it is not always convenient to the traders to arrange for physical delivery according to the terms of a standard contract. Hence, contracts can be arranged in such a way as to allow the parties to settle the agreement through a financial transaction, unbundled from the actual delivery of energy. A Contract for Difference (CfD), named as such because the settlement is based upon transferring the differences between two prices, is a convenient way to do that. A CfD is normally structured as a call option with specified strike price and volume of energy. It is agreed that the buyer owes the seller an amount equal to the strike price whereas the seller owes the buyer an amount equal to the spot price. The buyer normally exercises the option when the spot price of electricity is higher than the strike price, and thus pays the strike price for the amount of energy in the contract. However, the seller is entitled to hand over the cash value of the energy contracted, as defined by the spot market price at the time of delivery, rather than physically delivering the electricity. These opposing forces should ensure that, in the long run, strike prices tend towards spot prices.

2.2.3 Market Timing

The prices for a wholesale electricity market may be set in advance (*ex-ante*), after (*ex-post*) or at the same time as the actual delivery of electricity. Ex-ante prices are typically determined on a day-ahead basis. This allows the market participants to respond to the prices and to adjust their bids and offers, based on commercial decisions. A market with ex-ante prices requires a balancing mechanism to address any deviations between actual and projected schedules. These differences may arise due to those price-responsive actions of the market participants and to contingencies such as generators and transmission outage failures. In ex-post markets, the system or market operator can accept increasing and decreasing bids and offers from market participants willing to change their scheduled profile at short notice. The prices can then be determined on the basis of the balance of the demand requirements and resources actually used on the delivering day. Hence, ex-post market prices reflect the actual operating state of the power system in each trading period. Real time prices could lead to a true interaction between the market participants but their practical implementation in electricity markets is still an enormous challenge.

2.2.4 Bidding Structure

Different bidding approaches have been adopted in electricity markets around the world to reflect to some extent the different plant mixes in their power system. While hydro plants are extremely flexible as they can be committed relatively quickly, large thermal units may require many hours to synchronise with the system and increase their output to full load. Starting up thermal plants is very costly because a large amount of fuel is required to bring them to the operational temperature. Nuclear plants are typically highly inflexible and are usually only switched off for maintenance or refuelling.

The bidding prices in some electricity markets are composed of various parameters designed to represent the different elements of the cost of running a generating unit, including incremental and fixed costs (start-up and no-load costs). Such a complex bidding structure reduces the generators' risks associated with their fixed costs, but does not necessarily lead to lowering the electricity prices. It considerably increases the complexity of the scheduling algorithm and the price setting mechanism and hence reduces the transparency of the market framework. In addition, the scheduling of complex bidding systems minimises the production cost, rather than the price of electricity. This flaw may jeopardise the equity and efficiency of the electricity market. Furthermore, the complexity of bidding structures provides the market participants a wider opportunity for "gaming" and manipulating prices. The market regulator of the EPEW has expressed its concern over the strong evidence of price manipulation by some generating companies [3].

In other electricity markets the bids involve simple quantity-price pairs for the amount of energy that the market participants are willing to trade. The generators are thus required to internalise their complex cost structure to produce bids that appropriately reflect their production cost and operational constraints. To do so, they need to estimate properly the part of the market that they are likely to win. In such a bidding structure, the generators have to assume the risks associated with proper allocation of their start-up and no-load costs. Simple bids certainly increase the simplicity and transparency of a wholesale electricity market and, in such markets, the minimisation of costs based on bids are more likely to lead to a minimisation of electricity prices.

2.2.5 Demand-side Bidding

In some wholesale electricity markets, demand is determined on the basis of a load forecasting algorithm and hence plays a passive role in the determination of the market-clearing price. In such markets, the generating units are scheduled to meet the demand forecast. In other electricity markets, the demand curve is determined by aggregating the purchase bids. The dispatch and price are then determined by the intersection of the aggregated supply and demand curves. In such a *two-sided* market, the consumers are given the opportunity to express the value they put on their load.

2.2.6 Firmness of Bids and Offers

In some electricity markets, the bids and offers are treated as firm commitments to trade electricity, and deviations from those commitments are charged on the basis of the cost of imbalances. It is expected that in such markets generators and consumers will pay more attention to the construction of their bids and offers. In other markets, participants are given the opportunity to adjust their bids as the trading period approaches.

2.2.7 Price Computation

In some markets, such as the EPEW, all scheduled generating units are paid the market-clearing price independently of their bidding prices. The market-clearing price is set at the price of the most expensive unit selected. This is analogous to the so-called "*second price*" auction in which the winners are paid the price of the first losing bid. This principle is claimed to lead to economic efficiency and is justified as follows: if the generators were paid their bid prices, in a so called "*pay as bid*" framework, they would try to forecast the market-clearing price and would bid at that price to maximise their profit. Hence cheaper generating units would bid higher than their costs. Occasionally, a generator would bid too high and would be left out of the schedule. To compensate for this potential loss of income, generators would rarely use their normal bids. Therefore, the total cost of generation would be higher [21].

2.2.8 Transmission Constraints and Losses

Transmission constraints arise whenever the network capacity is not sufficient to meet all transmission requirements. Transmission losses vary over different parts of the system with the pattern of the flows on the network. Transmission constraints and losses may favour the scheduling of expensive generators over cheaper ones due to their locations in the grid. Prices in a wholesale electricity market may either be determined on a nodal basis, in which the marginal price of network constraints and losses are taken into account, or be calculated neglecting any locational variation. The costs of losses and constraints in such electricity markets are then averaged and recovered from market participants through a common uplift charge.

2.2.9 Capacity Payments

In centrally planned electricity systems, security of supply is sought by specifying the minimum acceptable margin of generating capacity over peak demand and constructing plant to ensure that this margin is maintained. Alternative approaches have been implemented to ensure security of supply in competitive electricity markets. In some markets, capacity payments are designed to encourage generators to keep marginally profitable generators available. Other markets rely solely on the energy price as an economic signal for market participants and potential new entrants. In such systems, generators should then construct their bids so as to recover their total costs over the expected hours of operation of their generating units.

2.3 Electricity Markets Around the World

In recent years, several countries have been undergoing the restructuring of their electricity industries to promote competition amongst generators and create market conditions in their power sector [22-38]. Electricity markets seek to encourage competition in generation and supply, maintaining the transmission and distribution services as natural monopolies, as there is no economic case for replicating the existing networks and no justification for implementing a transmission link for each generator-load pairing. Each country has adopted a specific model and it is useful to identify the common characteristics and the differences.

2.3.1 Background and Historic View

Historically, Chile was the pioneer country in the process of deregulation and created market conditions in the electricity sector in 1978. The process evolved into a new electricity law promulgated in 1982 [29].

England and Wales followed with the Electricity Act of 1989 and started trading through the Pool on the 31st of March 1990. The main features of the EPEW will be presented and discussed in more detail in Section 2.4.

The first step towards deregulation of the power sector in Argentina was taken by the enactment of the State Reformation Law in 1989. The process evolved into the Electric Power Regulation Frame of Law 24065 in 1990. This law created the National Regulatory Agency (ENRE) and officially established the Argentinean wholesale power market (MEM). The privatisation process leading to the commercial organisation of the market started in 1992 and hence this year is used as a reference regarding the beginning of the transformation of the Argentinean electricity industry [32].

Several South American countries promulgated deregulating laws in line with the Argentinean and Chilean initiatives: Peru in 1993, Bolivia and Colombia in 1994, Panama, El Salvador, Guatemala, Nicaragua, Costa Rica and Honduras in 1997 [31].

Deregulation and market competition in Norway were introduced with no privatisation by the Electricity Act of June 1990, which was effective from January 1st, 1991. The deregulated spot market started operation in May 1992. Sweden joined the market in January 1996 through the Nord Pool, which owns and operates the spot and futures markets. The Nord Pool is an entity equally owned by the Norwegian and Swedish independent system operators (ISO) Statnett and Svenska Kraftnät, respectively. The ISOs are owned by their respective national governments. Finland introduced supply competition for consumers with maximum demand over 500 kW through an electronic power exchange, El-Ex, in August 1996 and extended the choice to all customers in 1997. Finnish players are eligible to trade in the Nord Pool. Finland and West

Denmark joined the Nord Pool in March 1999 and July 1999, respectively. East Denmark is expected to merge with the Nord Pool in the future.

The United States Electricity Act was promulgated in 1992, but the electricity restructuring in the United States is essentially being undertaken state by state. To date, the most significant reforms have occurred in the state of California, where the deregulation of the electric power industry began with the ruling by the California Public Utilities Commission (CPUC) in April 1994. The California independent system operator started commercial operation on March the 31st, 1998.

Electricity restructuring in Australia has followed different approaches in each state. Victoria was the first state to introduce an open electricity market in 1993, with the break up of the vertically integrated state-owned utilities and the creation of unbundled state entities: (*i*) an independent company to administer the electricity market (the Victorian Power Exchange), (*ii*) five groups of power stations operating as independent producers, (*iii*) a company responsible for the transmission network (PowerNet Victoria), and (*iv*) five regional and eleven municipal distribution companies. Victoria's wholesale electricity market (VicPool) began operation in October 1994, and evolved into the National Electricity Market (NEM) that was introduced in May 1997 involving two interconnected regions (Victoria and South Australia are included in Region 1, whereas Region 2 comprises New South Wales and the Australian Capital Territory) [39]. Due to software problems and late changes to the market and system operation rules, the NEW actually stared operating on the 13th of December, 1998 [40].

The New Zealand electricity sector has undergone a series of reforms since 1987 that evolved into the creation of the New Zealand wholesale electricity market (NZEM), which started operating on the 1st of October 1996.

More recently, the Spanish wholesale electricity market was implemented on January the 1st, 1998 with a new Electricity Law (54/1997) published in late 1997. This law establishes a new legal and institutional framework to guarantee the electricity supply to all customers, under certain quality conditions at minimum cost to end-consumers.

The recent deregulation process in Brazil is being undertaken at a gradual pace through a succession of laws and decrees: in February 1995, the Law of Concessions defined important attributions for the concession authority to regulate the granted service; The national Agency for Electrical Energy (ANEEL) was created in December 1996 by the Law 9427; in May 1998, the Law 9648 created the Brazilian wholesale energy market (MAE) on the basis of an auction mechanism on the generation side; and finally, the Brazilian independent system operator (ONS) was created in July 1998 by the Decree 2655 [41, 42].

Several other countries are undergoing similar processes of introducing deregulation in the power sector. In 1993, the government of Malaysia decided to initiate the introduction of a private power sector.

The following sub-sections provide a few more relevant aspects regarding the electricity markets of particular countries.

2.3.2 The Chilean Market

Participants in the wholesale electricity market in Chile can either trade through the spot market or enter into bilateral contracts. Electricity is traded on the basis of short-term marginal costs (STMC) in this predominantly hydro system. The system operator, called Economic Load Dispatch Centre (CDEC), manages the dispatch, reliability and other pool functions. It forecasts the global demand, updates it monthly, and dispatches generators based upon audited costs and reservoir levels. By law, agreements in the CDEC are to be achieved unanimously, otherwise a regulatory body, the National Energy Commission (CNE), intervenes to solve the disputes. Dispatched generators receive payments for supplied energy based on nodal ex-post electricity prices. All available generators receive capacity payments [30, 43].

2.3.3 The Argentinean Market

The voluntary Argentinean wholesale electricity market (MEM) is run by *Compañía Administradora del Mercado Mayorista de Electricidad S.A.* (CAMMESA).

CAMMESA is a private non profit-making company that is responsible for scheduling, dispatching, price setting, settling payments and defining the reserve levels. Six companies own and maintain the transmission network. Contracts between market participants are essentially physical, but any difference between the contracted volume and the actual trade is settled in the wholesale market. Only generators and large customers can participate in the spot market in which hourly ex-ante energy prices are determined for the day-ahead. Unlike other markets, generators do not submit bids and offers. Instead the market operator determines marginal generation costs using pre-defined algorithms and seasonal fuel prices submitted. Demand plays a passive role in this market. Currently generators receive capacity payments by MEM whenever they are scheduled for output or reserve during peak demand, but this scheme is under review. A factor associated with the expected energy not supplied and the failure risk in energy payments has been proposed to replace the capacity remuneration.

2.3.4 The Nord Pool

More than 99% of the Norwegian generating system consists of hydro plants. The state-owned generator, Statkraft, with a market share of 30%, is the largest generating company amongst some 70 others, which are active market participants. Private utilities, such as Norsk Hydro, account for around 20% of the generation. The remaining 50% is supplied by some thirty municipally owned power plants. In Sweden, three large companies, Vattenfall (50%), Sydkraft (15%) and Stockholm Energi (6%), own the majority of the capacity.

The wholesale market operator (Nord Pool) is jointly owned by the Norwegian and Swedish grid companies. Nord Pool is responsible for the market-clearing process in the spot market (24-hour market) and for the futures market. It is also responsible for accounting and invoicing. System operation in Norway and Sweden is the responsibility of the owners of the national transmission networks, Statnett and Svenska Kraftnät respectively.

Three so-called organised markets are centralised and based on a standardised bidding structure: the spot market, the regulating market and the futures market [44, 45].

Currently, around a third of electricity is traded through these organised markets whereas the remaining two thirds are traded through bilateral contracts.

In the *spot market*, which is not mandatory, participants submit their bids on a day-ahead basis in the form of hourly price and quantity curves. The individual bids for generation and demand are aggregated by the MO to form supply and demand curves. The hourly ex-ante market-clearing prices and quantity are then determined by the intersection of these curves. The daily spot market is settled at noon for delivery during the 24 hours following midnight. Market participants have no opportunity to revise their bids and the accepted bid volumes become firm commitments at the ex-ante market price. Prices in the spot market have been largely determined by the water supply in the Norwegian hydro-dominated power system. Must-run generating units, those that are downstream with no storage capacity and use the water released from upstream reservoirs, can strategically bid zero for their generation and thus accept the market-clearing price.

In the *regulating market*, generators submit bids on how much they are willing to regulate up and down, including prices and duration. The system operator balances the real-time operation of the market, by selecting the cheapest available regulator from the merit order list. The regulating market prices are thus settled ex-post and all the regulators receive the price of the marginal regulator.

When the market started operation in May 1992, market participants were eligible to enter into future contracts for physical delivery, but in October 1995 the *futures market* became merely a financial market. The future contracts are settled against the average spot price of electricity for the delivery week.

Nord Pool does not incorporate any explicit capacity payments, since Norway's electricity system is energy rather than capacity constrained.

2.3.5 The Californian Market

In California, the restructuring process has led to the creation of a wholesale power exchange (PX) and an independent system operator (ISO). The PX, the ISO and the Scheduling Co-ordinators (SCs) are the main elements of the Californian market. The PX and ISO are public benefit, non-profit entities. The Californian market structure aims to entirely separate the PX and the market participants from the ISO [46]. The three large vertically integrated private utilities (the Pacific Gas & Electric - PG&E, Southern California Edison, and San Diego Gas & Electric) own and maintain the transmission assets.

The Californian ISO owns no transmission, generation or distribution facilities and has no financial interest in the power exchange or in any generation or load [47]. The primary responsibilities of the ISO are to: (*i*) operate the transmission network as an integrated system ensuring its reliability; (*ii*) provide non-discriminatory and open access to the network; (*iii*) schedule all power through the network and balance the grid operation; (*iv*) manage transmission congestion and constraints; (*v*) procure and operate ancillary services in a competitive way; (*vi*) provide information to market participants; and (*vii*) settle the real-time energy and ancillary services. Updated information on the Californian ISO can be found in [48].

The PX has no financial interest in generation and it is mainly responsible for (*i*) providing a competitive spot market for energy; (*ii*) determining day- and hour-ahead market-clearing prices for energy, based on a least-cost balance schedule; (*iii*) procuring adequate ancillary services on a least-cost basis; (*iv*) acting as a SC for PX participants; and (*vii*) performing settlements for the market. The main responsibilities of SCs are very similar to those of the PX.

Participation in the PX wholesale market is voluntary, i.e.; market participants may either trade through the PX or bilaterally with each other. However the three large vertically integrated private utilities must trade through the PX until the year 2002. The bilateral transactions between market participants are managed by the ISO through an SC.

Demand and supply are treated equally in the day-ahead PX market. Market participants submit simple price-volume bid curves for each hour of the following day. Fixed costs (start-up and no-load costs) are not included in the generators' bid files. The optional PX wholesale market determines the ex-ante market-clearing prices and quantities by the intersection of the aggregate supply and demand curves. The accepted bids represent firm commitment and deviations are settled in the balancing market.

An iterative auction is implemented in the Californian market. In this process, market participants are allowed to submit their bids and offers up to five times and, in each iteration, the PX publishes provisional prices and schedules and informs the participants whether or not their bids have been accepted. The purpose of this is to allow generators to establish plant output profiles that satisfy their operational constraints. It may also enable customers to adjust their demand patterns in the most economical way.

Real time system balancing is carried out by the ISO via a balancing market based on the hour-ahead adjustment bids. The ISO can alternatively call upon ancillary services to balance the system if these are cheaper than the adjustment bids. An ex-post balancing price is set for each trading period and is used to settle any divergences from day-ahead scheduled quantities. The California PX has no explicit payments for capacity.

The market participants may either provide themselves the ancillary services needed to support their energy schedules or request the ISO to procure the ancillary services on their behalf.

2.3.6 The Australian Market

With effect from 13 December 1998 the operation of VicPool ceased and responsibility for the management of the Victorian Electricity Market was passed to the National Electricity Market Management Company (NEMMCO), which is a non profit-making entity that acts as market and system operator. Grid ownership and control have been separated, PowerNet, Victoria is the owner and maintainer of the transmission network

[49]. Like the Vicpool, the Australian national electricity market (NEM) is a mandatory market.

Market participants submit bids and offers for a seven-day rolling period for each half-hourly trading interval. Generators are entitled to offer up to ten incremental prices for output above a minimum generation level. Generators' start-up and no-load costs are not included in the bid files. Demand side bidding is encouraged through load reduction bids. Market participants can revise their submitted quantities, but not their bidding prices, up to five minutes before real time. An analysis of generators' bidding behaviour in the face of the existing contracts can be found in [16].

The market operator runs the scheduling program every five minutes for the following five-minute period, determining a real-time market-clearing price during that period. Prices are thus determined on an ex-post five-minute basis. Deviations from the projected schedule are addressed via ancillary services. No capacity payments are incorporated in the NEM. However, pool prices are set at the value of loss of load (*voll*) whenever there is insufficient generation to meet demand.

In the wholesale electricity market in Victoria, market participants trade electricity through three main mechanisms: vesting contracts, bilateral contracts and the spot market [50]. It is estimated that vesting contracts, hedge contracts and the spot market account for 50%, 40% and 10% respectively of the total electricity traded in the NEM. *Vesting contracts* are financial instruments that were arranged between power stations and energy retailers before the introduction of the open electricity market. They aim at market stabilisation and at ensuring the recovery of the power stations' long-term marginal costs. *Hedge contracts* are also financial mechanisms that insulate market participants from the volatility of the spot prices. The *spot market* balances the supply and demand in a bidding system for the energy that is not traded through the above contracts.

2.3.7 The New Zealand Market

The Electricity Market Company (EMCO) is the market operator of the New Zealand electricity market (NZEM). The system operator, Trans Power, is also the owner and developer of the national grid. It is responsible for (*i*) providing a reliable national network and co-ordinating the grid operation; (*ii*) scheduling and dispatching generation to meet demand; (*iii*) purchasing ancillary services and resources for the economic and secure delivery of electricity; and (*iv*) providing information to grid users in an open and non-discriminatory way. Market participants can either trade through NZEM or through physical bilateral contracts. Volumes on bilateral contracts are notified to the system operator and deviations are settled at the prices established by NZEM [51].

A trading day starts at midnight and contains 48 half-hourly periods. Participants in the spot market submit simple price-volume bids, which are not firm bids, for each half hour of the day ahead. The bids can include up to five trading blocks for generators and ten for purchasers. Generators' ramp rates are part of the bids and are taken into account by the clearing process. Forecasted prices are issued and made available to market participants on the day-ahead to allow them to re-offer their bids for "*bona fide*" reasons only, such as plant failures. The market rules require concurrent dispatch of energy and reserve bids, which are cleared simultaneously based on location, quantities and prices offered when competing for the same resource. Generators' bids are used to determine the plant dispatch on the basis of forecast demand and to calculate ex-post spot prices using the metered demand. There are no separate payments for capacity.

2.3.8 The Spanish Market

The Spanish power exchange market operator (*Compañía Operadora del Mercado Español de Electricidad* - COMEL) is responsible for the management of the market and for the economic settlement of all transactions between market participants. The market is composed of four independent, though inter-related, markets and processes: the daily market, the ancillary services market, the hour-ahead market and the balancing market [52, 53]. The following paragraphs provide additional information on the main features of the Spanish market. Further information on this market can be found in [54].

The daily market is the responsibility of both the market operator (MO) and the system operator (SO). In this market, bidders offer prices for buying and selling electricity on a given day by 10:00 a.m. on the previous day. The bids in this day-ahead market are submitted for each generating unit or consumer point on an hourly basis and contain up to 25 blocks of energy. No portfolio of bids is allowed in an attempt to discourage the exercise of market power. An unusual feature of this market is that the bids can include a *minimum income condition* that allows the generators to cover their fixed costs. For each hour, aggregated selling and buying bidding curves are constructed. These curves are superposed and the market-clearing price is determined by the intersection of these respectively monotonically increasing and decreasing curves. The blocks of energy are treated as indivisible, as no interpolation between blocks is allowed. As such, one block of energy at a given price is either accepted in full or rejected. This may lead to some small imbalances when an indivisible bid sets the market-clearing price. The matching algorithm does not attempt to optimise the solution but simply tries different feasible solutions in the search for the one that increases the participants' satisfaction. This is done by minimising the benefit that generating units not selected would achieve if they were to receive payments based on the market-clearing price of the sub-optimal solution. This is an interesting approach to help achieving the equity of the electricity market. The unconstrained solution is thus determined by the MO.

This solution is then forwarded to the system operator (SO) who determines a constrained solution by using a contingency analysis algorithm. The security schedule modifications are sent back to the MO to balance the daily market again. At the end of this process the "*feasible daily schedule*" is made available to the market participants, and the ancillary services bids are requested and assigned by the SO.

The hour-ahead market is voluntary and any agent that had participated in the corresponding session of the previous daily market can offer several bids. The bidding structure and the price determination are similar to the day-ahead market. This market is again the responsibility of the MO who performs several iterations. The maximum number of sessions to date is five.

The unconstrained hour-ahead schedule is also sent to the SO to assess the viability of the transactions and to balance the schedule, respecting the economic merit order of the hour-ahead bids.

2.3.9 The Brazilian Market

The Brazilian power system is characterised by a high proportion of hydro resources – more than 90% of the total capacity are hydro plants - with several plants in the same river. This requires a sophisticated algorithm for hydro-thermal co-ordination. Hence, the spot price is not determined by a generation bidding process like in other electricity markets, but by the simulation of a hydro-thermal optimisation program based upon the generators' technical data. This program also produces the dispatch and exchanges between the generating companies as a by-product. The centralised scheduling and dispatching are performed by the Brazilian independent system operator (ONS), which is also in charge of the definition of the necessary investments in transmission services, including ancillary services. The ONS aims to guarantee non-discriminatory access to the transmission network, of which users pay a wheeling charge. A long-term marginal cost approach is used to define nodal transmission prices.

Independent power producers, brokers, distribution/retail companies, and consumers demanding more than 3 MW are eligible to participate in the Brazilian wholesale electricity market (MAE). They can enter into bilateral contracts and apply for access to the transmission and distribution systems, but it is compulsory that energy transactions be accounted for through the MAE. The Brazilian wholesale electricity market can thus be regarded as a forward market in which freedom only exists in negotiating bilateral contracts. Additional information regarding the main characteristics of the Brazilian power system, the privatisation program of the state- and federal-owned electrical utilities and the Brazilian wholesale electricity market can be found in [55, 56].

Table 2.1 summarises the main characteristics of the electricity markets discussed in this section.

Table 2.1: Main characteristics of some electricity markets around the world

	Chile	**Argentina**	**Nord Pool**	**California**	**Australia**	**New Zealand**	**Spain**	**Brazil**
Market operation (MO) and system operation (SO)	MO and SO functions carried out by CDEC, with disputes solved by NEC	CAMMESA accumulates the function of MO and SO	SO (national transmission networks) and MO (jointly owned company) are separate entities	PX and ISO are separate entities. ISO owns no transmission assets. SCs and PX have similar responsibilities	One non profit-making entity accumulates the functions of MO and SO	MO and SO are separate entities. SO owns the national grid.	MO and SO are separate entities.	MO and SO are separate entities.
Trading options	Optional market	Optional market	Optional market	Voluntary market, but the 3 large utilities must trade through PX.	Mandatory market	Optional market	Voluntary market	Compulsory market
Bidding structure	Audited costs instead	Seasonal fuel prices	Simple hourly price-quantity bids on a day-ahead basis.	Simple hourly price-volume bids on a day-ahead basis. They can be re-submitted up to five times.	Seven-day half-hourly bids up to ten incremental prices. No fixed prices Included	Simple half-hourly price-volume bids on a day ahead basis	Simple hourly price-quantity bids on a day-ahead basis, up to 25 blocks. Minimum income bids.	No bidding scheme. Generators submit only technical data
Demand role	Passive: forecasted by the SO	Passive: forecasted by the MO	Active: also offer bids	Active: also offer bids	Load reduction bids	Passive: forecasted by the SO	Active: also offer bids	Passive: forecasted by the SO
Firmness of bids	–	–	Yes	Yes	Yes	No	Yes	–
Price computation	Nodal ex-post STMC	Ex-ante energy prices	Ex-ante market-clearing prices from the intersection of the aggregated supply and demand curves	Ex-ante market-clearing prices from the intersection of the aggregated supply and demand curves	Ex-post clearing prices on a five minutes basis	Ex-post spot clearing prices based on metered demand	Ex-ante market-clearing prices from the intersection of the aggregated supply and demand curves.	Hydro-thermal optimisation algorithm
Capacity payments	To all available generators	Yes, but under review	No	No	No	No	Yes	No

2.4 The Electricity Pool of England and Wales

The previous structure of the nationalised industry in England and Wales was characterised by having one large generating and transmission company, the Central Electricity Generating Board (CEGB), selling electricity in bulk to twelve area distribution boards through a nation-wide transmission system (the National Grid). The electricity would then be delivered to the customers by the electricity boards through their own distribution networks. This monopolistic system was characterised by a centrally planned investment, an engineering-led approach and a cost-plus pricing mechanism. A detailed description of the re-structuring process of the electricity market in England and Wales can be found in [22, 23].

On the 31st of March 1990 the EPEW was established for the trading of electricity between generators and suppliers. The new system has been designed to stimulate competition in the generation and supply business as well as to ensure that monopoly power was not abused in transmission and distribution, which were both kept as natural monopolies. The market for electricity created by the pool trading arrangements establishes prices for sales and purchases of electricity, taking into account the variable supply and demand for the product. The “Pool Rules” in conjunction with other provisions of the “Pooling and Settlement Agreement” specify the way the market participants trade [57, 58].

The electricity industry of England and Wales has thus undergone two radical changes: privatisation of the electricity companies and introduction of competition in the power sector. Under the new restructuring, the CEGB was split into four parts. The fossil-fuelled power stations were divided into two large generating companies: National Power and PowerGen, which were privatised. The nuclear stations along with some gas turbine and hydro plant were transferred to Nuclear Electric, which remained in the public sector. The ownership and operation of the high voltage transmission system were transferred to the newly-created National Grid Company (NGC), which was also given the responsibility of administering the financial settlements following the trading of electricity in the wholesale competitive market. The twelve Regional Electricity Companies (RECs), privatised successors of the area boards, became the majority joint owners of NGC until December 1995 when NGC was floated in the stock market. In the new structure of the EPEW, (*i*) *generation* is regarded as the production of electricity, (*ii*) *transmission* is related to the business of transferring it in bulk across the country, (*iii*) *distribution* is considered the delivery of electricity over local networks, and (*iv*) *supply* is the term used for the acquisition of electricity and its re-sale to final customers.

2.4.1 Current Market Framework

The current EPEW can be described as a *"compulsory, one-sided, non-firm market in which complex offers are used to set market prices on a marginal ex-ante basis with the cost of imbalances averaged on an additional capacity payment levied. It is governed by its members and the functions of the Market Operator (MO) and System Operator (SO) are not fully separated."* [59]. Some additional information on the main features of the above definition is presented in the following paragraphs.

Compulsory Pool: Generators must sell their output into the Pool and suppliers must purchase their electricity from the Pool. Hence, trading outside the Pool is not an option for customers, suppliers or generators. However, market participants can enter into financial contracts to limit their exposure to price volatility.

One-sided market: Suppliers do not bid to take a particular quantity of electricity. Instead, a single forecast of the national demand is made on the day ahead by NGC on behalf of the Pool.

Non-firm market: The bidders are not required to meet the physical commitment to generate electricity. Hence, they are not exposed to financial risks, such as a "cash-out" for price imbalances or a financial penalty, for failure to generate.

Complex Bids: The bids are composed of several parameters designed to represent (*i*) the various elements of the costs of running a generating unit, including incremental, start-up and no-load costs, and (*ii*) the technical limitations on the operation of generating units, such as minimum up and down times, minimum stable generation and maximum capacity.

Marginal Prices: The price for the most expensive generating unit scheduled in each half hour sets the market-clearing price for the energy, named the System Marginal Price (*smp*). Payments to generators and suppliers for each MWh are mainly based on the *smp* of the corresponding scheduling period.

Ex-ante prices: The prices are determined and made available to all market participants before the day of the actual trade of electricity. This gives the bidders the opportunity to change their availability offer based on a commercial decision and allows the customers to adjust their activities and their demand profile.

Average Costs of Imbalances: The dispatch of the generating units on the day may not match the day-ahead dispatch due to a number of factors (e.g., differences between the actual and the forecasted demand, transmission constraints, generators' forced outages), and thus lead to additional costs (e.g., when a generating company is called to remedy imbalances, it is paid its bid price rather than the market-clearing price). The costs of the imbalances are aggregated into Energy Uplift, which is averaged over all suppliers on a per unit basis.

Capacity Payments: They are paid to generators in addition to the *smp*, regardless of the amount of electricity that their generating units actually produce, in an attempt to encourage them to make their plants available over the long term and at times of high demand. Even generating units not included in the schedule receive capacity payments whenever they are available.

Governance: The Pool is underpinned by a multi-lateral contract amongst generators and suppliers, which defines the trading rules and procedures that control the competitive bidding process, but does not act as a market maker, buying or selling electricity.

Combined Functions: NGC is the transmission asset owner, who builds, operates and maintains the transmission network, and is also the System Operator, who schedules and dispatches generation and ancillary services. The Pool is the Market Operator, who receives offers and bids, calculates prices and disseminates information, but important parts of these functions are carried out by NGC on behalf of the Pool.

Arrangements for trading on a given day start at 10:00 a.m. on the previous day, by which time the generators with a capacity greater than 100 MW submit their offer prices. Each day every generator bids prices at which it is willing to generate from each of its generating units at its power stations, and their corresponding available outputs. Also by 10:00 a.m. each day, the dispatcher produces a forecast of national demand for every half hour of the Availability Declaration Period (ADP). The ADP is the 39-hour period running from 21:00 on day 0 to 12:00 noon on day 2. The settlement day, the period running from 0:00 on day 1 to 0:00 on day 2, is split into 48 half-hourly periods called settlement periods. The schedule day is the period running from 05:00 a.m. on day 1 to 05:00 a.m. on day 2. A simplified model of the current framework of the EPEW is shown in Fig. 2.1. The main features of each major block of the model are presented in the following sections.

2.4.1.1 Load Forecasting

The demand forecast is produced on a day-ahead basis by the grid operator based mainly on historic demand data and weather forecasts. Total demand does not take into account demand from large customers, external pool members and the demand of pumped storage units, which is estimated separately and added to the total demand. The customers' response to prices is restricted to a mechanism in which demand-side bids can be treated as negative generation.

It has been argued that the scheduling problem in a pool-based electricity market such as the EPEW should be modified to properly represent the demand-side bidding aspects [60]. A simple way to show how generating companies owning a portfolio of units or controlling some demand-side bidding can manipulate prices is presented in [61].

A method to integrate the short-term elasticity of demand for electricity with a generation scheduling algorithm in a pool-based electricity market is presented in [62]. The customers' reaction to prices is modelled using a matrix of self- and cross-elasticities, which relates the demand change during one scheduling period due to deviation in the electricity price of that period, and other periods; respectively. For the rest of this thesis, however, demand is considered fixed.

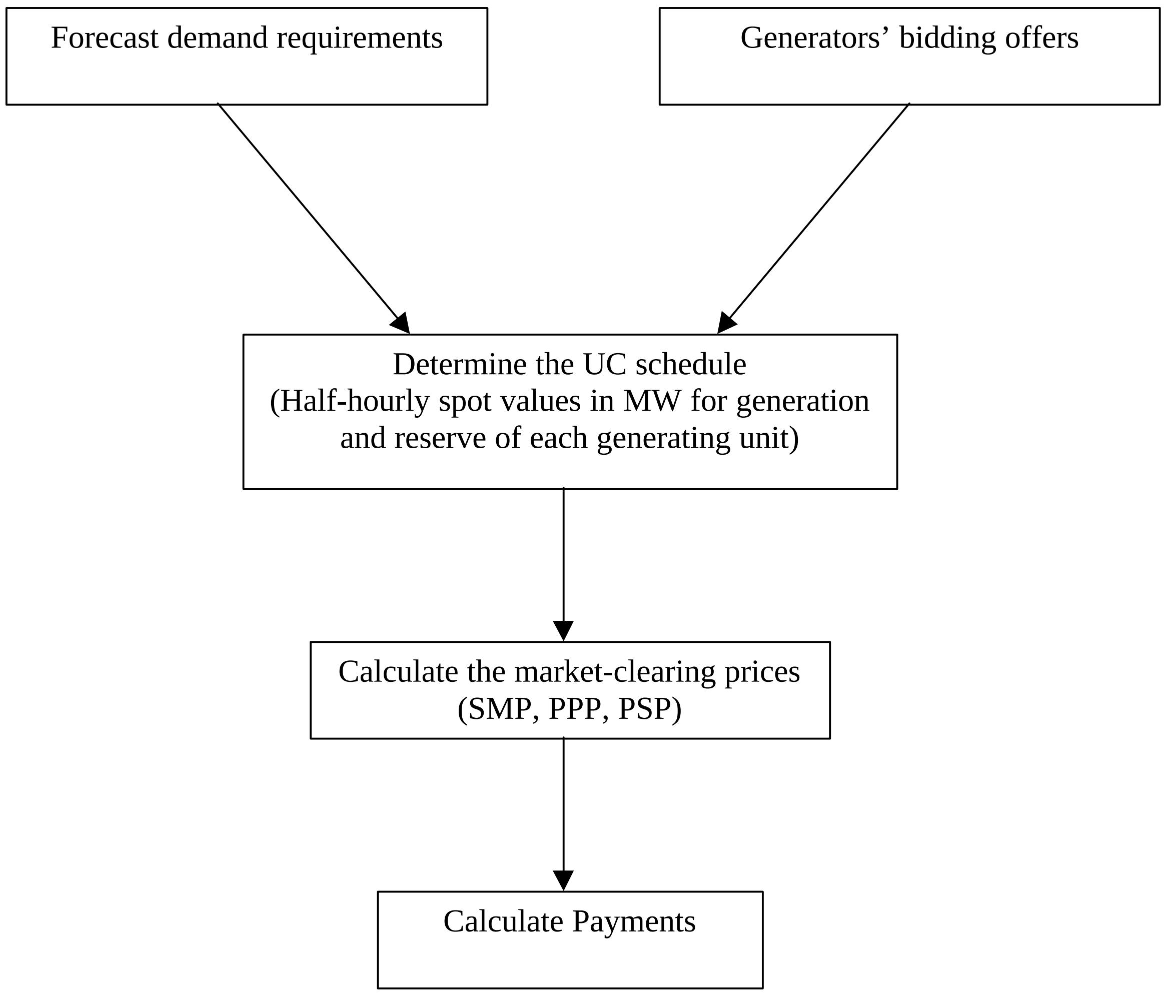

Fig. 2.1: Simplified Structure of the Electricity Pool of England and Wales

2.4.1.2 Generators' Bidding Offers

The generators' offer files are composed of the bidding prices and the units' operational characteristics such as run-up rates, run-down rates, and synchronising generation. The bidding files can contain up to a maximum of three incremental prices (inc_i^1, inc_i^2 and inc_i^3), one no-load price (nl_i^1) and one start-up price. A piece-wise linear price function is used to represent the offer prices. Such a price function is known as *Willans Line* and an example of it is shown in Fig. 2.2, in which $P_i^{\min}$ and $P_i^{\max}$ are the lower and upper generation limits and e_i^1 and e_i^2 are the first and second elbow points, respectively. Fig. 2.3 shows an alternative way to present the incremental bid prices. Only the first no-load price (nl_i^1) is actually part of the bidding file. The other two no-load parameters (nl_i^2 and nl_i^3), which can be negative, are calculated as follows:

$$nl_i^2 = nl_i^1 + (inc_i^1 - inc_i^2) \times e_i^1 \tag{2.1}$$

$$nl_i^3 = nl_i^2 + (inc_i^2 - inc_i^3) \times e_i^2 \tag{2.2}$$

Obviously, the bidding prices in the EPEW are represented on the basis of pounds sterling (£). However, for the rest of this thesis, any reference to prices, costs and payments will be done on the basis of the U.S. dollars ($). This is simply because the test systems used to illustrate the ideas and concepts of this thesis were originally published in the latter currency. Hence no treatment of the data was required and the results presented throughout this thesis can easily be compared to those of previous publications.

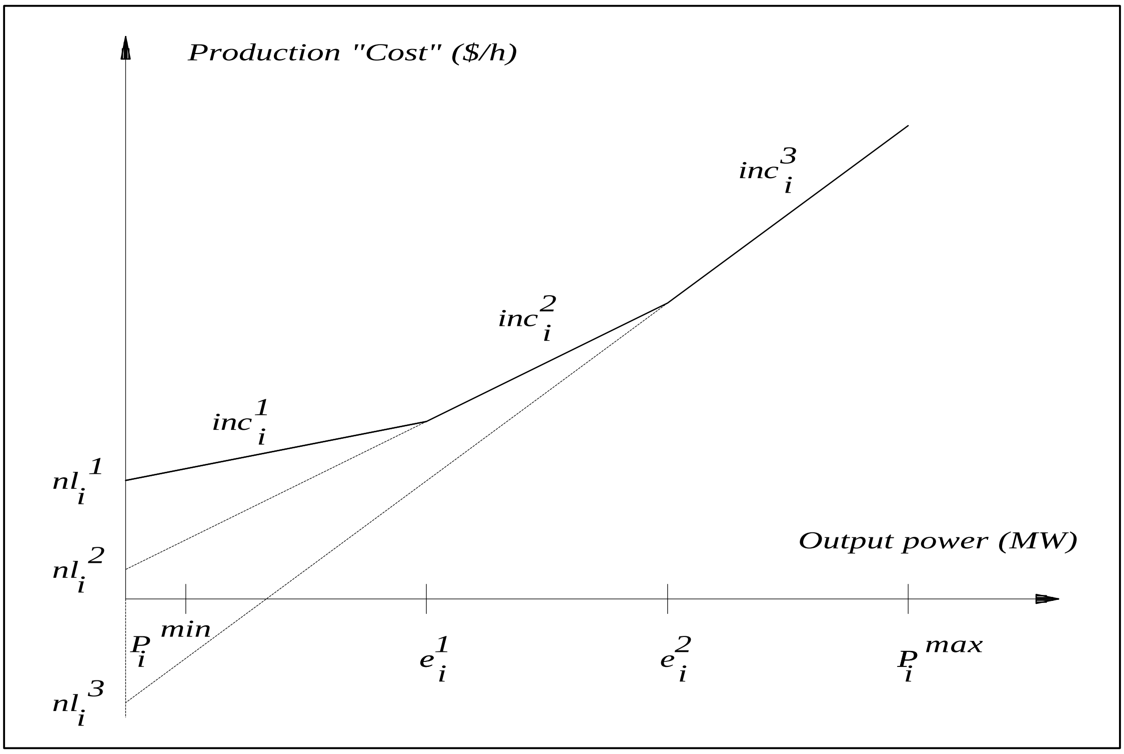

Fig. 2.2: Generators' offer price (*Willans line*)

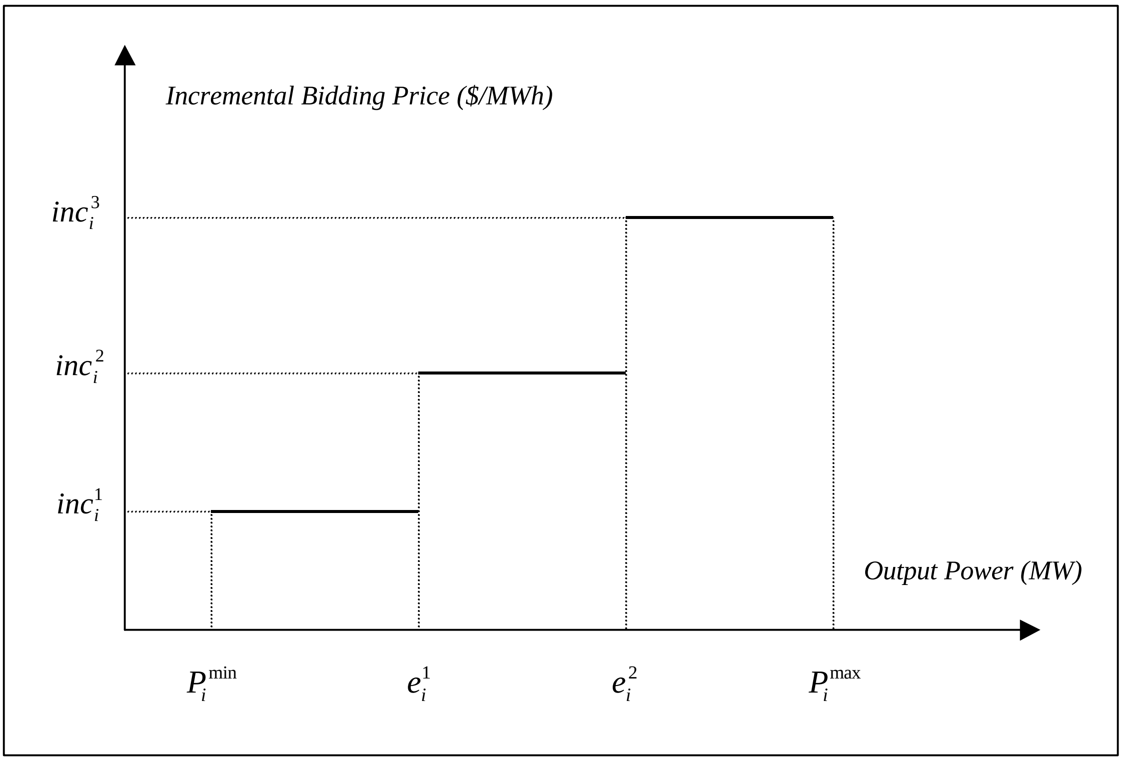

Fig. 2.3: Incremental bidding prices per quantity

2.4.1.3 Generation Scheduling

The generation ordering and loading (GOAL) program schedules generating units in a merit order that is derived from generators' offer prices. The schedule also takes into account the generators' offered availability, declared inflexibility and dynamic running characteristics. The output of the unconstrained schedule consists of half-hourly spot values in MW for generation, reserve and availability of each generating unit. At this stage the scheduling program ignores transmission constraints.

Due to the generators' start-up costs, reducing the output of a number of generating units for a few hours when the system demand is falling, and then increasing it when demand rises may be cheaper than shutting down a small number of generating units and starting them up later. The unit operational constraints, such as minimum up and down times, may also contribute to it. Hence, there are some periods, usually of low demand, when there is plenty of spare generating unit capacity. These periods are called Table B periods in the EPEW. There are a maximum of 20 Table B periods in a day. Other periods, near to demand peaks, when there is less spare generating unit capacity, are named Table A periods.

2.4.1.4 Price Computation

- **Unit Price**

In an attempt to produce lower prices when there is plenty of spare capacity and hence encourage demand during these periods, the pool operator in the EPEW determines the prices of the generating units scheduled during Table A and Table B periods in different ways. In Table A periods, the start-up prices (S_i^t) and the accumulated no-load prices for continuous scheduling periods are amortised over the total output during Table A periods, and then augmented to the unit incremental price. In Table B periods, those prices are not included in the unit prices, which only accounts for the unit incremental prices. Hence, the unit prices (gp_i^t) of scheduled units are determined as follows:

$$gp_i^t = inc_i^t + \frac{s_i^{t_{on}} + \sum_{t=t_{on}}^{t_{off}} (nl_i^t \times pl)}{\sum_{\substack{t=t_{on} \\ t \in \Omega_A}}^{t_{off}} (P_i^t \times pl)} \qquad \forall\, t \in \Omega_A \qquad (2.3)$$

$$gp_i^t = inc_i^t \qquad \forall\, t \in \Omega_B \qquad (2.4)$$

where:

t_{on} is the period at which the unit is started up;

t_{off} is the last period before the one at which the unit is shut down;

Ω_A is the set of Table A periods from t_{on} to t_{off} , inclusive; and

Ω_B is the set of Table B periods from t_{on} to t_{off} , inclusive.

When a generating unit is scheduled to generate for two or less than two consecutive hours, it is classified as a pulsing unit. The amortisation of the fixed costs over the accumulated output of at most two periods is felt in the EPEW to provide an incorrect price signal. Hence, the unit price in Table A periods is adjusted by amortising its fixed costs over the total capacity rather than over the total output power of the unit during the continuous running period. The unit price is then calculated as follows:

$$gp_i^t = inc_i^t + \frac{s_i^{t_{on}} + \sum_{t=t_{on}}^{t_{off}} (nl_i^t \times pl)}{\sum_{\substack{t=t_{on} \\ t \in \Omega_A}}^{t_{off}} (\boldsymbol{P_i^{max}} \times pl)} \qquad \forall\, t \in \Omega_A \qquad (2.5)$$

If a generating unit is scheduled to operate in the above way and if its operating costs are not recovered by the market-clearing price, then a side payment is made to the unit to guarantee cost recovery. Additionally, if a unit is scheduled to generate only during Table B periods, the cost recovery is not verified because the fixed costs are not taken into account when calculating the unit prices in those periods. Hence, another side payment is made to the unit to ensure cost recovery. The detailed description of these special payments can be found in [58].

As it can be seen in Fig. 2.2 the no-load prices (nl_i^t) can be negative, hence the second term of (2.3) or (2.5) can be negative. Therefore, the unit prices (gp_i^t) in a Table A period can actually be lower than the unit prices in a Table B period. Hence, if demand were to respond to prices, customers would modify their behaviour by increasing consumption in periods of less spare capacity (Table A periods) and reducing it in periods where plenty of spare capacity is available (Table B periods).

- **Market-clearing Price**

The market-clearing price, named the System Marginal Price (*smp*), is set at the highest unit price of the flexible generating units in any scheduling period. Hence,

$$smp^t = Max\,(gp_i^t) \qquad \forall\ i = 1, \ldots, \mathrm{N} \qquad (2.6)$$

where N is the total number of generating units.

- **Pool Purchase Price**

The Pool Purchase Price (*ppp*) is the price at which part of the generators' revenues is derived. It is the sum of the *smp* and the Capacity Element, which is defined on the basis of the Loss of Load Probability (*lolp*) and The Value of Loss of Load (*voll*). Hence,

$$ppp^t = smp^t + lolp \times (voll - smp^t) \qquad (2.7)$$

Loss of Load Probability is defined as the probability that the load becomes greater than the available energy. To date, with a few notable exceptions, *lolp* has never been significantly greater than zero. The calculation of *lolp* including a composite generation-transmission reliability evaluation of the system may result in a significant change in this framework.

The Value of Loss of Load is the price consumers are assumed to be willing to pay to avoid loss of supply. It is intended to reflect the cost to customers of an electricity outage. Its value was set by the British government at 2,000 £/MWh in 1990 and increased each April by the annual rate of inflation (RPI) measured by the preceding December. The current figure for 1998/99 is 2,694 £/MWh [40].

- **Pool Selling Price**

The Pool Selling Price (*psp*) is paid by the customers for their metered demand and metered station load for net importers. It is calculated to maintain a balance between payments to generators and the costs of ancillary services, reserve, deviations in availability from the original offers (including breakdowns) and changes in the unconstrained schedule due to transmission constraints. These costs are incorporated in a component named *uplift*, which is augmented to the *ppp* during Table A periods. In Table B periods, the *psp* is set equal to *ppp*. As the uplift does not affect the issues discussed in this research project, the customers' payments will be determined on the basis of the *smp*, and this component of the price will not be discussed any further. The formal definition of the *psp* is the following:

$$psp^t = ppp^t + uplift \qquad \forall\, t \in \Omega_A \tag{2.8}$$

$$psp^t = ppp^t \qquad \forall\, t \in \Omega_B \tag{2.9}$$

2.4.1.5 Payments

The Pool is a non profit-making institution that must balance its cash flow with no mismatches. Hence it collects payments from the suppliers on the basis of the *psp* and forwards them to the generators on the basis of the *ppp*. The Pool is responsible for administering all the imbalances.

For each settlement period the generators receive payments that are the sum of up to eight different components. For the purposes of this research, only the energy payment will be taken into account when computing the generators' revenue. Since the other components of the generators' revenue do not affect the concepts discussed in this work, they are briefly described in the following paragraphs but will not be discussed any further.

- **Energy payment (*EP*)**

This payment is made to every scheduled generating unit on the basis of its Revised Unconstrained Generation (W_i^t).

$$EP_i = \sum_{t=1}^{T} u_i^t \times ppp^t \times W_i^t \tag{2.10}$$

The capacity payments, the second term in (2.7), do not affect the concepts and results discussed in this research. Therefore, for the purposes of this work the payments to generators will be determined on the basis of the *smp* rather than on the formal definition of the *ppp*. In addition, the half-hourly spot values of MW should be converted into energy in MWh for each scheduling period. The precise formulation of this conversion can be found in [57, 58]. For the sake of simplicity, it is assumed that the amount of energy produced by each scheduled unit during one interval is given by the product of its spot production in MW (P_i^t) and the length of the scheduling period (pl). Hence,

$$EP_i = \sum_{t=1}^{T} u_i^t \times smp^t \times P_i^t \times pl \tag{2.11}$$

- **Reserve payment (*RP*)**

This payment is an incentive for the cost saving brought about by being constrained to operate at a lower level of output while holding reserve.

- **Marginal set adjustment payment (*MSA*)**

This payment is made to flexible generating units whose operating costs have not been recovered through the *smp*. This happens when the prices of pulsing generating units are adjusted as described in Section 2.4.1.4.

- **Availability payment (*AP*)**

This payment aims to encourage generators to make plants available beyond that necessary to meet demand, to cover breakdowns, unexpected high demand, etc. It is based on costs incurred if the availability were to be used in full.

- **Metered payment (*MP*)**

This payment attempts to cover the differences between the Metered Generation and that scheduled in the Revised Unconstrained Schedule.

- **Maxgen payment (*GMP*)**

This payment is designed to cover the additional costs of a generating unit, when it is requested, at short notice, by the Grid Operator to generate beyond its normal operation range.

- **Table B start-up payment (*TBP*)**

This payment covers the start-up and no-load costs of flexible generating units when they are scheduled to run only in Table B periods.

- **Non-dispatched payment (*GNY*)**

This payment is made to non-dispatched units. It is based on the Metered Generation.

2.4.2 New Market Arrangements

The EPEW is undoubtedly the most discussed deregulation process of the electricity industry around the world. Throughout these last ten years of operation, several drawbacks regarding the way the EPEW is promoting fair and transparent competition in the electricity market have been highlighted.

During the first year of the pool's operation, around 95% of all electricity sold was covered by some form of contract. The most significant effect of all these contracts was to protect the principal generators from low pool prices. Therefore, the obvious strategy for National Power and PowerGen was to bid low into the pool. There was a famous weekend in July 1990 when, for several hours, the pool price was zero. The group most immediately affected by low pool prices is the independent electricity producers. Helm and Power [63] have investigated the effects of contracts on the pool prices behaviour and found empirical evidence that the contracts for differences had led to low and volatile pool pricing during the early years of the operation of the EPEW. They add that these low prices constitute a significant barrier for an independent producer trying to secure investment capital for a new plant.

In [10], it is argued that the capacity payment mechanism ($voll \times lolp$) of the EPEW creates incentives to reduce plant declarations and thereby augment revenue through capacity credits. The authors claim that this mechanism is an effective incentive to new entrants but does not create an incentive for incumbent generators to invest.

According to Bunn [64] a key role of business simulation, optimisation methods and economic analysis has been identified as a basis for an effective regulatory framework to manage the transition to more efficient market competition. He suggests that the electricity market in England and Wales may have been designed with emphasis upon the operational conditions of the power system and have modelled the financial aspects in a rather immature and inefficient fashion. After discussing several issues regarding the re-structuring of the electrical industry in the early stages, the author concludes that privatisation has created a greater need for a new range of analytical models that encompass traditional electricity optimisation methods. These models need to be extended to deal with the softer more speculative issues of business strategy and market regulation.

Lucas and Taylor [65] have made an empirical analysis of the pool prices to assess the relationship between the demand levels and the system-marginal price (*smp*) during Table A and Table B periods. They argue that the implicit logic of the calculation of the *smp* in Table A periods is that the start-up costs of the next plant need to be reflected in the price by averaging those costs over units generated during the operation of the plant. However, the marginal cost of the increment of electricity which brings forward the operation of the next plant is simply the incremental cost of that plant, because that plant will be called upon to run later and so the start-up costs will be incurred anyway. This argument applies to all except the last plant. An incremental of electricity taken at peak will require an increment of plant to be started and at that point the start-up costs are a genuine component of the system marginal cost. The authors add that if the *smp* reflected the system marginal cost, it would be represented by a continuous supply curve monotonically increasing with demand. However, in practice this was not so and a larger scatter of *smp* was seen across all levels of demand. The scattered nature of the *smp* was then attributed to the way in which the generating unit price is calculated for Table A and Table B periods. It follows that the rules for calculating *smp* in Table A periods cannot be justified within the framework of the efficient allocation of resources by marginal costs pricing. Even if generators bid at their true short-run marginal cost, the pool rules will still not lead to an *smp* that accurately reflects the system marginal cost. The authors recommend an alternative derivation of *smp* whereby only the last plant to meet peak demand should contribute start-up costs to the *smp*. The review of the mechanism for generating pool prices may also involve the suppression of Table A periods.

Several other authors have contributed to the discussion on key issues of the implementation and operation of the market structure in the electricity industry of England and Wales [66-72]. More recently, it has been claimed that the price setting is complex and has facilitated the exercise of market power at the expense of consumers. It has also been argued that the capacity payments are problematic, and that the bids and pool prices have not reflected costs [2-4, 73].

On the light of the above criticisms and many others, new trading arrangements that are expected to provide significant advantages over the current ones have been proposed. They would offer lower prices from more efficient and competitive trading; greater choice of markets; more scope for demand management; sharper incentives to manage risks; transparency from simple bids; forward price curves to facilitate new entry; avoidance of discrimination against fuel sources, and more liquid contracts markets; scope for greater co-ordination and consistency with gas; and more flexible and effective governance. At present, the plants are scheduled based on generators' bids on the day ahead, and the most expensive unit sets the market-clearing price. The Pool is to be replaced by new trading arrangements that include (*i*) forward markets to the extent required by market participants, (*ii*) an organised short-term bilateral market, operating from at least 24 hours ahead to 4 hours ahead of each trading period, (*iii*) a balancing market operating from 4 hours ahead to the end of each trading period, and (*iv*) a settlement process for imbalances. The system operator will work within this framework to balance the system, deal with transmission constraints and maintain security of supply. More details about the review of the trading arrangements in England and Wales can be found in [49, 59, 74-80]

2.5 Summary

This chapter has presented and discussed the key features regarding the design and implementation of a wholesale electricity market in a competitive environment. Its main objective was to describe the principal mechanisms of electricity trading arrangements and market operation. The focus of this chapter was to show that the generating units are scheduled based on their bids and offers, and that the payments, collected from the consumers and forwarded to generators, are based on the system clearing price, which are determined from the spot price of the generating units.

The implementation of simple bids is a current trend in electricity markets around the world. The generation scheduling problem becomes a simple optimisation problem, which requires a simple solution algorithm. In such a framework, the equity and efficiency of the electricity market is likely to be easily assessed. However, simple bids increase the generators' risks associated with converting various elements of a complex cost structure, including incremental and fixed costs, into hourly pairs of quantity-price bids. There is no indication that the current complex bidding structure of the EPEW will be replaced by simple bids in the new electricity trading arrangements of England and Wales.

The remaining chapters of this thesis will focus on three major aspects of the general modelling of competitive electricity markets. Generation scheduling is the subject of Chapter 3 and Chapter 5; Chapter 4 concentrates on price computation mechanisms; and Chapter 6 provides an assessment of generators' bidding strategies.

CHAPTER 3

Generation Scheduling

3.1 Introduction

The determination of the most economical commitment schedule for thermal generating units, which satisfies all operating constraints, is regarded as one of the major power systems operation problems. The scheduling of generating units in a hydro-thermal power system brings additional difficulties because it requires a long-term forecast of the availability of water and a co-ordination between the hydro and thermal problems. In this chapter the generation scheduling problem will be restricted to thermal units.

Scheduling generating units over a short-term horizon is a complex, large-scale, mixed-integer, non-linear optimisation problem. This problem is referred to in the literature as the unit commitment (UC) problem [5], which has been the subject of a large number of publications [81]. To *commit* a generating unit means *turn it on*, that is to start it up, bring it up to speed, synchronise it to the system, and connect it so that it can deliver power to the network.

In the old vertically integrated structure of the power system sector, sub-optimal solutions of the UC problem were compensated in different ways sometimes making use of cross-subsidies. In the new era of electricity markets there is no scope for those mechanisms and therefore the quality of the solution of the UC problem is important to maintain the equity and efficiency of those markets. In a pool-based market, this problem is formulated and solved either by a centralised independent system operator (ISO) or by the market operator (MO).

Scheduling generating units is a simple optimisation problem when simple bids are offered. It can be performed by a simple solution method in which the individual bids are aggregated to form the total supply curve. Where this curve intersects the demand curve a clearing price is determined (see Fig. 3.1). All the blocks of power whose bid prices are lower than or equal to that clearing price are scheduled. Only a fraction of the last selected block will be dispatched. Adjustments are often required to find a solution that satisfies the operational constraints of the generating units.

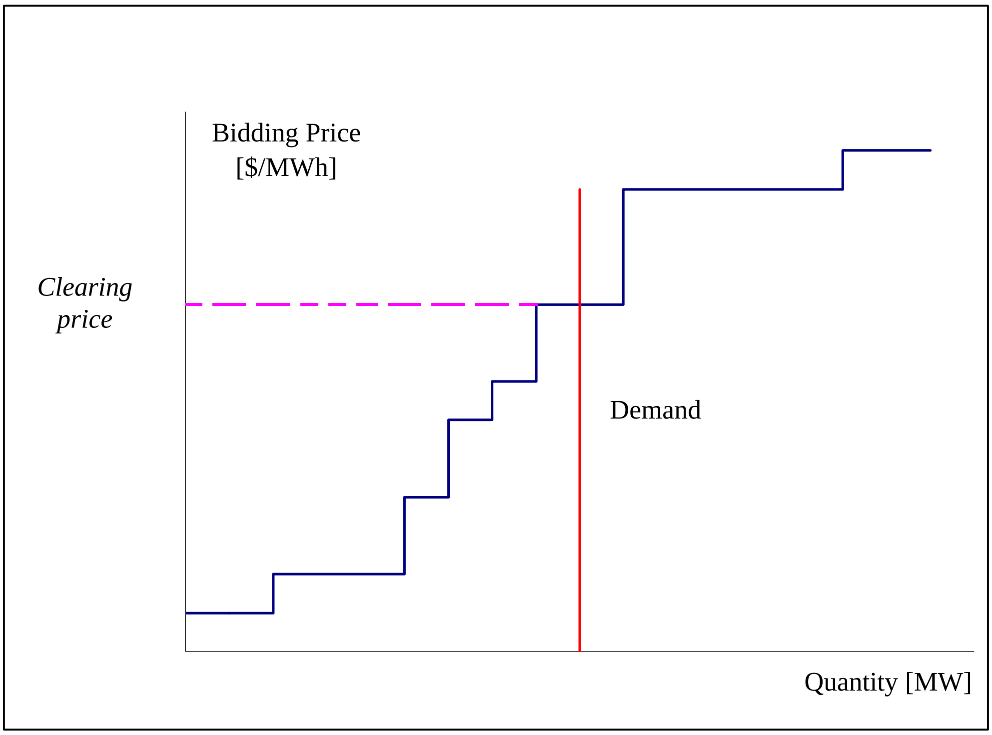

Fig. 3.1: Aggregate bidding price curve and forecasted demand to schedule generating units in a simple bidding framework

As discussed in the previous chapter, in several electricity markets the bids are composed of various parameters designed to represent the different elements of the cost of running a generating unit, including incremental costs and fixed costs (start-up and no-load costs). In contrast, in some other electricity markets the bids involve simple quantity and price for the energy offered.

The problem of scheduling thermal generating units in an electricity market with a complex bidding structure is the subject of this chapter. Throughout the chapter, the generating units' bidding prices are referred to as unit costs, bearing in mind that they are expected to represent the true production costs of the generators, but this is not a compulsory requirement imposed on generators.

This chapter reviews the basic features of some optimisation techniques currently available to solve the UC problem. It also presents a hybrid algorithm that combines the Lagrangian relaxation and the Dynamic Programming technique for scheduling thermal generating units in electricity markets.

3.2 Problem Formulation

The UC problem consists in minimising an *objective function* subject to a variety of *system constraints* and *unit constraints*. The objective function is usually non-convex and represents the total cost of producing electricity. The system constraints comprise demand and spinning reserve requirements in the classical UC problem. However, they can also incorporate transmission and environmental constraints in more generalised formulations. The unit constraints include all operational limitations of the generating units, such as minimum stable generation, maximum capacity, minimum up time, minimum down time, crew constraints and ramp rates. The list of system and unit constraints presented here is by no means exhaustive because each individual electricity market or power pool may impose different rules on the scheduling algorithm, depending on the structure of their power systems and market regulations.

3.2.1 Objective Function

The objective of the UC problem is to minimise the system operating costs, which is the sum of production, start-up and shut-down costs of all units over the entire study time span. Mathematically, the objective function, or the total operation cost of the system ($\mathsf{F}(\overline{P}, \overline{U})$), can be written as follows:

$$\mathsf{F}(\overline{P}, \overline{U}) = \sum_{t=1}^{T} \sum_{i=1}^{N} u_i^t [F_i(P_i^t) + S_i^t] \tag{3.1}$$

where:

P_i^t : output power of unit *i* at period *t* (MW);

$F_i(P_i^t)$: fuel cost of unit *i* when its output power is P_i^t (\$);

S_i^t : start-up price of unit *i* at period *t* (\$);

u_i^t : commitment state of unit *i* at period *t*:

(u_i^t =1: unit is on-line and u_i^t = 0 unit is off-line);

N: total number of generating units;

T: total number of scheduling periods;

Fuel cost functions may be represented by a polynomial cost functions, such as:

$$F_i(P_i^t) = a_i(P_i^t)^2 + b_i P_i^t + c_i \tag{3.2}$$

or as piece-wise linear cost functions such as the *Willans line* presented in Fig. 2.2, for which the following verifies:

$$F_i(P_i^t) = inc_i^1 P_i^t + nl_i^1 \qquad \text{"} \; P_i^{\min} \le P_i^t < e_i^1 \tag{3.3}$$

$$F_i(P_i^t) = inc_i^2 P_i^t + nl_i^2 \qquad \text{"} \; e_i^1 \le P_i^t < e_i^2 \tag{3.4}$$

$$F_i(P_i^t) = inc_i^3 P_i^t + nl_i^3 \qquad \text{"} \; e_i^2 \le P_i^t \le P_i^{\max} \tag{3.5}$$

The start-up costs are usually represented by exponential functions of the time that the units have been shut down ($X_{off,i}^t$), such as:

$$S_i^t = a_i + b_i[1 - \exp(\frac{-X_{off,i}^t}{t_i})] \tag{3.6}$$

The longer this period, the colder a unit is and the more expensive it is to start up. In (3.6), the crew start-up costs and the equipment maintenance costs, which are in part proportional to the number of start-ups, are integrated in the term a_i; the costs associated with the required fuel to start-up the unit from the cold condition are represented by the term b_i; and the unit cooling rate is expressed by the term t_i.

3.2.2 System Constraints

Demand requirements: as discussed in Chapter 2, the scheduling problem is assessed in the context of a one-sided electricity market where the demand is considered given and is determined using a load forecasting program by the market operator. It is common practice to divide the study time span into smaller time intervals (periods) of equal duration, in which the load demand is assumed constant. In the Electricity Pool of England and Wales a half-hourly interval is used. In such a market, the UC schedule should then provide the exact amount of power to balance the customers' demand (D^t) in every time interval. Hence

$$\sum_{i=1}^{N} u_i^t P_i^t = D^t \qquad \forall\ t = 1, \ldots, \mathrm{T} \qquad (3.7)$$

Spinning reserve requirements: *Spinning reserve* is the term used to describe the total amount of generation available from all units synchronised (i.e., spinning) on the system minus the load and losses being supplied. The UC schedule should guarantee that sufficient generating spinning reserve is available to ensure the reliable and secure operation of the power system during emergency conditions. This means that in the event of a contingency where one or more units are disconnected from the system, there must be enough reserve in the remaining units to make up for the generating outages. In the transitory period the loss of one or more units should not cause too big a drop in the system frequency, whereas in steady-state conditions there should be enough spinning reserve to satisfy the demand requirements [5]. The spinning reserve requirements (R^t) can be represented as the following:

$$\sum_{i=1}^{N} u_i^t P_i^{\max} \geq D^t + R^t \qquad \forall\ t = 1, \ldots, \mathrm{T} \qquad (3.8)$$

It is common practice among utilities to deterministically set the reserve requirements as equal to, or greater than, the maximum capacity of the biggest unit scheduled in one particular scheduling time. Other criteria for setting the reserve requirement are based on a percentage of the peak load or even as a hybrid function of the these two quantities [5].

While the deterministic criteria are easy to implement, they do not represent the stochastic nature of the operation of power systems and do not take into account the intrinsic reliability of the generating units. Probabilistic techniques for computing spinning reserve requirements, where a UC "risk index" determine the probability of failing to meet demand at some instant in time, have been proposed in [82]. The integration of the concept of a "risk index" with an LR-based UC program is described in [83], where it is proposed that the optimal risk index be determined by a balance between the cost of carrying a certain amount of reserve and the expected cost of energy not supplied. The first factor of the cost/benefit analysis is a by-product of the UC package while the second is simply the product of the expected energy not supplied by the value of lost load (*voll*). The work described in [83] relied on the LR-based UC developed as part of this project.

3.2.3 Unit Constraints

Generating limits: the generating units should only be scheduled to supply power within the limits set by their minimum stable generation and maximum capacity.

$$u_i^t P_i^{\min} \leq P_i^t \leq u_i^t P_i^{\max} \tag{3.9}$$

Minimum up time: once a generating unit is committed, it should remain on-line for a minimum period of time (T_i^{up}). Hence, considering the amount of time a unit has been running ($X_{on,i}^t$), the minimum up time constraint can be written as follows:

$$(X_{on,i}^{(t-1)} - T_i^{up})\,(u_i^{(t-1)} - u_i^t) \geq 0 \tag{3.10}$$

Minimum down time: similarly, once a generating unit is de-committed, it should not be re-committed before a minimum amount of time (T_i^{down}). Hence, considering the amount of time a unit has been off-line ($X_{off,i}^{t}$), the minimum down time constraint can be formulated as follows:

$$(X_{off,i}^{(t-1)} - T_i^{down})\,(u_i^t - u_i^{(t-1)}) \geq 0 \tag{3.11}$$

3.3 Optimisation Techniques

Several optimisation techniques have been proposed to solve the UC problem [81]. They can be categorised into three main groups: (*i*) mathematical methods, which include exhaustive enumeration, Dynamic Programming, integer and mixed programming, branch-and-bound methods, Lagrangian relaxation and augmented Lagrangian; (*ii*) heuristic methods, such as priority list, simulated annealing, and genetic algorithms; and (*iii*) artificial intelligent methods like artificial neural networks and expert systems. This section presents the basics of these techniques.

3.3.1 Mathematical Methods

The mathematical methods are the classical methods used to solve the UC problem. Each of them requires adequate modelling of the problem, that is, the capabilities of the algorithms have to be considered while formulating the system model.

3.3.1.1 Exhaustive Enumeration

Exhaustive enumeration is a natural approach to solve the UC problem: The optimal solution can be obtained by a complete enumeration of all feasible combinations of generating units' statuses, followed by the determination of the economic dispatch of each feasible solution. This approach requires the evaluation of (2^N - 1) combinations for each scheduling interval and $(2^N - 1)^T$ combinations for the scheduling time horizon when scheduling *N* generating units over *T* scheduling intervals. This, of course, leads to an enormous theoretical number of combinations. In practice, while the *system* and *unit* constraints of typical power systems significantly reduce that number, evaluating all possible combinations is not possible. In general, the major challenge of the UC problem for realistic power systems is this “curse of dimensionality”.

3.3.1.2 Dynamic Programming

The Dynamic Programming (DP) technique is a systematic multi-stage searching procedure that achieves the optimal solution without assessing all the possible combinations. The search of the optimum solution can start from the last time interval and proceed backward to the initial one. This procedure does not consider the previous history of the generating units and so cannot take into account the time dependent start-up costs and the minimum up and down time constraints. Alternatively, a forward search attempts to find the most economical schedule by starting at the initial time interval, accumulating operating costs and then backtracking from the last to the first time interval to trace the optimal schedule.

If the search for the optimal solution is done through the complete combination of generating unit statuses, then a total of $(2^N - 1) \times T$ states and $(2^N - 1)^2 \times T$ transitions must be assessed. Therefore, the DP technique has the advantage of reducing the dimensionality of the UC problem when compared to the method of complete enumeration. Considering that some states do not satisfy the demand and reserve constraints and that some transitions violate the minimum up and down constraints, the dimensionality of the UC problem is further reduced. Nevertheless, the DP technique still suffers from the “curse of dimensionality” as the size of the problem increases. Early works where the DP technique has been applied to the UC problem are reported in [84-88].

Several heuristics have been adopted to reduce the search space and hence the dimension of the UC problem. If a priority list is used to search for the optimal solution, only N states and N^2 transitions for each time interval are required to be examined [5]. In the truncated methodology a small part of the solution space is considered for the search of the solution. The units are normally grouped into three main categories: units that are automatically committed; the search range units; and excess units, which are not considered at all. The most used criterion to place the units in the above priority levels is the full-load average production cost [86, 87]. A variable window size approach is presented in [89, 90] and will be further discussed in Section 3.6.

3.3.1.3 Integer and Mixed Programming

The UC problem is a mixed-integer non-linear problem that involves the scheduling of generating units, sometimes called a "pure" commitment problem, and the search for the most economical allocation of the demand amongst the scheduled units – the economic dispatch problem. Based on the Benders' decomposition method, the UC problem can be divided into a pure integer non-linear commitment problem and a non-linear economic dispatch problem. The former can be solved by a branch-and-bound algorithm that reduces the solution space through discarding unfeasible subsets, whereas the latter can be solved by the Lagrangian methods. The application of such a mixed-integer programming technique to the UC problem is described in greater detail in [91-96].

3.3.1.4 Branch-and-Bound

The branch-and-bound (B&B) technique solves a discrete constrained optimisation problem by searching for the solution of a sequence of simpler problems derived from the original one. The search is organised via a branch-and-bound tree whose leaves correspond to all the feasible solutions. The B&B technique is based upon the idea of determining upper and lower bounds to the series of constrained versions of the optimisation problems, and then using these bounds to eliminate sets of possible solutions during the search of the near-optimum solution. In the search of the solution it is assumed that whenever the solution of a lower-bound problem is greater than a feasible solution or an upper bound to the original problem, it is not necessary to carry on evaluating the nodes below the respective lower-bound problem. The lower bound can be determined from a dual optimisation problem that can use Lagrangian relaxation, and the upper bound can be generated by selecting the most economical solution from a sequence of feasible solutions.

The application of the B&B technique to the UC problem requires the partitioning of the solution space into subsets. This allows the construction of a constraint tree whose nodes correspond to the unit constraints and in which moving down the tree corresponds to adding constraints. While the solution of the dual problems in the lower bound computation does not produce a feasible UC schedule, it provides valuable information that can be used to produce a very good UC schedule, helping the computation of an upper bound of the solution.

A more detailed explanation on the application of the B&B technique to UC problem can be found in [97-99]. More recently, the B&B techniques have been combined with constraint satisfaction techniques and with logic programming (CLP) to speed up the search of the optimum or near optimum solution of the UC problem [100].

3.3.1.5 Lagrangian Relaxation

The Lagrangian relaxation technique decomposes the UC problem into one dual and one primal subproblem that are solved independently, in an iterative process. It is a systematic and efficient method to schedule generating units in the short-term, but the sensitivity of the unit statuses to adjustments in the Lagrangian multipliers may cause the process to oscillate around the optimum solution. The LR technique may therefore produce sub-optimal solutions with unnecessary commitments or even fail to find a feasible solution within an acceptable number of iterations. Nevertheless, the Lagrangian relaxation technique has become almost the current industry standard for the solution of the UC problem as the size of scheduling problem, measured in terms of the number of generating units and the number of scheduling hours, increases [101]. The application of the Lagrangian relaxation technique to solve the UC problem is reported in many publications [83, 96, 101-110]. A more detailed description of the Lagrangian relaxation technique is provided in Section 3.4.

3.3.1.6 Augmented Lagrangian Relaxation

The augmented Lagrangian method introduces quadratic penalty terms associated with the demand constraints to the Lagrangian function to help improving the convergence. Those penalty terms are linearised around the solution obtained in the previous iteration to maintain the decomposition ability of the Lagrangian relaxation technique. Applications of the augmented Lagrangian method have been reported in [108, 111, 112].

3.3.2 Heuristic Methods

The main objective of the use of heuristics to handle the UC problem is to reduce the dimension of the original problem as much as possible without rejecting the optimal solution. However, this goal is not always achieved and hence the optimality of solution cannot be guaranteed, due to the incomplete search of the solution space.

3.3.2.1 Priority List

This method makes use of heuristics to arrange the generating units in a priority list, which is then used to sequentially commit the units such that the system demand is satisfied. The traditional economic index used to rank the units is the full-load average production cost. This index has been combined with a commitment utilisation factor, which is a quantitative measure of the effectiveness of each unit in supplying the system reserve requirements, to determine the priority commitment order in [113]. This priority list method seldom provides a satisfactory solution, as it does not examine the total solution space but a small subset of it. The rigorous application of this method is restricted to systems where the no-load costs are zero, the unit cost curves are linear from zero to full load, and the start-up costs are independent of the time the units are off-line [5].

3.3.2.2 Simulated Annealing

An analogy has been made between the UC problem and the process of annealing a metal. Annealing, physically, refers to the process of heating up a solid to a high temperature and subsequently cooling it by decreasing its temperature step by step. At each step the temperature is kept constant for a period of time long enough for the solid to reach thermal equilibrium. At equilibrium the solid can have many configurations (states), each corresponding to different spins of electrons and to specific energy level [114, 115].

Solutions of the UC problem are equivalent to states of the physical process and the cost of the solution is analogous to the energy of a state. The solution of the UC problem by a simulated annealing algorithm (SAA) is based on the idea of choosing a feasible solution at random and then finding a neighbour to this solution. The move to this neighbour solution is performed either if its cost is lower than the cost of the previous solution or if this state is associated with an acceptable probability. The probability of accepting a more expensive neighbour solution is decreased by a control parameter, which plays the role of the temperature in the annealing process. The core of the SAA is a good rule for finding a diversified and intensified neighbourhood to ensure that a large portion of the solution space can be explored. Choosing the initial value of the control parameter as well as determining its decrement function are also important aspects of the algorithm.

The simulated annealing algorithm is claimed (*i*) to be able to find a high quality solution that does not strongly depend on the choice of the initial solution; (*ii*) not to require complicated mathematical models of the UC problem; (*iii*) to be able to start from any given solution, therefore being able to improve a sub-optimal solution from another method; (*iv*) to converge to the optimal solution; and (*iv*) not to require large computer memory. Nevertheless, it has the disadvantage of requiring a long processing time, which might be reduced by parallel processing.

3.3.2.3 Genetic Algorithms

The UC problem has recently been dealt with by Genetic Algorithms (GA), which are search techniques based on principles inspired on the evolutionary theory of genetic processes of biological organisms. Holland is regarded as having first described the theoretical foundations of GA in [116]. The application of GA to solve the UC problem is reported in [117, 118]. The basic principle of GA is the survival of a population of solutions that evolve in time. Each solution, termed *chromosome* or *genotype*, is an encoded individual, and the evolution is based on the laws of natural selection (survival of the fittest) and genetic recombination (crossover and mutation) within the population.

Initially, a population of solutions, each of them encoded in binary strings representing a solution of the real problem, is generated randomly. Then, a *fitness* figure is assigned to each solution by a function that provides a measure of the quality of the solution. Subsequently, a stochastic sampling process is used to select pairs of solutions to generate *offspring* solutions through recombination. The likelihood of selecting a solution is proportioned to the fitness of the solution. Hence, high-quality solutions are more likely to be selected and therefore more likely to become *parents* of *offspring* solutions than are the low-quality solutions. The genetic-like operators used to generate *offspring* solutions are *crossover* and *mutation*. The former simply combines the parents' binary strings forming a new chromosome, which inherits characteristics from both parents; whereas the latter randomly alter the parents' binary strings, therefore allowing the *offspring* solutions to contain information not present within the population. When a population of *offspring* solutions with the same size as the initial population is generated, the latter is replaced by this new generation as a process of evolution. The convergence to the optimum or to a near-optimum solution is achieved after many generations.

When solving the UC problem for N generating units over T scheduling periods using GA, one solution can be represented by a matrix containing the statuses of the units. The operation schedule of each unit is represented by a binary string of T bits, in which the bit "1" denotes that the unit is on-line whereas the bit "0" denotes that the unit is off-line. The strings of the N units are then concatenated to form a *chromosome* of length N x T representing the respective solution of the UC problem. The fitness value of the solution is set as the sum of the total operation cost of the solution $F(P_i^t, u_i^t)$ as in (3.1) and the penalty function associated to the violations of system and unit constraints. The penalty function should be proportional to the magnitude of the violation in order to highlight the differences among unfeasible solutions. It should also be large enough to discourage the selection of unfeasible solutions.

One disadvantage of GA is that performing selection, mutation and crossover on chromosomes of large-scale power systems can become excessively time-consuming and the convergence to an optimal solution cannot be guaranteed. Since genetic algorithms work with populations, they are naturally adaptable to parallel processing where each individual in the population can be assessed by one processor.

3.3.3 Artificial Intelligence Methods

One of the main advantages of the artificial intelligence methods is that the complexity of the mathematical formulation of the UC problem can be avoided. However, the shortcoming of these methods is generally associated to the excessive computational resources that they require. With the advent of fast processors with larger memory, artificial intelligence methods have become promising and are still evolving. A brief description of the basic concepts of some of artificial intelligence methods and of their application to the UC problem is presented in the following sub-sections.

3.3.3.1 Artificial Neural Networks

Artificial Neural Networks (ANNs) have also been used to solve the UC problem [119-121]. They represent a new class of computing systems formed by hundred of thousands of simulated neurons, connected to each other in a similar way as the brain neurons are interconnected. Each neuron has multi-inputs from other neurons with assigned weights to represent the strength of the input connections. The output of a neuron is determined by a signal, which is proportional to the sum of all the inputs. The operation of an ANN is based upon the presentation of a set of inputs and a subsequent forward propagation of this information through the network. A multi-layer ANN comprises an input layer, an output layer and a number of hidden layers.

For the problem of scheduling N generating units over T scheduling periods, the input layer consists of T neurons and should be configured to adapt to a load demand profile. The neurons of the output layer form the output schedule in an N x T matrix format. The ANN should be trained to produce the corresponding scheduling pattern for any input load profile. The selected demand profiles represent typical operating scenarios of the power system under consideration and the corresponding UC schedule is obtained off-line by mathematical methods, which may include heuristic knowledge. Once the network is trained, the on-line operation would involve a sequence of simple arithmetic operations to generate the most economical UC schedule for the given demand load curve. If this solution is not feasible, it is used as an initial starting point for a near-optimum solution. The major advantage of using ANNs is that the previous knowledge for the solution of the UC problem and its behaviour in different circumstances can be used extensively for obtaining new solutions. Conversely, as an artificial intelligence system, ANNs are less efficient than mathematical programming in performing complicated calculations.

3.3.3.2 Expert Systems

Some research has been done to handle the UC problem by using a set of rules implemented in an expert system [122]. Expert systems are regarded as the implementation of an automation procedure for replacing or reducing the amount of human thinking in critical circumstances. Solutions of the various power system operation problems are usually determined based upon the human expertise of the power system operators as well as upon the mathematical programming techniques. In general, the experience and judgements of those experts can be formulated as a set of heuristic rules, which are applied by an inference engine to solve these problems according to the procedure that is followed by the human experts. Expert systems can then be used to support a mathematical programming technique by introducing a clever search within a small solution space to replace a blind search within a larger set of possible solutions. Therefore, they combine existing mathematical techniques with the knowledge of experienced power system operators and programmers to create a rule-based computational tool, which can be used to improve a previous UC schedule retrieved from a database. Not much work has been reported on the use of expert systems to solve the UC problem mainly because it is hard to develop an appropriate set of rules to search for the optimal solution of such an economic-related problem where the optimisation is performed with respect to complex constraints. The advantage of the expert systems though is that a new load profile slightly different from a previous one is not treated as new problem, but as one load pattern for which small modifications of the previous UC schedule is expected not to deviate significantly from the optimal schedule.

3.4 The Lagrangian Relaxation Technique

The Lagrangian relaxation method is based on a dual optimisation approach in which the primal cost function is augmented by the system (coupling) constraints through the Lagrangian multipliers to form the Lagrange function, or simply the *Lagrangian*. This dualisation procedure involves two separate optimisation problems that are solved in an iterative process. First, by taking an initial set of Lagrangian multipliers as constant, the system constraints are *relaxed* and the Lagrangian is minimised with respect to the other problem variables (production power and statuses of the units), subject to the unit constraints. Secondly, using the values of the problem variables, the dual solution is determined by the maximisation of the Lagrangian with respect to the Lagrangian multipliers. The convergence of the dual optimisation method can be measured by the relative size of the "gap" between the primal and dual solutions, henceforth called *relative duality gap*. For a convex problem with continuous variables, the relative duality gap becomes zero at the end of the iterative process [5].

The presence of integer variables (units' statuses) in the UC problem makes it clearly non-convex, and thus, the Lagrangian is not differentiable. Furthermore, even in not strongly convex problems the differentiability of the dual function cannot be guaranteed and the Lagrangian becomes unstable. Therefore, a subgradient algorithm is used to maximise the Lagrangian with respect to the Lagrangian multipliers, reducing the convergence rate of the dual optimisation [111]. Moreover, due to the non-convexity of the UC problem, the maximisation of the dual problem does not lead to the minimisation of the primal problem. Thus, the iterative search for the solution should be interrupted based on the value of the relative duality gap. During the solution of the UC problem for large-scale power systems, the duality gap does become quite small as the optimisation proceeds. In general, the final value of the duality gap tends to be smaller as the size of the problem increases.

In addition, due to the integer variables, it is very unlikely that the dual solution will satisfy the equality constraints (system demand requirements). Nevertheless, from a dual solution that does not satisfy the demand constraints but satisfies the spinning reserve requirements, a feasible solution can be generated by adjustments in the output of the generating units. This approach focuses on the determination of Lagrangian multipliers that generate solutions that satisfy the spinning reserve constraints rather than carrying out a rigorous maximisation of the dual function [123].

In the remainder of this section the main features of the Lagrangian relaxation method and the computational model developed in this project will be described in more details. This algorithm is illustrated in Fig. 3.2.

3.4.1 The Primal Problem

The minimisation of the system operating costs, as in (3.1), subject to the system and unit constraints, as in (3.2-3.11) is referred to as the primal problem. Some operating constraints, such as crew constraints, transmission limitations and ramp rates, are not included in the LR-UC developed in this project. Nevertheless, the most common features of the power system have been taken into account. Hence the primal problem involves the determination of the primal solution (***J***) as in the following:

$$\boldsymbol{J} = \min_{P_i^t,\, u_i^t} \mathbf{F}\,(\overline{P}, \overline{U}) \tag{3.12}$$

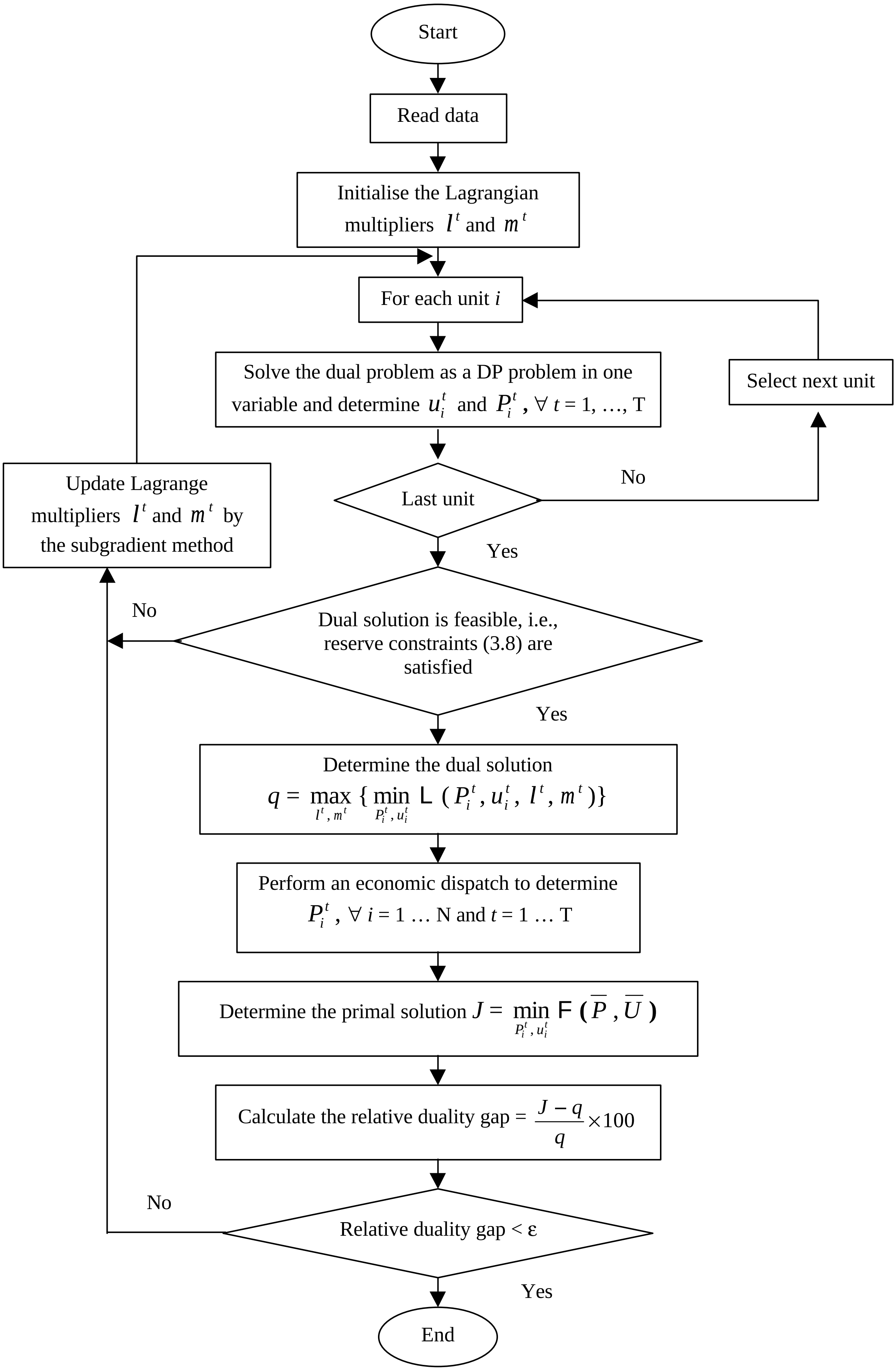

Fig. 3.2: The Lagrangian Relaxation algorithm

3.4.2 The Dual Problem

The system (coupling) constraints are incorporated into the objective function through the Lagrangian multipliers l^t and m^t to form the Lagrangian (L), as

$$\mathsf{L}(P_i^t, u_i^t, l^t, m^t) = \mathsf{F}(\overline{P}, \overline{U}) - \sum_{t=1}^{T} l^t \sum_{i=1}^{N} (u_i^t P_i^t - D^t) - \sum_{t=1}^{T} m^t \sum_{i=1}^{N} (u_i^t P_i^{\max} - D^t - R^t) \quad (3.13)$$

which can be rewritten as

$$\mathsf{L}(P_i^t, u_i^t, l^t, m^t) = \mathsf{F}(\overline{P}, \overline{U}) - \sum_{t=1}^{T} l^t \sum_{i=1}^{N} u_i^t P_i^t - \sum_{t=1}^{T} m^t \sum_{i=1}^{N} u_i^t P_i^{\max} + \sum_{t=1}^{T} l^t D^t + \sum_{t=1}^{T} m^t (D^t + R^t)$$

(3.14)

The dual procedure attempts to maximise the Lagrangian with respect to the Lagrangian multipliers l^t and m^t, while minimising it with respect to the other variables in the problem subject to the unit constraints. The dual problem is thus the search of the dual solution (q) expressed as

$$q = \max_{l^t, m^t} \{ \min_{P_i^t, u_i^t} \mathsf{L}(P_i^t, u_i^t, l^t, m^t) \} \qquad \forall\ l^t ³ 0 \text{ and } m^t ³ 0 \quad (3.15)$$

3.4.3 The Search for the Dual Solution

When the Lagrangian multipliers l^t and m^t are fixed, the last two terms of the Lagrangian (3.14) are constant and can be dropped from the minimisation problem. Hence, the system (coupling) constrains can be *relaxed* and the search for the dual solution can be done through the minimisation of the Lagrangian as:

$$\min_{P_i^t, u_i^t} \mathsf{L}(P_i^t, u_i^t, l^t, m^t) = \min_{P_i^t, u_i^t} \left(\sum_{t=1}^{T} \sum_{i=1}^{N} u_i^t [F_i(P_i^t) + S_i^t] - \sum_{t=1}^{T} l^t \sum_{i=1}^{N} u_i^t P_i^t - \sum_{t=1}^{T} m^t \sum_{i=1}^{N} u_i^t P_i^{\max} \right),$$

(3.16)

which can be rewritten as

$$\min_{P_i^t, u_i^t} L(P_i^t, u_i^t, l^t, m^t) = \min_{P_i^t, u_i^t} \sum_{i=1}^{N} \sum_{t=1}^{T} \{u_i^t[F_i(P_i^t) + S_i^t] - l^t u_i^t P_i^t - m^t u_i^t P_i^{\max}\} \quad (3.17)$$

Here, the goal of *separating* the units from one another is achieved. Hence, the dual problem can be solved separately for each generating unit, regardless of what happens to the other generating units. The minimum of the Lagrangian is then found by solving for the minimum for each generating unit over all time periods subject to the unit constraints (3.9-3.11); that is

$$\min_{P_i^t, u_i^t} \sum_{t=1}^{T} \{u_i^t[F_i(P_i^t) + S_i^t] - l^t u_i^t P_i^t - m^t u_i^t P_i^{\max}\} \quad (3.18)$$

At this point it is worthwhile to highlight one of the advantages of the LR technique: the size of the problem increases linearly, and not exponentially as in the DP algorithm, with the number of units. Therefore, the LR technique for solving the UC problem overcomes the dimensionality problem of the DP method, which is a time-consuming method not well suited for large-scale power systems due to the large number of states that must be tested in each scheduling period.

The dual problem is then easily solved as a dynamic-programming problem in one variable that can take two values, as shown in Fig. 3.3.

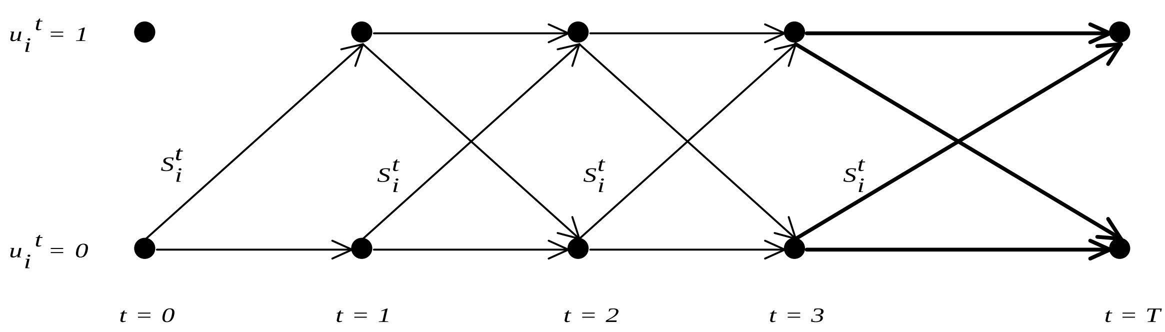

Fig. 3.3: Dynamic Programming problem of one variable

When the unit is off-line, i.e., u^t

and equal to zero. When the unit is on-line, i.e., $u_i = 1$, the function to be minimised is

$$\min_{\;i^t\;t} \;(\;_i^t,\;^t,\; l^t,\quad) = \min_{\;i^t\;t} \quad F_i(P_i^t) - l^t P_i^t - m^t P_i^{\max}) \tag{3.19}$$

Where the start-up cost S_i^t is dropped since the minimisation is with respect to P_i^t.

When the units' fuel cost functions are represented as polynomial functions as in (3.2), the minimum of (3.19) can be found by taking its first derivative.

$$\frac{\partial\,[F_i(P_i^t) - l^t P_i^t - m^t P_i^{\max}]}{\partial P_i(t)} = \frac{\partial F_i(P_i^t)}{\partial P_i^t} - l^t = 0 \tag{3.20}$$

Hence,

$$P_i^t = \frac{l^t - b_i}{2a_i} \tag{3.21}$$

If the values of the output power of each unit (P_i^t) obtained from (3.21), do not satisfy the generating limits as in (9), i.e., if they are below the unit's minimum stable generation or beyond its maximum capacity, they are set as follows:

- If $P_i^t < P_i^{\min}$ then $P_i^t = P_i^{\min}$ (3.22)

- If $P_i^t > P_i^{\max}$ then $P_i^t = P_i^{\max}$ (3.23)

However, if the units' fuel cost functions are represented by piece-wise linear cost functions as in (3.3-3.5), the minimisation of (3.19) is performed bearing in mind that the Lagrangian multipliers can naturally be interpreted as economic indicators. Hence, the output power of each unit (P_i^t) can be determined as follows:

- If $l^t < inc_i^1$ then $P_i^t = P_i^{\min}$ (3.24)

- If $inc_i^1 \le l^t < inc_i^2$ then $P_i^t = e_i^1$ (3.25)

- If $inc_i^2 \le l^t < inc_i^3$ then $P_i^t = e_i^2$ (3.26)

- If $inc_i^3 \le l^t$ then $P_i^t = P_i^{\max}$ (3.27)

The values of the system variables P_i^t, u_i^t, l^t, m^t are substituted back into the Lagrangian (3.13) to determine the dual solution (q) whenever the dual solution is feasible with respect to the spinning reserve constraints (3.8) and the following constraint regarding the minimum output power of the scheduled units is verified:

$$\sum_{i=1}^{N} u_i^t P_i^{\min} \le D^t \qquad \forall\, t = 1, \ldots, \mathrm{T} \tag{3.28}$$

3.4.4 Determining the Primal Solution

From a dual solution that satisfies the spinning reserve constraints, it is possible to generate a solution that satisfies the demand constraints by adjusting the output power generated by the scheduled units, without starting up additional units. Thus, whenever the spinning reserve constraints are satisfied, an economic dispatch algorithm is used to establish the exact power output of the scheduled units in MW to satisfy the demand constraints. The system variables u_i^t, l^t, m^t and the updated units' output P_i^t are substituted in (3.12) to compute the primal value (J) for the current iteration. The dual value is always smaller than the primal value and thus constitutes a lower bound for the system operating cost [98].

The economic dispatch problem can be expressed as the problem of determining the most economical set of output power of the units connected to the power system, i.e., the scheduled units, to supply the demand at one particular scheduling period. Mathematically speaking, the objective of the economic dispatch problem is to minimise the production cost of the scheduled units as follows:

$$\text{Min } g\,(P_i^t, u_i^t) = \sum_{i=1}^{N} u_i^t \; F_i(P_i^t) \tag{3.29}$$

Subject to the demand constraints and to the unit generating limits as expressed in (3.7) and (3.9) respectively.

The economic dispatch is part of the UC problem and, as such, should be performed for every single scheduling period during the iterative search for a feasible solution. Since this implies that the economic dispatch may have to be solved a very large number of times, a computationally efficient algorithm is required.

The lambda search [5] is an adequate method for solving the economic dispatch problem when the units' fuel cost functions are represented by polynomial functions as in (3.2). If equivalent linear representations can be derived from the polynomial cost functions, then linear programming techniques can also be used to perform the economic dispatch. Network flow programming has also been used as an alternative way to determine the most economical allocation of the system demand requirements amongst the scheduled generating units, whose cost functions are linearised [108].

For the purpose of this research, the units' fuel cost functions are represented by piece-wise linear cost functions as in Fig. 2.2 and thus the scheduled units are economically dispatched in increasing order of incremental costs [5]. This procedure goes as follows:

i. Dispatch the minimum stable generation $P_i^{\min}$ of all units scheduled to generate;

ii. Find the unit with the lowest incremental cost segment;

iii. Determine the total output of all scheduled units ($\sum_{i=1}^{N} u_i^t \; P_i^t$);

iv. If the demand is satisfied, i.e., if $\sum_{i=1}^{N} u_i^t \ P_i^t = D^t$, then stop. Otherwise, continue from step *v*;

v. Raise the output of the unit with the lowest incremental cost segment up to the right end of the segment at which the lowest incremental cost applies, or up to the point where the total output of all scheduled units equals the demand;

vi. Find the unit with the next lowest incremental cost segment, and go back to step *iii*.

Note that if there are two or more units with identical incremental costs they should be loaded equally.

To speed-up this procedure, a table giving each segment of the units' piece-wise linear cost function and its corresponding MW contribution is created (See Table 3.1). This table is then ordered in increasing order of incremental costs (first column) and searched from top down to successively dispatch the scheduled units until the demand is satisfied, after dispatching the minimum stable generation $P_i^{\min}$ of all units scheduled to generate.

Table 3.1: Incremental cost and MW contribution of piece-wise linear cost functions

Incremental costs (\$/MWh)	***MW contribution***
inc_1^1	$e_1^1 - P_1^{\min}$
inc_1^2	$e_1^2 - e_1^1$
inc_1^3	$P_1^{\max} - e_1^2$
...	...
inc_N^1	$e_N^1 - P_N^{\min}$
inc_N^2	$e_N^2 - e_N^1$
inc_N^3	$P_N^{\max} - e_N^2$

3.4.5 Checking for Convergence

The convergence of the LR algorithm can be measured by the *relative duality gap* between the primal and dual solutions.

$$\textit{Relative duality gap} = \frac{J-q}{q}\times 100 \tag{3.30}$$

The process stops when the relative duality gap is smaller than a pre-specified tolerance, or when a maximum number of iterations is reached. A relative duality gap of 0.5 % is acceptable for the accuracy of the solution of the UC problem.

The sensitivity of the integer variables corresponding to the generating unit statuses (u_i^t) to small adjustments in the Lagrangian multipliers may cause the algorithm to oscillate around the optimal solution. As such, there is no guarantee that the solution achieved in the last iteration of the iterative process will be feasible or optimal. Hence, in the computational model developed here, a running record of the feasible solutions is kept so that the final solution is the one corresponding to the most economical schedule, i.e., the one with the minimum primal solution (J).

3.4.6 Adjusting the Lagrangian Multipliers

If convergence is not achieved, the Lagrangian multipliers are adjusted by the subgradient method proposed in [123], which is based on the observation that when the solution of the decomposed problem satisfies the spinning reserve constraints it is easy to modify this solution to balance the system demand requirements.

The use of the subgradient method is mainly justified by its simplicity and by the fact that the subgradient of the dual function, i.e., the mismatches in the system demand and spinning reserve constraints, is easily determined. Slow convergence is one of the shortcomings of the subgradient method, but it can be improved by adequate initialisation of the Lagrangian multipliers and by proper choice of the heuristic coefficients in the adjustment procedure.

A good initial estimate of the Lagrangian multipliers can be obtained by an algorithm in which the units are dispatched according to the increasing order of their incremental costs. This is actually an economic dispatch algorithm, as described in Section 3.4.4, of all available units, assuming that their minimum output power are equal to zero. The initial values of the Lagrangian multipliers for the demand constraint l_0^t are set equal to the incremental cost of the last unit dispatched to satisfy the demand requirements of the corresponding scheduling period. The initialisation of the Lagrangian multipliers for the spinning reserve constraint is based on the mismatch of reserve provided by the dispatched units. The initial values of the Lagrangian multipliers for the spinning reserve constraint m_0^t are then determined by the following

$$m_0^t = \max \{ f \ (D^t + R^t - \sum_{i=1}^{N} u_i^t \ P_i^{\max}) \quad , 0\} \tag{3.31}$$

where u_i^t =1, if unit *i* is dispatched or u_i^t = 0, otherwise, and f is a system dependent constant, which is determined heuristically.

Alternative approaches to update the Lagrangian multipliers have been reported in the literature [124]. The variable metric method [125] takes into account that adjustments in a Lagrangian multiplier affect not only the corresponding subgradient vector. Hence the Hessian inverse matrix is approximated and multiplied by the subgradient vector. This new vector is then used to adjust the Lagrangian multipliers. Most recently, an optimal distance method, which is based on the Kuhn-Tucker optimality conditions, has been proposed [126]. The basic idea of this method is to update the Lagrangian multipliers trying to find the solution of the primal problem directly. To do so, a new function that represents the distance between the dual solution and the primal optimal solution is defined. The value of this function can be determined when the dual problem is solved and the Lagrangian multipliers are updated in order to nullify this function.

3.4.6.1 The Lagrangian Multiplier for the Demand Constraint

When the system spinning reserve constraints (3.8) are satisfied, the output power of each unit scheduled to generate is determined by the economic dispatch algorithm described in Section 3.4.4 to satisfy the system power balance constraints (3.7). Therefore there is a *marginal unit* for each scheduling period, which is the last unit loaded by the economic dispatch algorithm in the corresponding period, for which the *marginal cost*[1] (MC_k^t) is known. This marginal cost represents the cost to the system to supply an additional MW of demand in that period. In order to gradually bring the value of the Lagrangian multipliers for the demand constraint l^t closer to the marginal cost of generation during that period MC_k^t, the following modification is then made:

$$l_{k+1}^t = s \; l_k^t + (1 - s)\, MC_k^t \tag{3.32}$$

where a value of s = 0.6 has been shown heuristically to give good results.

Conversely, if the system spinning reserve constraints (3.8) are not satisfied, it is very likely that the system power balance constraints are also unsatisfied. This is so because no adjustments in the output of the units (P_i^t) have been made, and because equality constraints (3.7) are more difficulty to satisfy. In an attempt to produce a commitment schedule that satisfies the system constraints, the following modification is made:

$$l_{k+1}^t = \max \{ l_k^t + r_k \, [D^t - \sum_{i=1}^{N} u_i^t \, P_i^t] \quad , 0\} \qquad \text{where } r_k = \frac{1}{d + q\,k} \tag{3.33}$$

The scaling factor d and the tuning constant q are system dependent parameters that are determined heuristically.

[1] Different from the concept of *system marginal price* described in Chapter 2.

3.4.6.2 The Lagrangian Multiplier for the Reserve Constraint

The inequalities related to the spinning reserve constraints (3.8) do not impose an upper bound on the amount of reserve. Nevertheless, there should not be too much reserve because it would certainly increase the cost associated with the corresponding dual solution. Therefore, a slack term (e_k^t) is included in the reserve constraints to assess the quality of the dual solution. The upper-bound limit introduced by the approximation term restricts the solution space and therefore may prevent the optimal solution to be found. In addition, the value of the approximation term may affect the convergence of the process. Unfortunately, there is no mathematical guideline for properly selecting the value of slack term e_k^t [127]. For the first iteration the approximation term e_k^t is set at zero.

The Lagrangian multiplier for the reserve constraints m^t and the approximation term e_k^t are left unchanged when the spinning reserve constraint is satisfied within the approximation term e_k^t, i.e., when

$$D^t + R^t \le \sum_{i=1}^{N} u_i^t \; P_i^{\max} \le D^t \; + R^t + e_k^t \tag{3.34}$$

Conversely, when the system spinning reserve constraints are not satisfied to within e_k^t, the Lagrangian multipliers for the reserve constraints m^t and the approximation term e_k^t are updated as follows:

$$m_{k+1}^t = \max \{ m_k^t + r'_k \, [D^t + R^t + e_k^t - \sum_{i=1}^{N} u_i^t \; P_i^{\max}] \quad , 0\} \qquad \text{where } r'_k = \frac{1}{d' + q' k} \tag{3.35}$$

$$e_{k+1}^t = e_k^t + e \tag{3.36}$$

where, e is a constant of the order of magnitude of the maximum capacity of the smallest generating unit of the system, and the scaling factor d' and the tuning constant q' are system dependent parameters that are determined heuristically.

3.5 Application of the Lagrangian Relaxation Algorithm

The algorithm for the solution of the UC problem using the Lagrangian relaxation technique developed for this research project has been applied to several test systems of different sizes. The original polynomial price functions were replaced by piece-wise linear functions. The method used to linearise those functions is described in Appendix A. The results obtained and the effectiveness of the model are discussed in this section.

3.5.1 10-Unit System

The 10-unit system has total capacity equal to 3,125 MW, and peak load and minimum load equal to 2,000 MW and 1,140 MW, respectively. The data for this system were obtained from [84] and are shown in Appendix B. The maximum capacity of the largest unit (550 MW) is used as the spinning reserve requirement.

A relative duality gap of 4% indicated that the LR-based UC program failed to produce an acceptable sub-optimum solution for the 10-unit system. This was expected because it is well known that the smaller the system is, the worse is the solution obtained using the LR technique. The sub-optimal UC schedule is presented in Table 3.2 where the figures in bold show the differences between this schedule and the optimal UC schedule, which was obtained using a Dynamic Programming-based UC program developed in the early stages of this project.

Table 3.2: Sub-optimal UC schedule of the 10-unit system (1=on-line, 0=off-line)

Unit	***statuses from hour 0 to hour 24***																								
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*
U60	0	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**
U80	1	1	1	1	1	1	1	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
U100	1	1	1	1	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	1
U120	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	**1**	**1**	**1**	1	1	1	1
U150	1	1	1	1	1	1	1	1	1	1	1	1	**1**	**1**	**1**	**1**	**1**	**1**	1	1	1	1	1	1	1
U280	1	1	1	1	1	1	1	1	1	**1**	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
U320	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U445	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U520	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U550	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

The convergence report of the iterative process is presented in Table 3.3 where the figure in bold represents the total generators' production cost associated with the sub-optimal UC schedule.

Table 3.3: Convergence report of the LR-based UC scheduling algorithm applied to the 10-unit system

Dual solution	$ 76,970
Primal solution (minimum)	**$ 80,095**
Relative duality gap	4.0636%
Maximum dual solution within feasible solutions	$ 76,970
Minimum relative duality gap	4.0636%
Number of iterations	70
Number of feasible iterations	36
Iteration of the first feasible solution	4
Iteration of the maximum dual solution	70
Iteration of the minimum primal solution	67
Iteration of the minimum duality gap	67

3.5.2 26-Unit System

The 26-unit system is derived from the IEEE-RTS [128], of which data are summarised in Appendix B. The total available capacity of the 26 units is 3105 MW. Four different load levels, as shown in Fig. B.3, were simulated for this test system. The spinning reserve requirements were set equal to the maximum capacity of the largest unit, except during few hours of Load level 4, to comply with the total available capacity of the system.

Table 3.4 shows the convergence report of the iterative process where the figures in bold are the total generators' production cost associated with the corresponding sub-optimal UC schedule. The dual solution shown in the first row of Table 3.4 is the value obtained in the last iteration of the process, whereas the maximum dual solution presented in the next row is related to the feasible solutions only. Thus, the fact that the dual solution is higher than the maximum dual solution of Load level 3 indicates that the solution obtained in the last iteration (iteration 70) is not feasible. The minimum relative duality gap was set equal to 0.4%, therefore the simulation of the cases presented in Table 3.4 were interrupted when the maximum number of iteration (the sixth row of the table) was reached.

As mentioned in Section 3.4.5, the minimum primal solution is not necessarily obtained in the last iteration, as can be seen for load levels 1 and 2. This shows the importance of keeping a running record of the feasible solutions so as to adopt the one corresponding to the most economical schedule, that is the solution with the minimum primal solution. Additionally, the minimum relative duality gap is not necessarily achieved in the same iteration as the minimum primal solution, as shown again for load levels 1 and 2.

Table 3.4: Convergence report of the LR-based UC scheduling algorithm applied to the 26-unit system

	Load level 1	**Load level 2**	**Load level 3**	**Load level 4**
Dual solution	\$ 719,230	\$ 577,534	\$ 580,890	\$ 756,724
Maximum dual solution within feasible solutions	\$ 719,230	\$ 577,534	\$ 580,481	\$ 756,724
Primal solution (minimum)	**\$ 722,719**	**\$ 584.649**	**\$588,821**	**\$ 763,375**
Relative duality gap	0.4861%	1.2332%	1.4408%	0.8789%
Minimum relative duality gap	0.4852%	1.2321%	1.4408%	0.8789%
Number of iterations	60	97	70	76
Number of feasible iterations	18	43	23	10
Iteration of the first feasible solution	38	30	35	32
Iteration of the maximum dual solution	60	97	58	76
Iteration of the minimum primal solution	56	96	69	76
Iteration of the minimum duality gap	60	97	69	76

The behaviour of the dual and primal solutions during the LR iterative search of the optimum solution for the Load level 1 can be observed in Fig. 3.4, in which the upper bound value ($ 725,000) is a hypothetical figure to represent unfeasible solutions. The purpose of the trendline is to show the theoretical decreasing behaviour of the primal solution. The sensitivity of the UC schedule to small variations in the Lagrangian multipliers can be observed from the oscillation between feasible and unfeasible solutions in consecutive iterations.

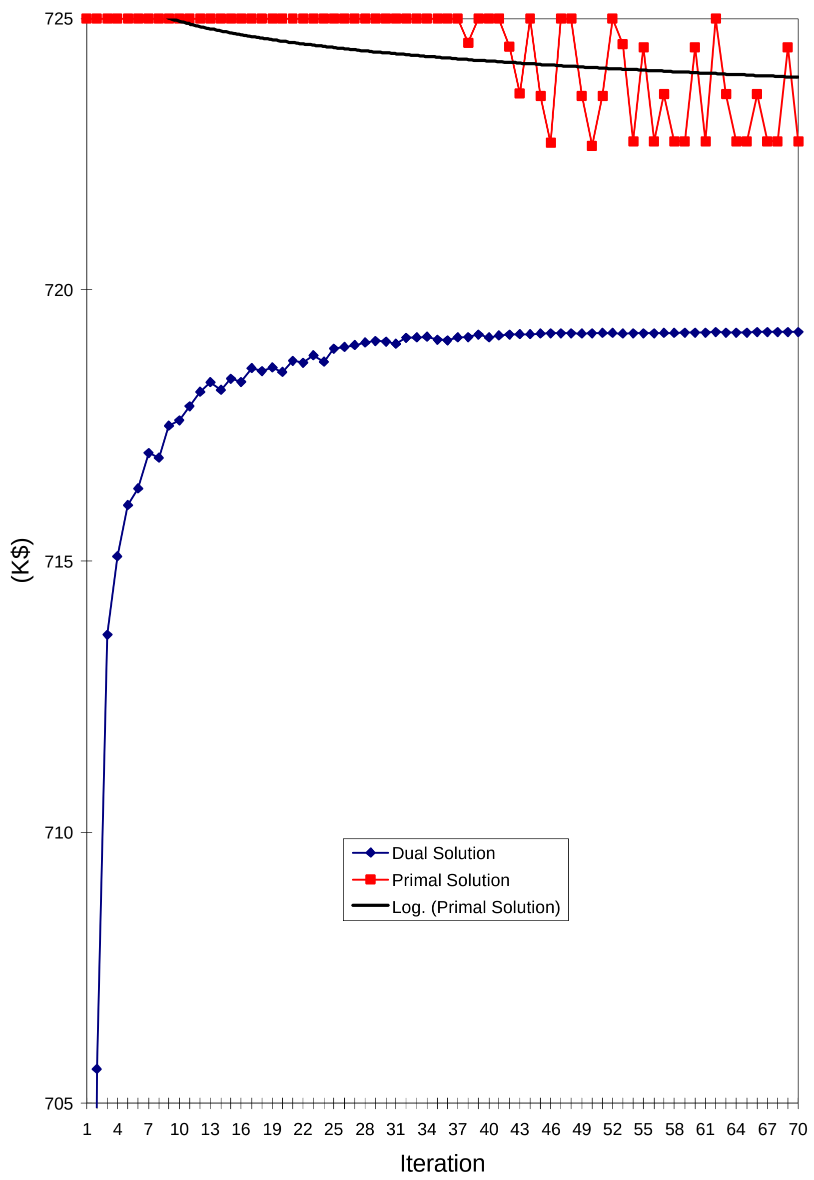

Fig. 3.4: Behaviour of the dual and primal solutions during the LR iterative search for the 26-unit system, Load level 1

3.5.3 110-Unit System

The 110-unit system has total capacity equal to 20,502 MW, and peak load and minimum load equal to 16,500 MW and 9,300 MW respectively. The data for this test system were collected from [118] and are presented in Appendix B. The maximum capacity of the largest unit (700 MW) is used as the spinning reserve requirement.

Table 3.5 shows the convergence report of the iterative process where the figure in bold is the total generators' production cost associated with the corresponding sub-optimal UC schedule. The sub-optimal UC schedule was obtained with a relative duality gap equal to 0.21%, which shows the high quality of the solution. However, it has been observed that a considerable excess of capacity is committed in the early hours of the scheduling period compared to the late hours of the scheduling day, as shown in Fig. 3.5.

Table 3.5: Convergence report of the LR-based UC scheduling algorithm applied to the 110-unit system

Dual solution	$ 3,744,195
Primal solution (minimum)	**$ 3,751,870**
Relative duality gap	0.2050%
Maximum dual solution within feasible solutions	$ 3,744,195
Minimum relative duality gap	0.2050%
Number of iterations	58
Number of feasible iterations	3
Iteration of the first feasible solution	4
Iteration of the maximum dual solution	58
Iteration of the minimum primal solution	58
Iteration of the minimum duality gap	58

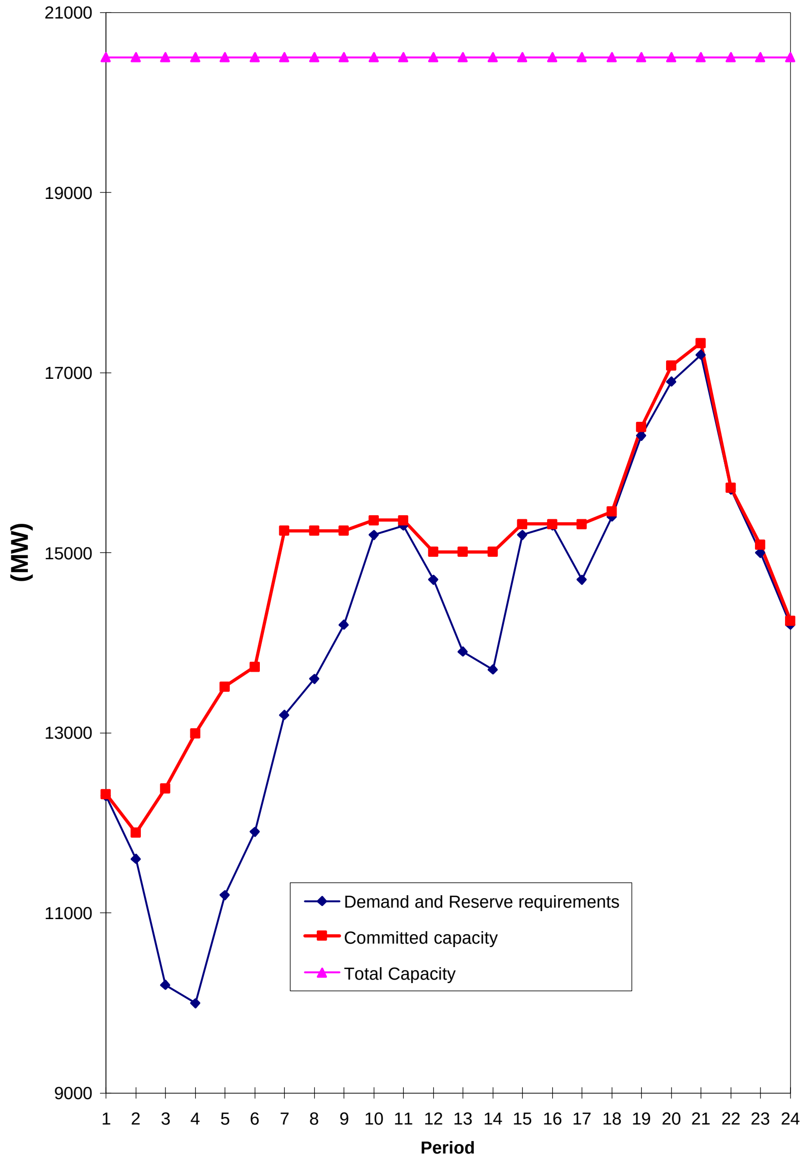

Fig. 3.5: Capacity committed and capacity requirements for the 110-unit system

3.6 A Hybrid Algorithm for Generation Scheduling

The implementation of competitive electricity markets has been raising some practical difficulties when trying to achieve the most economical operation of actual power systems. It has been argued that global schedules with negligible discrepancies in total production costs may introduce considerable changes in the total payments by the consumers, as the last acceptable bid sets the market-clearing price [6]. It has also been claimed that a centralised scheduling may be inequitable to some generation companies as their profit can be significantly reduced by a sub-optimal UC solution produced by the Lagrangian relaxation technique [6, 7].

In addition, the search of the LR UC schedule is very sensitive to small adjustments in the Lagrangian multipliers, especially when scheduling generating units with similar or identical cost functions. Therefore the LR iterative search can oscillate around the solution. It is also very likely that the final UC schedule obtained by using the LR technique contains over-commitments. Hence, the equity of an electricity market may be jeopardised by using the LR technique in a centralised scheduling system [109].

3.6.1 Review of Related Works

The idea of restricting the search by truncating the window in the DP approach is not novel. In [86, 87] the average incremental production cost is used to create a priority list, which is then used to define the search range.

Further improvements in the techniques to establish the variable window size were reported in subsequent publications. The application of linear programming to improve the feasible sub-optimal solutions obtained by the LR approach is reported in [127]. A revised economic dispatch routine incorporated as a post-processor in the LR algorithm to refine the sub-optimal solution by assessing unnecessary commitments has been proposed in [129]. In the above publications, some criteria are used to identify the inefficient units, which are committed in the original schedule and when de-committed would introduce savings in the total generators' cost, without violating the system constraints.

Intelligent systems and mathematical programming have been combined to solve the UC problem. In [120, 121] an artificial neural network is used to generate an initial schedule according to the input load profile, and then perform a limited search of the scheduling hours where the commitment states of some units are not certain.

3.6.2 The Combined Lagrangian Relaxation-Dynamic Programming Approach

A hybrid Lagrangian relaxation–Dynamic Programming (LR-DP) technique for scheduling thermal generating units in electricity markets is presented in this thesis. The solution of the UC problem is achieved in two steps. First, an LR-based unit commitment program determines a preliminary schedule that attempts to minimise the generators' production cost. A variable window size is defined for each time interval by classifying the units into *must-run, must-not-run* and *candidate* units. This is done by assessing the behaviour of the units whose statuses are uncertain during the iterative search of the LR-based solution, and by identifying the units that might have been over-committed in the preliminary schedule. A new schedule, which further improves the preliminary solution, is then obtained by using a DP-based algorithm as a post-processor. The criteria used to determine the variable window size described in this thesis can produce a global schedule where not only de-commitments but also commitments actually occur. This is a step forward compared to previous works reported in [120, 121, 127, 129], in which improvements over a preliminary schedule were restricted to the identification and thus de-commitment of unnecessarily scheduled units.

The idea behind this post-processor is to examine a number of iterations to the best solution obtained with the LR-based UC. To do this systematically, a DP-based algorithm has been adopted. This algorithm focuses on the scheduling hours where the LR-based UC appears to have difficulties deciding which solution is best and on hours where it may have scheduled too many units. A variable size window is therefore defined for each interval by classifying the units into *must-run*, *must-not-run* and *candidate* units based on their behaviour during the search of the LR-based solution and on the preliminary schedule itself. For each time interval, candidate units include:

Blinking units - units that are turned on and off in successive iterations during the solution process;

Small units - units that are on-line in the preliminary schedule and whose maximum capacities are smaller than the excess reserve for that particular time interval. Therefore if such units were de-committed, the system demand and reserve constraints would not be violated.

Pulsing units - units that are committed for very short time intervals (e.g., one hour) in the preliminary schedule.

At each hour, the *must-run* and *must-not-run* units are those which are respectively committed and not committed in the preliminary schedule and which are not candidate units.

When selecting the candidate units their operational constraints (e.g., minimum up and down time constraints) are taken into account to avoid generating unfeasible states in the DP algorithm. If a unit is classified as *must-run* in one time interval, the number of hours this unit has been running in the preliminary schedule is compared to its minimum up time. This will indicate how many time intervals the unit must be kept running before an attempt to shut it down is made. This unit then will not be classified as candidate but as *must-run* for those time intervals. Similarly, if a unit is classified as *must-not-run* in one time interval, the number of hours this unit has been off-line is compared to its minimum down time. This again will indicate how many time intervals the unit must be kept off-line before being started up. Then this unit will be classified as *must-not-run* for those time intervals.

The LR algorithm may sometimes face some difficulties to achieve the first feasible solution, depending upon the initialisation and adjustment of the Lagrangian multipliers. A slow convergence can also be verified for system whose generating units have similar cost functions. For each time interval, the *blinking units* are selected in decreasing order of the number of times they "blink" after the first feasible solution is achieved. The units that blink most often are assumed to be those more sensitive to small adjustments in the Lagrangian multipliers. It is likely that they are marginal units and that their cost curves are similar or even identical.

The *blinking units* can be on-line or off-line in the preliminary schedule. Hence the final schedule may have commitments of units that were previously not scheduled to generate. This post-processor, therefore, does not constitute a simple process of de-committing generating units from a schedule with over-commitments.

If the *small units* and the *pulsing units* are to be considered as candidate units, their contribution for the reduction of the total generators' cost will happen due to de-commitments because they are initially scheduled to generate in the preliminary schedule. As an upper limit of the number of candidate units is set, the *small units* and the *pulsing units* are selected in decreasing order of their full load average cost. It is likely that this criterion will produce higher savings in the total generators' cost.

The number of units in the window can vary from one time interval to another. The window during peak hours is likely to be smaller than the window for the hours of low demand.

The search for the solution by the DP-based post-processor is done by complete enumeration of the possible states in every scheduling period, i.e., the maximum number of states in a generic period *t* is $(2^{N_c^t} - 1)$, where N_c^t is the number of candidate units at period *t*. The number of strategies, or paths, saved in each step is also limited to $(2^{N_c^t} - 1)$.

3.7 Application of the Hybrid LR-DP Algorithm

The application of the hybrid LR-DP algorithm for scheduling generating units of three test systems is illustrated in this section. An upper limit of ten units is imposed on the window size to avoid an excessive increase in the computation time.

3.7.1 10-Unit System

The post-processor managed to determine the optimum UC schedule with no more than 4 units in a window, introducing considerable savings in the total generators' production cost (0.51 %). Table 3.6 shows that the savings introduced by the post-processor would amount to $ 150,380 on an annual basis.

Table 3.6: Generators' production costs (GC) of the 10-unit system

LR	Hybrid LR-DP	Savings	Annual Savings
$ 80,095	**$ 79,683**	0.51%	$ 150,380

Table 3.7 shows the optimal UC schedule obtained with the hybrid LR-DP algorithm, in which the underlined figures show the units that were selected as *candidate* units. The number of candidate units in each scheduling period is shown in the last row of the table. The figures in bold character represent the units whose status have changed from the LR UC schedule, presented in Table 3.2, to the optimal UC schedule by the post-processor.

Table 3.7: Optimal UC schedule of the 10-unit system (1=on-line, 0=off-line)

Unit	***statuses from hour 0 to hour 24***																								
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*
U60	0	**0**	**0**	**0**	**0**	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	**0**	**0**
U80	1	1	1	1	1	1	1	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
U100	1	1	1	1	**0**	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	1
U120	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	**0**	**0**	**0**	1	1	1	1
U150	1	1	1	1	1	1	1	1	1	1	1	1	**0**	**0**	**0**	**0**	**0**	**0**	1	1	1	1	1	1	1
U280	1	1	1	1	1	1	1	1	1	**0**	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
U320	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U445	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U520	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U550	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Candidate		1	2	4	4	4	3	4	4	4	4	4	4	4	4	4	4	4	4	3	4	1	3	3	3

The optimal UC schedule for such a small-scale test system can certainly be achieved by using a DP-based UC program, performing a full enumeration of the possible states in every scheduling period. Moreover, one may argue that some heuristics can be used to avoid assessing all possible combination of units' statuses. For example, the number of states to be tested can be considerably reduced by assessing the maximum capacity of the system, the hourly demand and reserve requirements and the units' maximum capacity. For this system, all units, except unit U60, can be considered as *must-run* units for period 1, considering that the maximum capacity of the system is 2,625 MW and the demand and reserve requirements in period 1 is 2,550 MW. This is truly a valuable information for stressed systems. However, it is not as useful for systems with plenty of spare capacity such as the 110-unit system.

The merit of the hybrid LR-DP algorithm is then related to its ability to systematically select few units as *candidate* units, considering the rest of them as *must-run* or *must-not-run* units, from the LR UC schedule. The DP-based post-processor is thus performed in a much faster and efficient way requiring much less computer resources. For example, when 4 units are considered as candidate units, only $(2^4 - 1) = 15$ states, instead of $(2^{10} - 1) = 1023$ in the full enumeration approach, are required to be assessed. Heuristics, such as the one discussed in the previous paragraph can be easily implemented in addition to the criteria used to determine the variable window size, as described in Section 3.6.2.

Other methods have been proposed to improve a preliminary UC schedule, but they tend to be restricted to the determination of inefficient units, which can be de-committed to reduce the generators' costs, without violating the systems' demand and spinning reserve constraints. These methods would never produce the optimal solution starting from the preliminary UC schedule presented in Table 3.2.

At this point it is worthwhile stressing one of the qualities of the hybrid LR-DP algorithm proposed in this project, which is its ability to identify an improved UC schedule in which a generating unit is scheduled to generate in one period when the same unit was off-line in an LR UC schedule.

One should remember that the production power of each selected unit is also part of the results of the UC problem. Table 3.8 shows the units' production power for the optimal UC schedule of the 10-unit system.

Table 3.8: Units' production power for the optimal UC schedule of the 10-unit system

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
U60																								
U80	60	60	60	60	60	60																60	60	60
U100	77	77	77																				77	77
U120	88	88	88	88	88	88	88	85	57												88	88	88	88
U150	150	150	150	150	150	150	150	117	98	83	83							83	83	150	150	150	150	150
U280	143	143	143	143	143	143	143	143											143	143	143	143	143	143
U320	187	187	187	187	187	187	187	187	187	187	159	182	122	120	120	120	182	187	187	187	187	187	187	187
U445	445	445	405	442	382	412	422	338	338	338	338	338	338	300	280	300	338	338	338	390	422	442	415	445
U520	450	430	430	430	430	430	430	430	430	402	340	340	340	340	340	340	340	372	409	430	430	430	430	440
U550	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400	400

3.7.2 26-Unit System

The results of the application of the hybrid LR-DP algorithm to the 26-unit system are summarised in Table 3.9. Again, the post-processor managed to produce schedules with lower generators' production cost for all load levels simulated. The effectiveness of the post-processor may be measured by its ability to improve sub-optimum solutions of such different qualities. In general, the higher the relative duality gap is, the higher is the efficiency of the post-processor. The solutions were obtained with a maximum of ten units in a window.

It is common practice to present the solution of the UC problem in a chart similar to Fig. 3.6, which shows the unit commitment and the output power of each selected unit of the 26-unit system, Load level 1.

The final UC schedule of Load level 2 is presented in Table 3.10, in which the digit "1" means that the unit is on-line and the digit "0" means that the unit is off-line. The underlined figures are the units selected as *candidate* units. It should be noticed that some units are off-line in the preliminary schedule but on-line in the improved schedule (e.g., unit U197a in hours 15, 16, 17 and 18). This highlights the ability of the LR-DP algorithm to produce a cheaper UC schedule in which commitments and not only de-commitments can occur.

Table 3.9: Generators' production costs (GC) of the 26-unit system

Load Level	**LR**	**Hybrid LR-DP**	**Savings**	**Annual Savings**
1	$ 722,719	**$ 722,049**	0.10 %	$ 244,550
2	$ 584,649	**$ 581,120**	0.60 %	$ 1,288,085
3	$ 588,821	**$ 586,588**	0.43 %	$ 815,045
4	$ 763,375	**$ 760,540**	0.37 %	$ 1,034,775

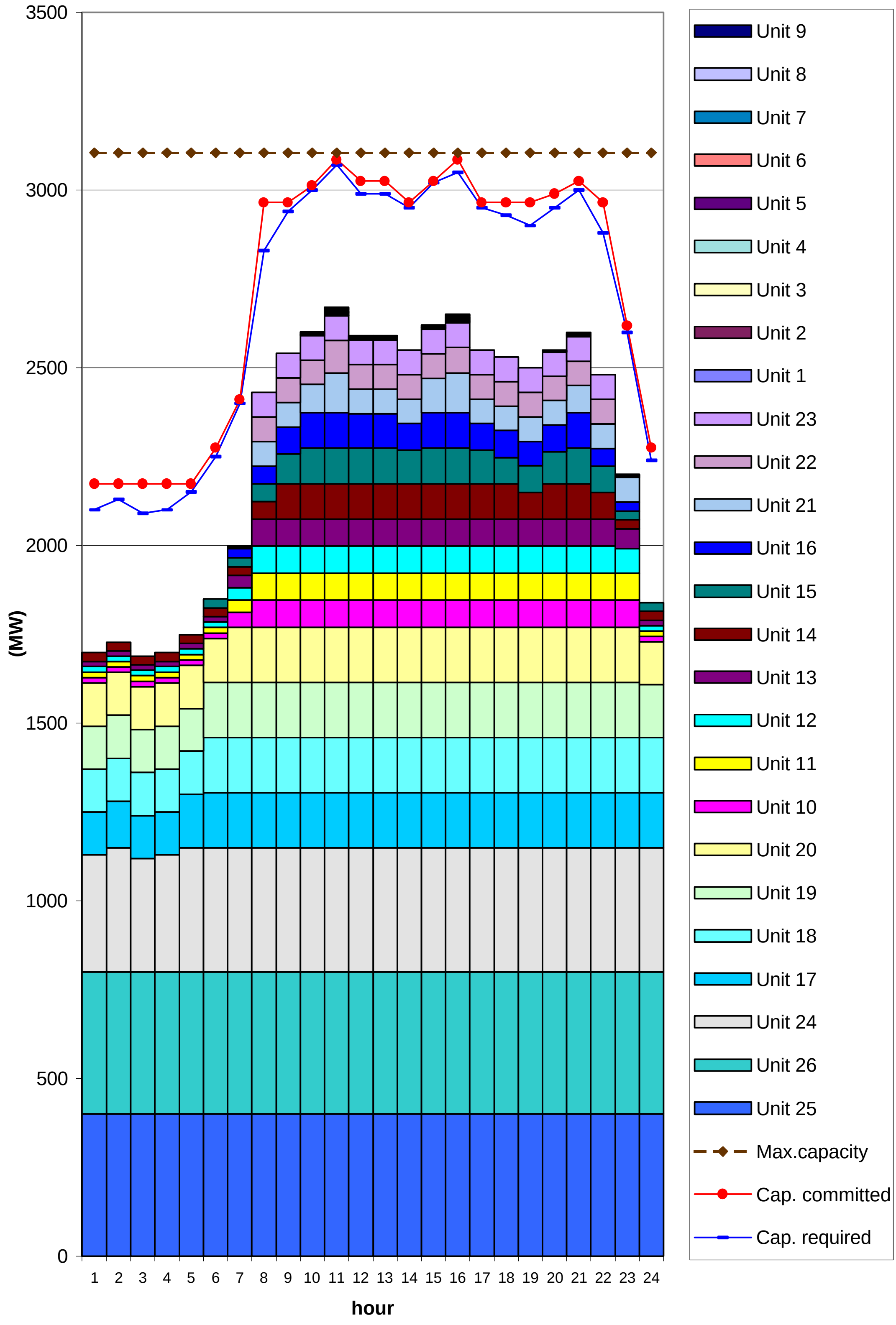

Fig. 3.6: UC Solution obtained by the hybrid LR-DR algorithm for the 26-unit system, Load level 1

Table 3.10: UC schedule obtained by the hybrid LR-DP algorithm for the 26-unit system, Load level 2

Unit	***statuses from hour 0 to hour 24***																								
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*
U12a	0	0	0	0	0	0	0	0	**0**	0	1	0	1	0	**0**	1	0	1	1	0	**0**	**0**	1	0	0
U12b	0	0	0	0	0	0	0	0	**0**	0	**0**	0	**0**	0	**0**	1	0	1	1	0	**0**	**0**	**0**	0	0
U12c	0	0	0	0	0	0	0	0	**0**	0	**0**	0	**0**	0	**0**	0	0	**0**	1	0	**0**	0	**0**	0	0
U12d	0	0	0	0	0	0	0	0	**0**	0	**0**	0	**0**	0	**0**	0	0	**0**	**0**	0	**0**	0	**0**	0	0
U12e	0	0	0	0	0	0	0	0	**0**	0	**0**	0	**0**	0	0	0	0	0	0	0	**0**	0	**0**	0	0
U20a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U20b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U20c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U20d	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U76a	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U76b	1	1	1	**0**	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U76c	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U76d	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U100a	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
U100b	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**0**	0
U100c	0	0	0	0	0	0	0	0	**0**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**0**	0
U155a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U155b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U155c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U155d	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U197a	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	**1**	**1**	**1**	**1**	1	1	1	**0**	**0**	0
U197b	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	**0**	**0**	**0**	**0**	1	1	1	1	**1**	0
U197c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U350a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U400a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U400b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

3.7.3 110-Unit System

Table 3.11 shows the savings introduced by the use of the hybrid Lagrangian relaxation-Dynamic Programming algorithm to the 110-unit system. The global UC schedule of this system is presented in Appendix D. The number of candidate units was limited to seven. The high quality of the preliminary sub-optimum solution can be measured by the small relative duality gap equal to 0.2 % (see Table 3.5). The post-processor, however, managed to further improve this solution introducing savings that would represent $ 398,580 on annual basis. The solution obtained by the hybrid LR-DP algorithm is 2.0 % cheaper than the solution obtained by a hybrid genetic algorithm reported in [118]. It is also 1.6 % cheaper than the solution obtained from the simulated competition algorithm presented in [130].

Table 3.11: Generators' production costs (GC) of the 110-unit system

LR	Hybrid LR-DP	Savings	Annual Savings
$ 3,751,870	**$ 3,750,778**	0.03 %	$ 398,580

3.8 Effects of the Improved UC Schedule on an Electricity Market

The applicability of the hybrid LR-DP algorithm to a competitive electricity market with a centralised scheduling entity is discussed in this section, where the improvement in final schedule is explored from the perspective of the customers' payments. The total customers' payments (PC) were calculated using the rules of the EPEW described in Section 2.4.1.4 and Section 2.4.1.5, however the feature regarding the adjustments of the pulsing was not considered. Hence,

$$PC = \sum_{t=1}^{T} smp^{t} \times D^{t} \tag{3.1}$$

Table 3.12 summarises the changes in the total customers' payments, or pool cost, resulting from the improvements in the solution of the UC problem.

Table 3.12: Total customers' payments for the hybrid LR-DP scheduling algorithm

		LR	Hybrid LR-DP	Difference
10-Unit system		118,552	96,062	- 19.0 %
26-Unit system	Load level 1	1,850,259	1,820,515	- 1.6 %
	Load level 2	1,249,915	1,257,824	+ 0.6 %
	Load level 3	1,302,614	1,223,685	- 6.1 %
	Load level 4	2,093,702	2,113,062	+ 0.9 %
110-Unit system		6,065,381	6,018,822	- 0.8 %

The results show that a minor improvement in the quality of the solution can lead to considerable changes in the customers' payments as the last acceptable bid sets the market-clearing price. In addition, it can be seen that a solution that is more economically attractive for the generators can be more expensive for the customers.

3.9 Discussion

In the traditional UC problem, the fixed costs of the generating units are allocated when they are incurred, in proportion to the units' output of the corresponding period and the minimisation is based on the idea of paying the units their actual bid prices, "*pay as bid*". The rules used to determine the unit prices and payments in the EPEW, described in Section 2.4.1.4, differ from the above criteria adopted by the scheduling algorithm. First, the payments are made on a market-clearing price basis and not on a "*pay as bid*" basis. In addition, in the price mechanism the fixed costs are amortised over continuous running periods rather being allocated on a single period. This contributes to the fact that the minimisation of the generators' production costs does not necessarily lead to the minimisation of the customers' payments. For example, let's assume that a unit out of the margin reduces its bids and consequently increases its market share. Let's also assume that the marginal unit loses part of its market share. Hence the prices of the marginal unit is likely to increase because its fixed costs will be amortised over a smaller quantity. Consequently, even though the total production costs decreases, the total customers payments is likely to increase. Therefore, the traditional scheduling problem is inadequate for simulating the operation of a competitive electricity market because it jeopardises its equity and efficiency.

3.10 Summary

This chapter has discussed the generation scheduling problem in a power system organised in a competitive market framework. Some techniques to solve the UC problem have been presented briefly and more attention has been given to the Lagrangian relaxation technique. A hybrid method that combines the Lagrangian relaxation and the Dynamic Programming techniques has been presented as an attempt to overcome some of the drawbacks of these techniques and to take advantage of their positive features. This proposed approach has been applied to several test systems of different magnitudes and the results were discussed. In general the efficiency of the combined LR-DP model has been proved to be satisfactory, introducing considerable savings over the total generators' production cost of the system. It has also been shown that an improved UC solution can considerably change the total customers' payments in a pool-based electricity market. The equity and efficiency of an electricity market, of which the scheduling problem is formulated as a standard UC problem, has also been discussed.

In this chapter, the rules of the EPEW have been used to calculate the unit prices and customers' payments. These pricing rules and alternative pricing mechanisms will be further discussed in Chapter 4. A re-dispatching algorithm that aims to reduce the total customers' payments is also presented in Chapter 4. An alternative generation scheduling algorithm that attempts to improve the equity and efficiency of electricity markets will be presented and discussed in Chapter 5.

The hybrid LR-DP computational algorithm developed for this research work has been included in a related work of scheduling the spinning reserve in large-scale power systems based on probabilistic criteria [83]. It has also been used in a project whose purpose was to assess how the short-term elasticity of the demand for electricity could be taken into account when scheduling generation and setting prices in a competitive electricity market [62].

CHAPTER 4

Auction Pricing Mechanisms

4.1 Introduction

In efficient electricity markets, prices should be cost-reflective and should provide economic signals for market participants and for prospective new entrants. In the Electricity Pool of England and Wales (EPEW) prices are determined by a centralised agent and payments are based on the system-clearing price and not on a "*pay as bid*" system. In addition, as explained in Chapter 2, prices in the EPEW are determined on the basis of complex bids that include not only a price for the energy produced by each generator but also the fixed costs associated with producing this energy, the no-load and start-up costs of the generators. The issue of pricing is made considerably more complex by these fixed costs. This chapter presents and discusses different ways to allocate these fixed costs in the pricing mechanism. It also shows how a re-dispatching algorithm can reduce the prices of the scheduled units, and consequently the market-clearing prices.

4.2 Allocation of Fixed Costs

In a traditional unit commitment, fixed costs are taken into account at the hour where they are incurred. In particular, start-up and shutdown costs are considered at the hour where the unit changes status and no-load costs are added at each hour the unit is committed. This approach is acceptable when the only item of interest is the total production cost. On the other hand, in a competitive environment, the allocation of these fixed costs and their influence on prices become problematic. One of the fundamental goals of competition in the electricity supply industry is indeed to reduce or eliminate cross-subsidies, i.e., to make each customer pay for the expenses that it causes in the system. In particular, it is the loads that cause the commitment of a particular generating unit that should pay the start-up costs of this unit.

Another important constraint is that, in a system based on complex bids, any pricing mechanism must ensure cost recovery, i.e. generators should never be forced to generate at a cost lower than the cost indicated by their bids. In other words, in a fair mechanism, the electricity prices are expected to allow the generating units to recover their costs based on their bidding prices.

In the pricing mechanism of the EPEW the fixed costs are allocated according to the Table A/B scheme, which attempts to produce lower prices of electricity during hours of low demand and higher prices during peak hours, therefore encouraging voluntary demand side management. However, because of the nature of the *Willans line*, prices during peak times may end up lower than prices during periods of low demand. In this chapter alternative schemes to allocate the generators' fixed costs in the pricing mechanism are proposed and discussed.

Without loss of generality, let's assume that unit i is scheduled to generate for N^{jm} continuous scheduling periods, starting at period j and running until period m. Hence, its production cost can be determined as follows:

$$[s_i^j + (nl_i^j \times pl) + (inc_i^j \times P_i^j \times pl)] + \ldots + [(nl_i^m \times pl) + (inc_i^m \times P_i^m \times pl)]$$

which can be rearranged as:

$$[s_i^j + \sum_{t=j}^{m}(nl_i^t \times pl)] + [\sum_{t=j}^{m}(inc_i^t \times P_i^t \times pl)] \tag{4.1}$$

Let's now assume that the start-up costs and accumulated no-load costs of this unit are amortised over a generic parameter k_i^t, in MWh. Hence the unit prices are as follows:

$$gp_i^t = inc_i^t + \frac{s_i^j + \sum_{t=j}^{m} (nl_i^t \times pl)}{k_i^t} \qquad \forall\ t = j, \ldots, m \tag{4.2}$$

If we assume that the price of unit *i* sets the market-clearing prices for the scheduling hours *j* until *m*, inclusive, the revenue of unit *i* will be:

$$[(inc_i^j + \frac{s_i^j + \sum_{t=j}^{m} (nl_i^t \times pl)}{k_i^j}) \times P_i^j \times pl] + \ldots + [(inc_i^m + \frac{s_i^j + \sum_{t=j}^{m} (nl_i^t \times pl)}{k_i^m}) \times P_i^m \times pl]$$

which can be rearranged as:

$$[(s_i^j + \sum_{t=j}^{m} (nl_i^t \times pl)) \times (\sum_{t=j}^{m} \frac{P_i^t \times pl}{k_i^t})] + [\sum_{t=j}^{m} (inc_i^t \times P_i^t \times pl)] \tag{4.3}$$

To ensure cost recovery for unit *i,* its production cost should be equal to its revenue. This can be achieved by making (4.1) equal to (4.3). Hence,

$$\sum_{t=j}^{m} \frac{P_i^t \times pl}{k_i^t} = 1 \tag{4.4}$$

Four fixed cost allocation schemes will be presented and discussed in the next subsections.

4.2.1 Scheme 1: Allocate the Fixed Costs when they are Incurred, in Proportion to the Units' Output of the Corresponding Period

The conventional way to take fixed costs into account when determining the generating unit prices is to allocate them to the periods in which they are incurred. In other words, the start-up cost is allocated only on the scheduling periods in which a unit is started-up, and the no-load costs are allocated in every period a unit is committed. This is the way the fixed costs are taken into account during the solution of the traditional UC problem.

The generating unit prices are thus by-products of the UC schedule and can be determined by simply dividing the units' hourly costs by its output in the corresponding scheduling period. Hence,

$$gp_i^t = inc_i^t + \frac{s_i^t + (nl_i^t \times pl)}{(P_i^t \times pl)} \qquad \forall\ t = 1, \ldots, \mathrm{T} \qquad (4.5)$$

The cost recovery for this cost allocation scheme can easily be verified. Following the same assumption of the previous section that unit *i* is scheduled to generate from period *j* to *m*, its revenue can be determined as follows:

$$[(inc_i^j + \frac{s_i^j + (nl_i^j \times pl)}{(P_i^j \times pl)}) \times P_i^j \times pl] + \ldots + [(inc_i^m + \frac{(nl_i^m \times pl)}{(P_i^m \times pl)}) \times P_i^m \times pl] \qquad (4.6)$$

which can be re-arranged as in (4.1). Thus, unit *i* is guaranteed to recover its bidding prices, which is assumed to reflect its operating cost.

4.2.2 Scheme 2: Amortise the Fixed Costs Only in Table A Periods, in Proportion to the Units' Total Output During those Periods (Table A/B Scheme)

As described in Section 2.4.1.2 the scheduling periods are classified as Table A or Table B periods depending on the amount of spare capacity scheduled at each period. The start-up cost and the accumulated no-load cost for continuous scheduling periods are amortised over the total output during Table A periods, and then added to the unit incremental price only during the Table A periods (refer to Section 2.4.1.3). Hence, re-writing (2.3) and (2.4) in accordance with the nomenclature of this chapter we have:

$$gp_i^t = inc_i^t + \frac{s_i^j + \sum_{t=j}^{m} (nl_i^t \times pl)}{\sum_{\substack{t=j \\ t \in \Omega_A}}^{m} (P_i^t \times pl)} \qquad \forall\ t \in \Omega_A \qquad (4.7)$$

$$gp_i^t = inc_i^t \qquad\qquad \forall\ t\ \text{Î}\ \Omega_B \qquad\qquad (4.8)$$

To verify the cost recovery for this cost allocation scheme, let's assume, again without loss of generality, that unit *i* is scheduled to generate for N^{jm} continuous scheduling periods, starting-up at period *j* and running until period *m*. Let's also assume that periods *k* to *l* inclusive are Table B periods, while the others are Table A periods. Hence,

$$\Omega_A = \{j, \dots, k\text{-}1, l+1, \dots, m\}$$

$$\Omega_B = \{k, \dots, l\}$$

By comparing (4.7) and (4.8) with (4.2), we obtain:

$$k_i^t = \sum_{\substack{t=j \\ t \in \Omega_A}}^{m} (P_i^t \times pl) \qquad\qquad \forall\ t\ \text{Î}\ \Omega_A \qquad\qquad (4.9)$$

$$k_i^t = \infty \qquad\qquad \forall\ t\ \text{Î}\ \Omega_B \qquad\qquad (4.10)$$

Substituting (4.9) and (4.10) into the left-hand side of (4.4) we obtain:

$$\frac{P_i^j \times pl}{\sum\limits_{\substack{t=j \\ t \in \Omega_A}}^{m} (P_i^t \times pl)} + \dots + \frac{P_i^k \times pl}{\infty} + \dots + \frac{P_i^l \times pl}{\infty} + \dots + \frac{P_i^m \times pl}{\sum\limits_{\substack{t=j \\ t \in \Omega_A}}^{m} (P_i^t \times pl)} = 1 \qquad\qquad (4.11)$$

If alternatively we assume that all periods from *j* to *m* are Table B periods, then the cost recovery does not verify because the substitution of (4.10) into the left-hand side of (4.4) yields to:

$$\frac{P_i^j \times pl}{\infty} + \ldots + \frac{P_i^m \times pl}{\infty} \; {\scriptstyle 1} \; 1 \qquad (4.12)$$

Hence, the generating units scheduled to generate only during Table B periods would need a side payment to recover their bidding prices. In the EPEW, this side payment is called Table B start-up payment, as described in Section 2.4.1.5.

4.2.3 *Scheme 3: Amortise the Fixed Costs Over Continuous Running Periods, in Proportion to the Units' Total Output During those Periods*

Fixed costs are amortised over every single period a unit is scheduled, in proportion to its total power output during the continuous running period. No distinction is made amongst the scheduling periods. This scheme is thus a variation of the previous Scheme 2, i.e., it is the particular case in which all periods are classified as Table A periods. Hence, the unit prices are determined as follows:

$$gp_i^t = inc_i^t + \frac{s_i^j + \sum_{t=j}^{m} (nl_i^t \times pl)}{\sum_{t=j}^{m} (P_i^t \times pl)} \qquad \forall\ t = 1, \ldots, \mathrm{T} \qquad (4.13)$$

The cost recovery condition is again easily checked. By comparing (4.13) with (4.2), we obtain:

$$k_i^t = \sum_{t=j}^{m} (P_i^t \times pl) \qquad \forall\ t = j, \ldots, m \qquad (4.14)$$

Substituting (4.14) into the left-hand side of (4.4) we obtain:

$$\frac{P_i^j \times pl}{\sum_{t=j}^{m}(P_i^t \times pl)} + \ldots + \frac{P_i^m \times pl}{\sum_{t=j}^{m}(P_i^t \times pl)} = 1 \tag{4.15}$$

4.2.4 Scheme 4: Amortise the Fixed Costs Over Continuous Running Periods, in Proportion to the Total Demand During those Periods And in Inverse Proportion to the Units' Market Share

Similarly to Scheme 3, the fixed costs are amortised over every hour a unit is scheduled. However, a more complex quantity is used to amortise the fixed costs. It takes into account the accumulated demand of the continuous running period and the unit's market share in each period, i.e., the ratio between the energy produced by the unit and the demand of the corresponding period.

$$gp_i^t = inc_i^t + \frac{s_i^j + \sum_{t=j}^{m}(nl_i^t \times pl)}{\sum_{t=j}^{m} D^t} \times \frac{D^t}{(P_i^t \times pl)} \qquad \forall\ t = 1, \ldots, \mathrm{T} \tag{4.16}$$

According to (4.16), the larger the share of the market a unit wins, the lower is its price.

The cost recovery is again easily checked. By comparing (4.16) and (4.2), we obtain:

$$k_i^t = \frac{(P_i^t \times pl) \times \sum_{t=j}^{m} D^t}{D^t} \qquad \forall\ t = j, \ldots, m \tag{4.17}$$

Substituting (4.17) into the left-hand side of (4.4) we obtain:

$$\frac{P_i^j \times pl}{\dfrac{(P_i^j \times pl) \times \sum_{t=j}^{m} D^t}{D^j}} + \ldots + \frac{P_i^m \times pl}{\dfrac{(P_i^m \times pl) \times \sum_{t=j}^{m} D^t}{D^m}} = 1 \tag{4.18}$$

4.3 Application of the Fixed Cost Allocation Schemes

The four fixed cost allocation schemes described in the previous section were applied to the test systems presented in Appendix B. This section discusses the numerical results.

4.3.1 4-Unit System

Table 4.1 shows the standard UC solution of the 4-unit system and the production cost associated with this solution. The unit prices (gp_i^t) and the customers' payments (PC) when the fixed costs are allocated according to schemes 1, 2, 3 and 4 are presented in Table 4.2, Table 4.3, Table 4.4 and Table 4.5, respectively. The prices in bold character are the market-clearing prices (smp^t). In the Table A/B scheme, hours 1 and 3 were classified as Table B periods, whereas hour 2 is a Table A period.

Table 4.1: UC solution for the 4-unit system

Unit	**Output (MW)**			**Production Cost ($)**
	hour 1	hour 2	hour 3	
1	740	740	740	30,360
2	-	340	160	7,700
3	60	70	-	2,459
4	1000	1000	1000	0
Total	1800	2150	1900	40,519

Table 4.2: Prices for the 4-unit system when fixed costs are allocated using Scheme 1

Unit	gp_i^t **($/MWh)**			**Total**
	hour 1	**hour 2**	**hour 3**	**Revenue ($)**
1	14.22	13.41	13.41	*39,580*
2	-	15.42	**15.36**	*8,291*
3	**20.97**	**17.16**	-	*2,459*
4	0	0	0	*53,486*
PC **($)**	37,740	36,888	29,189	103,817

Table 4.3: Prices for the 4-unit system when fixed costs are allocated using Scheme 2

Unit	gp_i^t **($/MWh)**			**Total**
	hour 1 (B)	**hour 2 (A)**	**hour 3 (B)**	**Revenue ($)**
1	13.00	15.03	13.00	*37,719*
2	-	16.15	**13.80**	*9,984*
3	**14.30**	**22.87**	-	*2,459*
4	0	0	0	*50,971*
PC **($)**	25,740	49,174	26,220	101,134

Table 4.4: Prices for the 4-unit system when fixed costs are amortised using Scheme 3

Unit	gp_i^t **($/MWh)**			**Total**
	hour 1	**hour 2**	**hour 3**	**Revenue ($)**
1	13.68	13.68	13.68	*39,930*
2	-	15.40	**15.40**	*8,895*
3	**18.92**	**18.92**	-	*2,459*
4	0	0	0	*53,231*
PC **($)**	34,048	40,668	29,260	103,976

Table 4.5: Prices for the 4-unit system when fixed costs are amortised using Scheme 4

Unit	gp_i^t **(\$/MWh)**			**Total**
	hour 1	**hour 2**	**hour 3**	**Revenue (\$)**
1	13.62	13.74	13.66	*39,936*
2	-	15.05	**16.15**	*9,032*
3	**18.86**	**18.97**	-	*2,459*
4	0	0	0	*53,968*
PC **(\$)**	33,943	40,776	30,677	105,395

The main objective of the application of the different fixed cost allocation schemes to such a small system is to clarify some complex issues regarding the definition of the pricing mechanisms in electricity markets. An important aspect of this problem is that the profitability of the generating units and the costs to the customers' can be considerably affected by different schemes. For example, the Table A/B scheme produces the smallest total customers' payments ($ 101,134). Moreover, it is the most convenient cost allocation scheme for Unit 2, which receives payments equal to $ 9,984. However, it is the least advantageous scheme from the perspective of Unit 4, of which revenue is $ 50,971. In addition, the profile of the market-clearing prices can vary significantly as can be seen in Fig. 4.1.

One should notice that the payments received by Unit 3 are the same ($ 2,459) whichever cost allocation scheme is used. This happens because Unit 3 sets the market-clearing prices in all periods that it is scheduled to generate in all cases. The cost recovery of Unit 3 is then ensured because its production costs equals its revenue in all cases. Since the marginal unit recovers its costs, so do the other units.

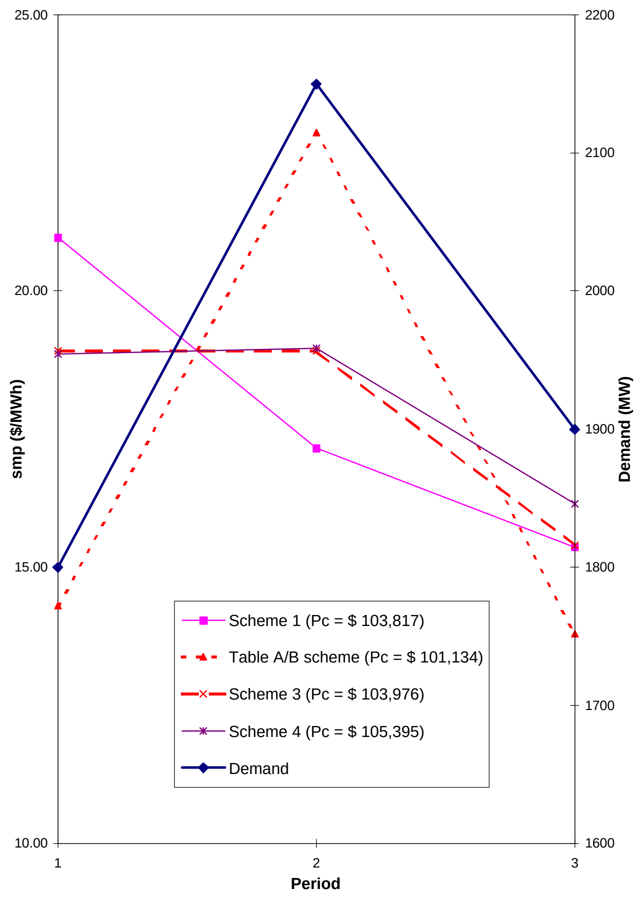

Fig. 4.1: Market-clearing prices of the 4-unit system for different cost allocation schemes

Table 4.6 presents the results of the application of the Table A/B scheme in an additional hypothetical situation in which hour 2 is also classified as a Table B period. The results of the others schemes would not change. However, in the Table A/B scheme, the market-clearing price in hour 2 is reduced and hence the revenue of all units committed in that hour. The payments received by Unit 3 in this situation are not enough to cover its production costs. In the current trading arrangements in the EPEW such situation is covered by a side payment made only to Unit 3.

Table 4.6: Prices for the 4-unit system when fixed costs are allocated as in Table A/B scheme and hour 2 is also a Table B period

Unit	gp_i^t **($/MWh)**			**Total**
	hour 1 (B)	**hour 2 (B)**	**hour 3 (B)**	**Revenue ($)**
1	13.00	15.03	13.00	*32,742*
2	-	**16.15**	**13.80**	*7,699*
3	**14.30**	14.30	-	*1,989*
4	0	0	0	*50,971*
PC **($)**	25,740	49,174	26,220	101,134

4.3.2 10-Unit System

The profiles of the market-clearing prices of the 10-unit system for the four fixed cost allocation schemes are presented in Fig. 4.2, Fig. 4.3, Fig. 4.4, and Fig. 4.5. The figures also show the demand plus reserve requirements, the committed capacities and the total capacity of the system. Table 4.7 shows the total customers' payments, or pool cost, (PC) of the 10-unit system for all fixed cost allocation schemes. The total generators' production cost (GC) for the UC solution is $ 79,683, as shown in Table 3.6.

The allocation of the generators' fixed costs in the actual periods they are incurred, in proportion to the units' output of the corresponding period (Scheme 1) produces price spikes in the hours a generating unit is committed. This is mainly a consequence of the concentration of the start-up costs in only one period. This effect is worsened when the unit that sets the market-clearing price delivers a small amount of energy in that period. The volatility of the electricity spot prices when this scheme is used can be observed in Fig. 4.2. In addition, the total customers' payments produced by Scheme 1 is considerably higher than the customers' payments when the other schemes are used. These flaws would prevent the implementation of this Scheme 1 in a real electricity market.

It has also been observed that the unit price tends to reduce as the output increases. This finding cannot easily be generalised because while an increase in the production power may contribute to reduce the no-load cost and thus the second term of (4.5), it also can lead to an increase of the incremental cost. Then, due to the above combination, the unit price can either increase or decrease as the unit increases its production power.

As discussed in Section 2.4.1.3, the second term of (4.7) can be negative because the no-load prices (nl_i^t) can be negative. Hence, the market-clearing price (*smp*) in a Table A period can actually be lower than the *smp* in a Table B period. This can be observed in Fig. 4.3 for the Table A/B scheme, where the electricity prices during Table B period 11 is higher than the prices in the Table A periods 10 and 12. Prices during Table B periods 14, 15 and 16 are also higher than the prices during Table A periods 13 and 17. This is in disagreement with the theoretical purpose of this price mechanism, which aims to encourage lower prices when there is plenty of spare capacity and higher prices when the system is stressed, thus encouraging demand side management. Additionally, it can be observed that while demand decreases from period 10 to 11, the market-clearing price increases. The same happens between periods 13 and 14. On the other hand, whereas demand increases from period 16 to 17, the electricity prices decreases. The Table A/B scheme is thus providing an incorrect price signal for market participants, even though this scheme produces the smallest total customers' payments compared to the other three schemes. Moreover, the Table A/B scheme requires side payments to guarantee the cost recovery of generating units that are scheduled to generate only during Table B periods.

An empirical analysis of pool prices for the period April-June 1992, examining the relationship between the level of demand and the *smp* for both Table A and Table B periods, is presented in [65]. The authors found that the *smp's* in Table B periods are generally below the costs of generation and is approximately constant over all levels of demand, while in Table A periods, the *smp's* are more scattered and contain much higher values. Their empirical examination for the related period revealed that *smp* is an unpredictable function of demand.

The cost allocation schemes 3 and 4 do not have the similar drawbacks albeit they yield to total customers' payments slightly higher than the Table A/B scheme does. As shown by Fig. 4.4, and Fig. 4.5, the pattern of the market-clearing prices when these schemes are used follows the demand profile.

Table 4.7: Total customers' payments of the 10-unit system

	PC (\$)
Scheme 1	101,256
Scheme 2	**96,062**
Scheme 3	96,276
Scheme 4	96,066

As discussed in Section 2.4.1.4, the fixed costs of pulsing units are amortised over the total capacity of the units rather than over their total output power in the Table A/B scheme of the EPEW. This mechanism is an attempt to reduce the volatility of the spot prices of electricity. It also reduces the total customers' payments: for the 10-unit system these payments drops from \$ 96,062 to \$ 95,395. This artificial mechanism, however, demands a side payment to guarantee cost recovery to the marginal unit, and hence it may compromise the efficiency and equity of the electricity market. Therefore, this intriguing feature of the EPEW will not be incorporated in the fixed cost allocation schemes presented in this chapter.

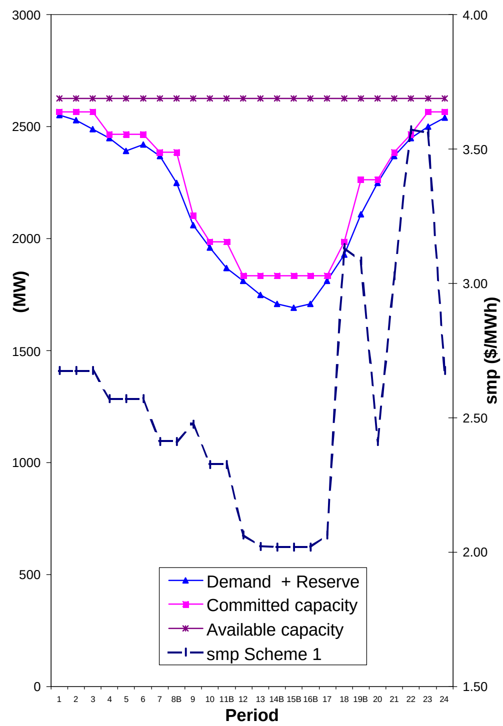

Fig. 4.2: smp, demand, reserve and capacities of the 10-unit system when fixed costs are allocated as incurred

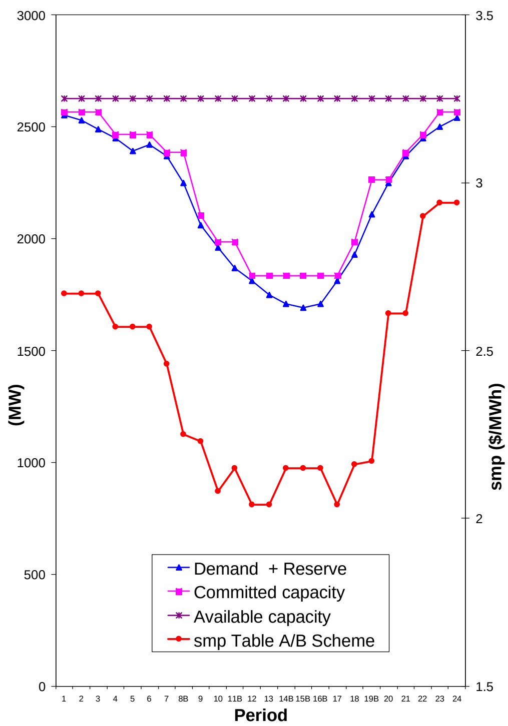

Fig. 4.3: smp, demand, reserve and capacities of the 10-unit system when the fixed costs are allocated as in the Table A/B scheme

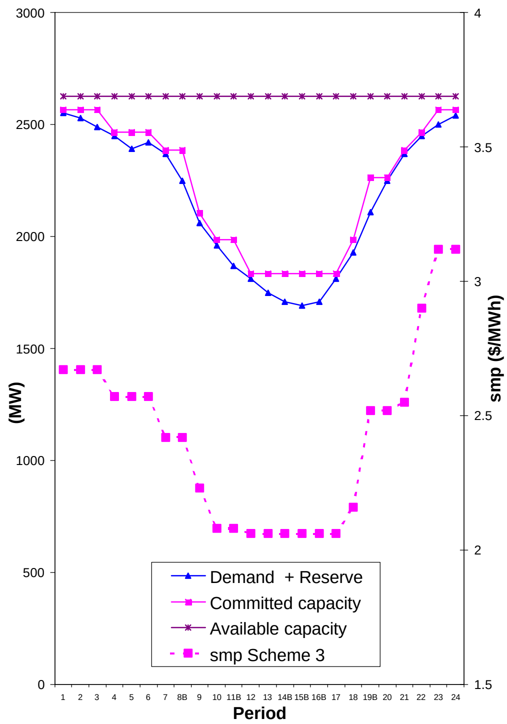

Fig. 4.4: smp, demand, reserve and capacities of the 10-unit system when fixed costs are amortised in proportion to the total output

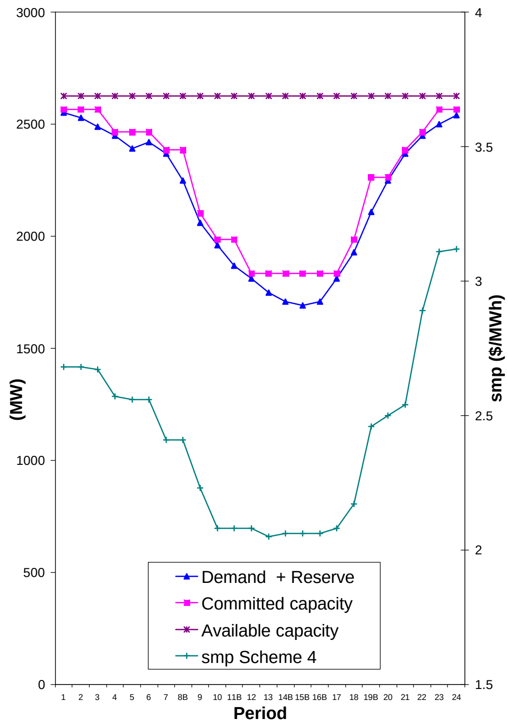

Fig. 4.5: smp, demand, reserve and capacities of the 10-unit system when fixed costs are amortised in proportion to total demand and in inverse proportion to the unit's market share

4.3.3 26-Unit System

Table 4.8 summarises the total customers' payments, or pool cost, (PC) of the 26-unit system for all fixed cost allocation schemes, for different load levels. The last column of the table shows the total generators' production cost (GC) for the UC solution. It can be observed that cost scheme 2 is not always the one that produces the smallest total customers' payments. In addition, the scheme 1 is the most costly to the customers for all load levels simulated. It has also been observed that the cost scheme 1 produced very high spike of prices in some periods.

Table 4.8: Total customers' payments and generators' production costs of the 26-unit system

	PC (\$)				GC (\$)
	Scheme 1	***Scheme 2***	***Scheme 3***	***Scheme 4***	
Load level 1	1,937,851	**1,820,515**	1,847,055	1,882,822	722,049
Load level 2	1,335,758	**1,257,824**	1,282,674	1,288,234	581,120
Load level 3	1,297,224	1,223,685	**1,214,294**	1,231,918	586,588
Load level 4	2,172,394	**2,113,062**	2,142,531	2,148,343	760,540

4.3.4 110-Unit System

Table 4.9 summarises the total customers' payments (PC) for the 110-unit system when the four cost allocation schemes are implemented. The total production cost for the UC solution is $3,750,778. The cheapest scheme in this case is the scheme 3 and the most expensive is still the cost scheme 1. The difference between them would represent an enormous amount of money around $ 170 Millions on an annual basis. These results and the others obtained on the previous sections indicate that the cost allocation scheme 1 has no practical applicability for real electricity markets. Hence this cost scheme will not be used any further in this thesis.

Table 4.9: Pool cost (total customers' payments) and generators' production costs for the 110-unit system

	PC (\$)
Scheme 1	6,485,547
Scheme 2	6,018,822
Scheme 3	**6,014,384**
Scheme 4	6,117,830

4.4 Re-dispatching Units to Reduce Prices

The idea of allocating the units' fixed costs using different approaches, as presented in the previous sections, can be further explored as an attempt to reduce the unit prices and hence the market-clearing prices. From the price formulation of the four cost allocation schemes presented in Section 4.2, it can be seen that the reduction of the unit prices can be achieved by amortising the fixed costs over a larger amount of energy. Hence, by increasing the output of the scheduled units, their prices can be reduced. However, one should notice that by increasing the unit's output power, the incremental costs (inc_i^t) may increase. This can be inferred from the *Willans line* presented in Fig. 2.2. Therefore, one should not argue that the units' prices could be reduced by simply increasing the units' output power.

In this section a re-dispatching algorithm that attempts to reduce the prices of the generating units, and consequently the market-clearing prices, is presented. This algorithm is illustrated in Fig. 4.6. The loading of the units is performed in a step-wise procedure, following the four "capital" points of the *Willans line* ($P_i^{\min}, e_i^1, e_i^2, P_i^{\max}$). The calculation of the unit prices depends on the choice of the cost allocation scheme. Therefore, gp_i^t is determined by using (4.7), (4.13) or (4.16) for cost schemes 2, 3 or 4, respectively. In the cost scheme 2 (the Table A/B scheme), the fixed costs are not allocated in Table B periods, hence a standard economic dispatch, as described in Section 3.4.4, is performed in those periods.

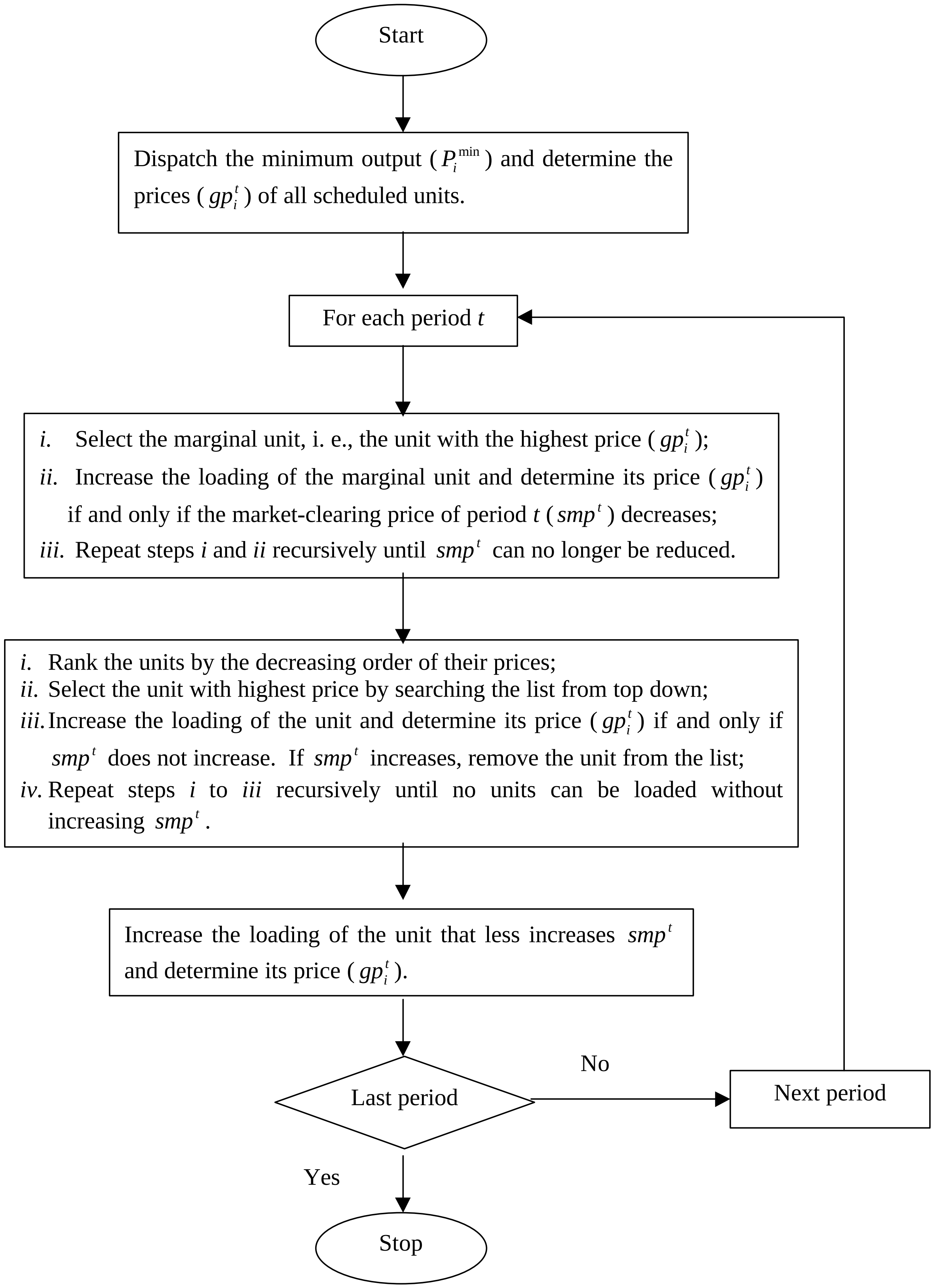

Fig. 4.6: The re-dispatching algorithm

4.5 Application of the Re-dispatching Algorithm

The 4-, 10-, 26-, and 110-unit systems described in Appendix B have been used to illustrate the applicability of the re-dispatching algorithm developed in this project.

4.5.1 4-Unit System

The unit output and prices after re-dispatching the units are presented in Table 4.10 Table 4.11 and Table 4.12 for the cost allocation schemes 2, 3 and 4, respectively. As suggested in Section 4.3, the scheme 1 is no longer used because of the volatility of the prices and high cost to the customers that result from its application. The market-clearing prices (smp^t) are shown in bold character. The underlined figures represent the total customers' payments (PC) and the percentage savings compared to the corresponding payments before re-dispatching, which are shown in Section 4.3.1. The figures in Italics represent the total production costs and their corresponding increase compared to the standard UC solution, which is shown in Section 4.3.1.

Table 4.10: Application of the re-dispatching to the 4-unit system for cost allocation Scheme 2

Unit	hour 1 (Table B)		hour 2 (Table A)		hour 3 (Table B)		*GC* ($)
	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	*40,643* *+0.30%*
1	740	13.00	645	15.33	740	13.00	29,125
2	-	-	340	16.15	160	**13.80**	7,700
3	60	**14.30**	165	**17.94**	-	-	3,818
4	1000	0	1000	0	1000	0	0
PC ($)	25,740		38,563		26,220		**90,523** **-10.49%**

Table 4.11: Application of the re-dispatching to the 4-unit system for cost allocation Scheme 3

Unit	hour 1		hour 2		hour 3		*GC* ($)
	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	*40,923* *+1.00%*
1	635	13.82	645	13.82	560	13.82	25,420
2	-	-	340	14.98	340	**14.98**	10,184
3	165	**16.12**	165	**16.12**	-	-	5,319
4	1000	0	1000	0	1000	0	0
PC ($)	29,014		34,654		28,455		**92,122** **-11.40%**

Table 4.12: Application of the re-dispatching to the 4-unit system for cost allocation Scheme 4

Unit	hour 1		hour 2		hour 3		*GC* ($)
	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	*40,923* *+1.00%*
1	635	13.73	645	13.85	560	13.87	25,420
2	-	-	340	15.05	340	**14.90**	10,184
3	165	**15.96**	165	**16.28**	-	-	5,319
4	1000	0	1000	0	1000	0	0
PC ($)	28,723		35,000		28,317		**92,041** **-12.67%**

Table 4.13 shows the sequential calculation of the re-dispatching algorithm applied to the 4-unit system when the cost allocation Scheme 3 is used. The bold figures are the market-clearing prices in the corresponding periods. The underlined figures represent the change in the units' output power and the associate change in the unit prices. The bidding prices of Unit 4 are zero and hence this unit is a *must-run* unit. It is scheduled

to generate its maximum output during all periods, but does not contribute to set the market-clearing prices.

Table 4.13: Sample calculation of the re-dispatching algorithm

i. Dispatch the minimum output and determine the unit prices of all scheduled units;						
	Production Power (MW)			Unit Prices ($/MWh)		
	hour 1	hour 2	hour 3	hour 1	hour 2	hour 3
Unit 1	130	130	130	16.85	16.85	16.85
Unit 2	-	45	45	-	22.69	**22.69**
Unit 3	15	15	-	**34.30**	**34.30**	-
ii. Increase the loading of the marginal unit (unit 3) in hour 1;						
Unit 1	130	130	130	16.85	16.85	16.85
Unit 2	-	45	45	-	**22.69**	**22.69**
Unit 3	165	15	-	**17.63**	17.63	-
iii. Increase the loading of the marginal unit (unit 1) in hour 1;						
Unit 1	635	130	130	14.68	14.68	14.68
Unit 2	-	45	45	-	**22.69**	**22.69**
Unit 3	165	15	-	**17.63**	17.63	-
iv. Increase the loading of the marginal unit (unit 2) in hour 2;						
Unit 1	635	130	130	14.68	14.68	14.68
Unit 2	-	340	45	-	15.88	**15.88**
Unit 3	165	15	-	**17.63**	**17.63**	-
v. Increase the loading of the marginal unit (unit 3) in hour 2;						
Unit 1	635	130	130	14.68	14.68	14.68
Unit 2	-	340	45	-	15.88	**15.88**
Unit 3	165	165	-	**16.12**	**16.12**	-
vi. Increase the loading of the marginal unit (unit 1) in hour 2;						
Unit 1	635	645	130	14.06	14.06	14.06
Unit 2	-	340	45	-	15.88	**15.88**
Unit 3	165	165	-	**16.12**	**16.12**	-
vii. Increase the loading of the marginal unit (unit 2) in hour 3;						
Unit 1	635	645	130	14.06	14.06	14.06
Unit 2	-	340	340	-	14.98	**14.98**
Unit 3	165	165	-	**16.12**	**16.12**	-
viii. Increase the loading of the marginal unit (unit 1) in hour 3;						
Unit 1	635	645	560	13.82	13.82	13.82
Unit 2	-	340	340	-	14.98	**14.98**
Unit 3	165	165	-	**16.12**	**16.12**	-

4.5.2 10-Unit System

Table 4.14 summarises the total customers' payments, or pool cost, (PC) and the total generators' production costs (GC) for the 10-unit system after the application of the re-dispatching algorithm. The figures in parenthesis represent the difference between the corresponding value and the solution obtained before re-dispatching, which is shown in Section 4.3.2. They show that the total costs to the customers can be reduced by changing the output power of the generating units. This, of course, increases the total generators' production costs. These results ratify that the minimisation of generators' production based on their bid prices and the minimisation of the customers' payments costs are conflicting objectives. Hence a model designed to simulate the operation of an electricity market should not rely on either of them.

Table 4.14: Total customers' payments and generators' production costs after the application of the re-dispatching algorithm to the 10-unit system

	GC ($)	PC ($)
Scheme 2	82,081 (+ 3.0 %)	94,936 (- 1.2 %)
Scheme 3	82,081 (+ 3.0 %)	95,857 (- 0.5 %)
Scheme 4	81,494 (+ 2.3 %)	95,001 (- 1.1 %)

4.5.3 26-Unit System

Table 4.15 shows the total customers' payments, or pool cost, (PC) and the total generators' production costs (GC) for the 26-unit system after the application of the re-dispatching algorithm. By comparing the figures below with the results presented in Table 4.8, it can be seen that the algorithm managed to considerably reduce the total customers' payments in all cases by re-dispatching the generating units and hence increasing the total generators' production costs. The savings experienced by the customers were as high as 22 % for the Load level 4.

Table 4.15: Total customers' payments and generators' production costs after the application of the re-dispatching to the 26-unit system

	Scheme 2		***Scheme 3***		***Scheme 4***	
	GC (\$)	PC (\$)	GC (\$)	PC (\$)	GC (\$)	PC (\$)
Load level 1	817.379	1,477,602	846,486	1,491,875	846,486	1,486,011
Load level 2	625,715	1,099,152	684,238	1,109,792	684,238	1,107,973
Load level 3	622,990	1,087,824	694,848	1,092,792	694,848	1,089,560
Load level 4	894,389	1,682,169	894,389	1,679,400	894,894	1,676,241

4.5.4 110-Unit System

Table 4.16 shows the total customers' payments, or pool cost (PC), and the total production costs (GC) for the 110-unit system after the application of the re-dispatching algorithm. The figures in parenthesis represent the increasing in generators' production costs and the reduction of the total customers' payments compared to the corresponding values of Table 4.9. Again, these results show the ability of the re-dispatching algorithm in reducing the cost to the customers, at the expenses of increasing the total generators' production costs.

Table 4.16: Total customers' payments and generators' production costs after the application of the re-dispatching to the 110-unit system

	GC (\$)	PC (\$)
Scheme 2	3,849,217 (+ 2.62 %)	5,788,431 (- 3.83 %)
Scheme 3	3,976,505 (+ 6.02 %)	5,812,355 (- 3.36 %)
Scheme 4	3,976,505 (+ 6.02 %)	5,814,118 (- 4.96 %)

4.6 Discussion

The allocation of the generators' fixed costs plays a crucial role in the price mechanisms of pool-based electricity markets. Four different fixed cost allocation schemes were presented in this chapter. They were formulated in such a way as to guarantee cost recover to generators, which is an important requirement for the establishment of a fair market. The simulation of the market operation has shown that, by changing the fixed cost allocation schemes, the total customers' payments can vary significantly. Different schemes yield significant changes in the generators' profitability. Some other features of those fixed costs allocation schemes are discussed in the following paragraph.

In Scheme 1, the fixed costs are allocated when they are incurred, in proportion to the units' output of the corresponding period. This produces spot prices with high volatility due to the concentration of the fixed costs in one single period. To flatten the price profile, the fixed costs are amortised over the hours the units are scheduled to generate in Schemes 2, 3 and 4. In the Scheme 2 (Table A/B scheme), those hours are classified into Table A and Table B periods according to their excess reserve, and the fixed costs are amortised only over the Table A periods, in proportion to the total output power during those hours. This is an attempt to produce lower prices when there is plenty of spare capacity and higher prices when the system is stressed, and hence to encourage demand side management. However, due to the nature of the *Willans line*, the prices of Table B periods can be higher than prices in Table A periods, which is in conflict with the theoretical purpose of this price mechanism. In addition, the Table A/B scheme requires side payments to guarantee the cost recovery of generating units that are scheduled to generate only during Table B periods. Moreover, the Table A/B scheme fails to provide a correct price signal for market participants because it can lead to situations in which the spot price increases from one period to another as the demand decreases, or vice-versa. In Scheme 3 there is no such a classification amongst the periods and the fixed costs are amortised over the continuous running hours in proportion to total output power during those hours. Likewise, in Scheme 4, the fixed costs are amortised over the continuous running hours, but in proportion to the accumulated demand during those periods and in inverse proportion to the units' market share. These two schemes produce spot prices of which profile is more "in tune" with the load profile albeit they may yield to total customers' payments slightly higher than the Table A/B scheme does, in some cases.

This chapter has also presented a re-dispatching algorithm to reduce the unit prices and hence the market-clearing prices. The start-up and no-load costs, which the generators are guaranteed to recover, are therefore simply amortised over a larger amount of energy. The results of the case studies show that the re-dispatching algorithm can reduce the total customers' payments, or pool cost, by up to 22 % at the expenses of the generators, of which the production costs increase by up to 18 %. This algorithm is paradoxical as it increases the loading of the units with higher incremental costs (inc_i^t) before the units with lower incremental costs. This paradox can be resolved by considering that the objective is to minimise market-clearing prices and thus the customers' payments, and not the total generators' production costs. This re-dispatching algorithm does not make sense from an economics perspective but it is applied to an artificial market and not a real one. However, it should be noticed that the mechanism to adjust the prices of pulsing units, described in Section 2.4.1.4, carries this basic idea of increasing the amount of energy over which the fixed costs are amortised. Similarly, the re-dispatching algorithm could also be used by the market operator only to determine the spot prices of electricity. The actual output power of the scheduled units would be calculated by the traditional economic algorithm. Of course, as shown in Section 4.3.2, the market-clearing prices would not be enough to ensure cost recovery to all units, and hence like in the EPEW, this mechanism would require side payments to those units.

4.7 Summary

In this chapter, alternative ways to allocate the generators' fixed costs in the pricing mechanism have been presented and discussed. The implementation of different fixed cost allocation schemes resulted in significant variations in the electricity prices, generators' revenue and customers' payments. The chapter has also shown how a re-dispatching algorithm can be used to reduce the prices of the scheduled units, and consequently the market-clearing prices.

CHAPTER 5

Generation Scheduling Based on Customers' Payment Minimisation

5.1 Introduction

In a centrally operated, non-competitive system, the generation scheduling problem is to find the commitment schedule that minimises the energy production costs over a specified time span, while satisfying the system constraints and unit constraints. This optimisation problem is known as the unit commitment (UC) problem. In the context of a pool-based electricity market, the pool's resources scheduling problem can still be formulated as a UC problem. In this problem however, the objective function is no longer expressed in terms of the operating costs of the generating units but in terms of the day-ahead bids provided by the generating companies. The reasoning behind this modification is that, in a perfectly competitive market, suppliers have no incentive to bid higher or lower than their production costs. Treating these bids as costs to be minimised thus appears to be a reasonable way to simulate a market. Furthermore, payments to suppliers are based on a system marginal price set by the last acceptable bid. This practice is intended to discourage suppliers from trying to predict the market price and encourage bids based on the actual production costs.

This approach suffers, however, from a number of practical difficulties. Johnson et al. [6] argue that centralised scheduling of multi-owned resources under imperfect information may face difficulties that do not arise when resources are centrally owned. The structure of the UC problem may yield several near optimum solutions, which are of equal quality in terms of total production costs but may vary significantly in terms of individual costs, profits and commitments. These effects are inherent when attempting to optimise UC from the perspective of a central operator because of the near-degeneracy of the UC problem and the presence of many near-optimal solutions. Johnson and his collaborators used an LR-based UC algorithm to show that variations in near optimum UC schedules with negligible effect on total generation production costs have a significant impact on the profitability of individual resources. They also show that centralised scheduling may be inequitable to some generation companies as their profit can be significantly reduced by a sub-optimal UC solution. The results are particularly volatile for marginal resources, but the changes in the payments to base-loaded resources due to changes in the price vectors for different solutions can become very large. These authors suggest that centralised scheduling by a mandatory power pool may be perceived by suppliers and consumers as unnecessarily volatile and even inequitable, and hence in the long run yield schedules that do not minimise costs. Their results support a more decentralised approach to UC such as physical scheduling of self-nominated transactions or a simple auction with single prices and self-commitment. As discussed in Chapter 2, simple pairs of quantity-price are already used in some electricity markets and the idea of implementing such a bidding structure has been under extensive discussion under the Review of the Electricity Trading Arrangements in the EPEW [40].

The results presented in Chapter 3 of this thesis ratify the argument that global UC schedules with negligible discrepancies in total production costs may result in considerable changes in the total payments by the consumers. The results have shown that a UC solution that is more attractive to generating companies can be more costly to customers. Furthermore, it has been discussed in Chapter 3 that the equity and efficiency of an electricity market can be affected by a scheduling algorithm because the scheduling criteria differ from the rules of the price mechanism.

Some examples that point out the failures of administrative mechanisms to mimic the operation of a power market have also been presented by Jacobs [7]. This author argues that the "least-cost" dispatch of a central authority combined with a payment rule based on a market-clearing price does not necessarily minimise the cost to consumers of electricity. This happens because the scoring rule, which is used to select resources based upon a production cost minimisation, differs from the payment rule. He suggests that the UC problem should incorporate in its objective the consumer cost instead of the production cost.

Hao et al. presented a new methodology for calculating optimal generating schedules that minimises energy payments by power pool consumers instead of minimising generators' production costs [8]. Payment adequacy constraints are introduced in the problem formulation to ensure that all units winning the auction recover their fixed costs as well as their incremental costs. The total fixed costs for each selected unit appears in the payment adequacy constraint and is computed in the solution process. The portions of start-up and no-load costs allocated to each hour of the scheduling period are decision variables in the optimisation process. The application of the methodology is illustrated by a simple example of scheduling four units over three hours, in which minimum and maximum generation levels are the only operational characteristics taken into account and the start-up costs are constant. The authors claim that the start-up and no-load costs are optimally allocated throughout the scheduling period, and hence the presented methodology can produce the lowest possible consumer payment under uniform pricing rule and payment adequacy requirements. Unfortunately, their paper does not provide enough information for the reader to reproduce their results or to study the effectiveness of their methodology when applied to real-size systems.

This chapter introduces an alternative generation scheduling algorithm in which the objective is to minimise the total customers' payments. A forward Dynamic Programming algorithm is used to search for the UC schedule. The proposed methodology attempts to improve the equity and efficiency of a pool-based competitive electricity market.

In the rest of this chapter, the standard problem of scheduling generation units seeking for the minimisation of the generators' production costs is called *cost minimisation scheduling problem*. Similarly, the problem of scheduling generation units by minimising the total customers' payments (or pool cost) is hereafter called *price minimisation scheduling problem*.

5.2 Formulation of the Price Minimisation Scheduling Problem

The objective of this problem is to minimise the total customers' payments, or pool cost (PC) over the entire time horizon, subject to the same system and unit constraints described in Section 3.2.2 and Section 3.2.3. As this problem combines *generation scheduling* and *price computation*, the pricing rules must be defined before formulating the optimisation problem. In several electricity markets the units' payments are based upon a "*second-price*" auction mechanism, i.e., the customers' payments (pool costs) are calculated on the basis of the market-clearing prices. This is different from the traditional DP-based UC algorithm where the generators' operating costs are calculated based on the idea of paying each selected unit its own bidding price. This poses a significant difference between the *price minimisation scheduling problem* described in this section and the *cost minimisation scheduling problem*. Mathematically, the *price minimisation scheduling problem* can be written as:

$$Min \quad PC \quad = \quad Min \quad (\sum_{t=1}^{T} smp^t \times D^t) \tag{5.1}$$

$$smp^t = Max\,(gp_i^t) \qquad \forall\ i = 1, \ldots, \mathrm{N} \tag{5.2}$$

To completely formulate the problem, the pricing rules should specify the scheme to allocate the generators' fixed start-up and no-load costs during the calculation of the unit prices (gp_i^t).

As discussed in Chapter 4, allocating the fixed costs when they are incurred, in proportion to the units' output power (Scheme 1) would lead to an undesirable price volatility. Therefore the fixed cost allocation Scheme 1 will not be used in the formulation of the *price minimisation scheduling problem*.

In the Table A/B scheme, the fixed costs are not allocated during Table B periods. The search of the optimum solution of the *price minimisation scheduling problem* can then be restricted to the set of solutions, in which all scheduling periods are classified as Table B periods. However, as discussed in Section 4.2.2, the cost recovery will not be verified for these trivial solutions. Hence, this cost allocation scheme will not be included in the formulation of the *price minimisation scheduling problem*.

If the fixed costs of a unit are amortised over every single period in proportion to its total power output (Scheme 3), the problem is fully formulated by the inclusion of the unit prices, as follows.

$$gp_i^t = inc_i^t + \frac{s_i^{t_{on}} + \sum_{t=t_{on}}^{t_{off}} (nl_i^t \times pl)}{\sum_{t=t_{on}}^{t_{off}} (P_i^t \times pl)} \qquad \forall\ t = 1, \ldots, \mathrm{T} \qquad (5.3)$$

The generators' fixed costs can also be allocated using the Scheme 4, in which the fixed costs are amortised over every hour a unit is scheduled in proportion to the accumulated demand and in inverse proportion to the unit's market share. The unit prices are thus given by:

$$gp_i^t = inc_i^t + \frac{s_i^{t_{on}} + \sum_{t=t_{on}}^{t_{off}} (nl_i^t \times pl)}{\sum_{t=t_{on}}^{t_{off}} D^t} \times \frac{D^t}{(P_i^t \times pl)} \qquad \forall\ t = 1, \ldots, \mathrm{T} \qquad (5.4)$$

5.3 Price Minimisation Scheduling Algorithm

5.3.1 Searching for the Solution

The search for the solution of the generation scheduling problem based on the customers' payment minimisation is performed using the forward Dynamic Programming technique [5]. Like the standard DP-based algorithm for the solution of the UC problem, this algorithm keeps only the best strategy to achieve each state in a generic period *t*. However, the accumulated pool costs (customers' payments), rather than the accumulated operating costs (generators' costs), are determined to assess the best strategy to achieve each state in each period. The accumulated pool cost (customers' payments) from period "1" to period "t_a" ($PC_1^{t_a}$) is given by:

$$PC_1^{t_a} = \sum_{t=1}^{t_a} smp^t \times D^t \tag{5.5}$$

Since the DP-based algorithm is not well suited for large-scale power systems due to the "curse of dimensionality", the hybrid LR-DP scheduling algorithm described in Section 3.6 is used for the solution of larger systems.

5.3.2 Determining the Units' Output Power

The standard ED algorithm, in which the generating units are dispatched according to the increasing order of their incremental costs, is described in Section 3.4.4. This section presents an alternative ED algorithm, in which the objective function is the customers' payments rather than the generators' costs.

The power produced by each scheduled unit at hour *t* is determined in such a way as to minimise the *accumulated pool cost from hour 1 to hour t* (PC_1^t). It can be seen from (5.3) and (5.4) that the unit prices can be reduced by increasing the amount of power output of a continuous running period. Therefore, the market-clearing price, and consequently the customers' payments, can be reduced. The step by step algorithm runs as follows, until the demand at hour *t* is satisfied:

i. Dispatch the minimum output of committed units and determine gp_i^t, as in (5.3) or (5.4). Calculate the smp^t from hour 1 to hour t and determine the *accumulated pool cost from hour 1 to hour t* (PC_1^t);

ii. Increase the loading of committed units which most reduce PC_1^t. This is done by loading unit *i* up to the next elbow point on its *Willans line* ($P_i^{\min}, e_i^1, e_i^2, P_i^{\max}$) and determining the updated gp_i^t, smp^t and PC_1^t. The unit that most reduces PC_1^t is then selected and the initial loading of the other units is restored.

iii. When PC_1^t cannot be reduced any further, increase the loading of committed units that do not increase PC_1^t. The idea is to reduce gp_i^t of the units that are likely to become marginal in subsequent hours. First, select the unit with the highest gp_i^t. Then increase the loading of the unit if and only if its gp_i^t reduces and if PC_1^t does not increase.

iv. Increase the loading of committed units which less increase PC_1^t. Determine gp_i^t, smp^t and PC_1^t. The unit that less increases PC_1^t is then selected and the initial loading of the other units are restored.

The ED algorithm proposed in this section is different from the "re-dispatch" algorithm presented in Section 4.4. In the former, the UC schedule is known over the whole time horizon and thus the calculation of the power production of one generating unit in a particular period takes into account the statuses of the unit in precedent and subsequent periods. The algorithm presented in this section is part of the search for the UC solution, which is performed in a sequential fashion. Thus, no information regarding the units' statuses in subsequent periods is available. Hence, the calculation of the unit prices is done by considering the statuses of the units in previous hours only.

5.3.3 Calculating the Units' Prices

In the forward search for the solution, the unit prices are calculated using (5.3) or (5.4), in which the indices of the summations are set in such a way as to account only for the current and past states. Therefore, the unit prices in one given scheduling period *t* are calculated by taking into account the unit's output power of previous periods *t-1*, *t-2*, etc. Hence, the unit prices in those previous periods are constantly updated. Therefore the market-clearing prices of those previous hours are also updated.

5.4 Illustration of the Price Minimisation Scheduling Algorithm

This *price minimisation scheduling algorithm* is rather complex and can be better understood through a small example. This section illustrates its application to the 4-unit system, which is described in Appendix B. In this example, the units' fixed costs are allocated using the fixed cost scheme 3, as described in Section 4.2.3.

5.4.1 The Search for the Solution

Fig. 5.1 illustrates the search for the solution when a standard ED algorithm is used to determine the units' output power. Similarly, Fig. 5.2 shows the searching process for the alternative ED algorithm. For both cases, the fixed cost allocation Scheme 3 was used to calculate the unit prices as in (5.3). The units' statuses are represented in a binary form in which the digit "1" indicates that the unit is "*on-line*", whereas the digit "0" indicates that the unit is "*off-line*". The circles denote hourly states, which are the combinations of the "*on-line*" and "*off-line*" units' statuses. The total capacity of each state is indicated next to the combination of the units' statuses. The dashed line shows the boundary between the feasible states (the states with enough capacity to satisfy the demand and reserve requirements) and the unfeasible ones. The figures next to the dashed line represent the system load at each period. The connections represent the most economical strategy to achieve each state. The connections in bold represent the solution obtained by the algorithm. The figures in parenthesis next to the connections are the sum of the start-up costs and the accumulated no-load costs up to the corresponding period for each unit, except unit 4 for which bid prices are zero (see Table B.1), in $. The figures within braces are the market-clearing prices in $/MWh for the corresponding period and for the previous periods. The figures below the market-clearing prices are the accumulated customers' payments up to the corresponding period, in $.

5.4.2 Determining the Units' Output Power

Table 5.1 presents the output power of units 1, 2 and 3 for all feasible states when the standard ED is implemented in the *price minimisation scheduling algorithm*. Similarly, Table 5.2 presents the units' output power when the alternative ED is used. Each set of three figures separated by the slash represents the output power of unit 1, 2 and 3, in that order. The figures in bold character indicate units' output power in each state of the solution (see Fig. 5.1 and Fig. 5.2).

Table 5.1: Units' output power of the 4-unit system determined by a standard ED algorithm

Unit Statuses	Total Capacity (MW)	hour 1 load = 1800 MW	hour 2 load = 2150 MW	hour 3 load = 190 MW
1111	2145	740 / 45 / 15	**740 / 340 / 70**	740 / 145 / 15
1101	2100	740 / 60 / 0	-	740 / 60 / 0
1011	1905	**740 / 0 / 60**	-	**740 / 0 160**

Table 5.2: Units' output power of the 4-unit system determined by the alternative ED algorithm, using the fixed cost allocation Scheme 3

Unit Statuses	Total Capacity (MW)	hour 1 load = 1800 MW	hour 2 load = 2150 MW	hour 3 load = 190 MW
1111	2145	295 / 340 / 165	**645 / 340 / 165**	395 / 340 / 165
1101	2100	460 / 340 / 0	-	**560 / 340 / 0**
1011	1905	**635 / 0 / 165**	-	735 / 0 165

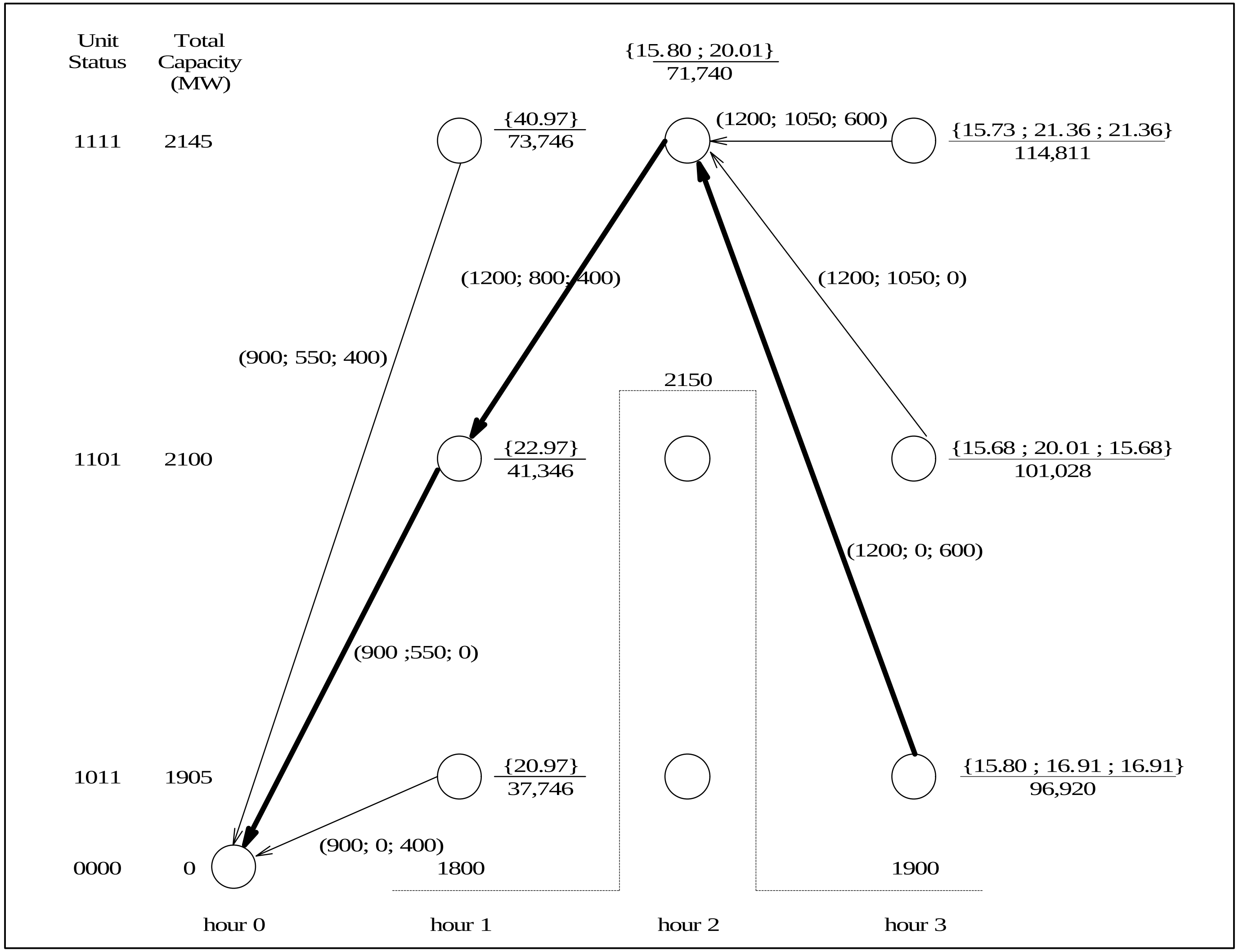

Fig. 5.1: Application of the price minimisation scheduling algorithm, with a standard ED to the 4-unit system

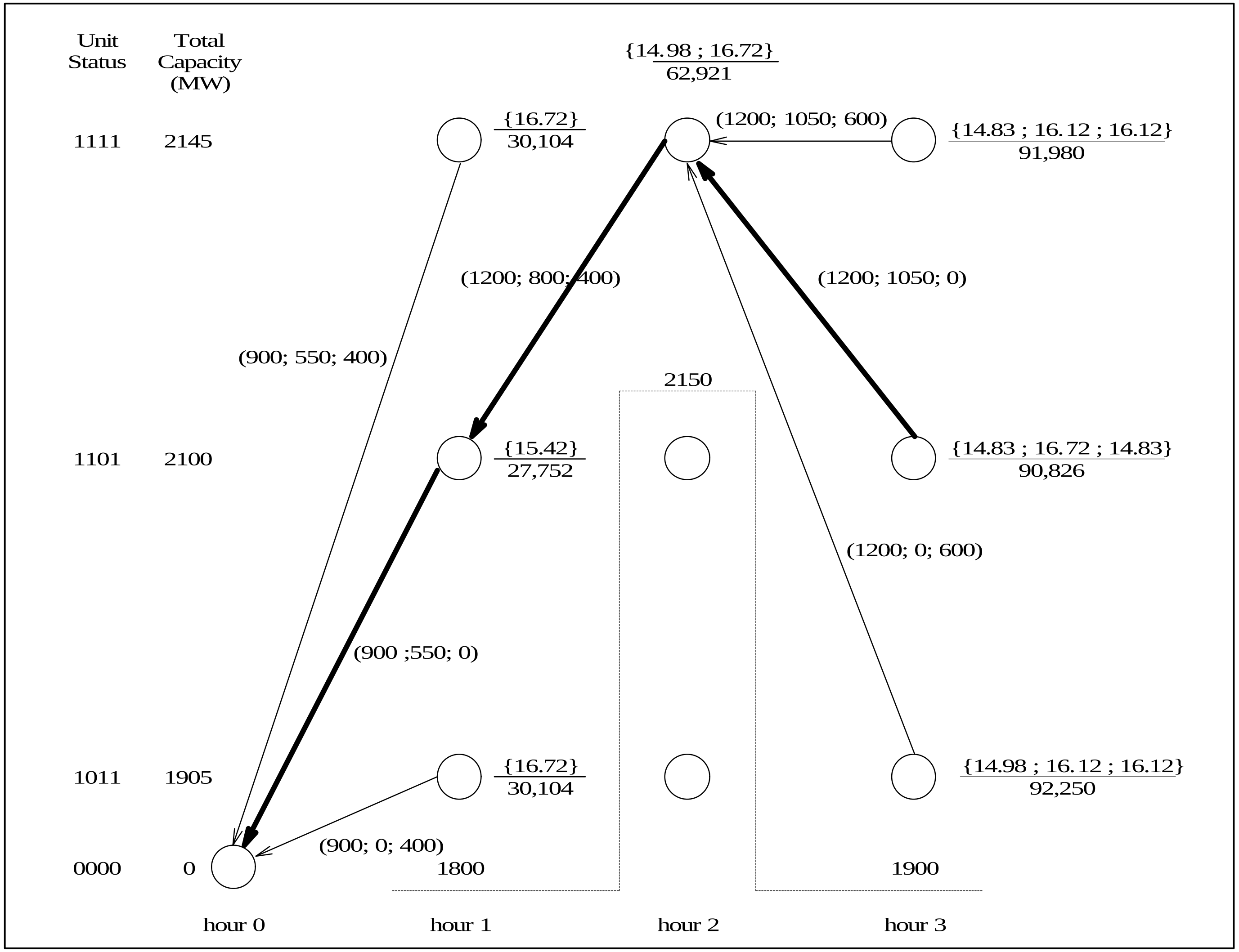

Fig. 5.2: Application of the price minimisation scheduling algorithm, with the alternative ED to the 4-unit system

5.4.3 Calculating the Units' Prices

By looking at Fig. 5.1, one should notice that the market-clearing price of state "1101" in hour 1, which is set by unit 2, is initially calculated as 22.97 \$/MWh. According to (5.3), the unit price of unit 2 is given by:

$$gp_2^1 = 13.8 + \frac{300 + 250}{60} = 22.97 \text{ \$/MWh}$$

The accumulated pool cost up to hour 1 is then given by the product of the clearing price of state "1101" and the demand in hour 1. Hence,

$$PC_1^1 = (22.97 \times 1{,}800) = \$\ 41{,}346$$

As the algorithm proceeds to hour 2, the market-clearing price of state "1101" in the previous hour 1, is still set by unit 2, but it decreases to:

$$gp_2^1 = gp_2^2 = 13.8 + \frac{300 + 250 + 250}{60 + 340} = 15.80 \text{ \$/MWh}$$

In hour 2, the market-clearing price of state "1111" is set by unit 3 and equals 20.01 \$/MWh. It is given by:

$$gp_3^2 = 14.3 + \frac{200 + 200}{70} = 20.01 \text{ \$/MWh}$$

The accumulated pool cost from hour 1 to hour 2 can be determined as follows:

$$PC_1^2 = (15.80 \times 1{,}800) + (20.01 \times 2{,}150) = \$\ 71{,}740$$

The transition from state "1111" in hour 1 to state "1111" in hour 2 leads to a higher accumulated pool cost (\$ 84,367). This is also true for the transition from "1011" in hour 1 to "1111" in hour 2, of which the accumulated pool cost is \$ 74,715. Hence, the best strategy to reach state "1111" in hour 2 is from state "1101" in hour 1. This strategy is then saved and the other two discarded.

As the unit prices of previous hours change while the algorithm moves forward, the strategy saved in previous hours may not be the best all the time. Therefore, there is no guarantee that this algorithm will always achieve the optimum solution.

5.4.4 Solution of the Example

Initially, a standard ED algorithm is used and the results of this exercise are presented in Table 5.3. The figures in bold represent the market-clearing prices for each scheduling period (smp^t). The underlined figures on the bottom right cell of the table are the total consumers' payments (PC) and the corresponding savings (- 6.79 %) compared to the solution of the standard UC algorithm, which is shown in Table 4.4. The other three cells of the last row show the hourly pool costs. The figures in italics in the top right cell represent the total generators' production costs (GC) and the corresponding increase (+0.12 %) compared to the solution of the standard UC algorithm, which is shown in Table 4.1. The following four cells of the last column show the units' production costs.

Table 5.3: Solution of the price minimisation scheduling problem for the 4-unit system, using the cost allocation Scheme 3 and a standard ED algorithm

Unit	hour 1		hour 2		hour 3		GC ($)
	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	*40,569* *+ 0.12 %*
1	740	13.82	740	13.82	740	13.82	30,360
2	60	**15.80**	340	15.80	-	-	6,320
3	-	-	70	**16.91**	160	**16.91**	3,889
4	1000	0	1000	0	1000	0	0
PC ($)	28,440		36,354		32,127		**<u>96,920</u>** **<u>- 6.79%</u>**

Similarly, Table 5.4 presents the results obtained when the alternative ED algorithm is used. One should notice that the solution of the scheduling problem when the alternative ED algorithm is used differs from the solution achieved when the standard ED is used not only by the units' output power but also by the unit commitment. This can be easily seen from the comparison of Fig. 5.1 and Fig. 5.2, in which the arrows in bold represent the solution of each case. The use of the alternative ED algorithm introduces further reduction in the total consumers' payments compared to the solution obtained when the standard ED algorithm is used. The total cost to the customers decreases from \$ 96,920 to \$ 90,826, representing additional savings of 6.29 %. The total generators' production costs, however, increases 6.07 %. It changes from \$ 40,569 to \$ 43,031. The cost recovery requirement can be easily verified by computing the units' revenues and comparing them with the units' production costs in the last column of Table 5.3 and Table 5.4. The units' revenues can be calculated using the market-clearing prices in bold character in Table 5.3 and Table 5.4 and the units' output power in Table 5.1 and Table 5.2, again in bold character.

Table 5.4: Solution of the price minimisation scheduling problem for the 4-unit system, using the cost allocation Scheme 3 and the alternative ED algorithm

Unit	hour 1		hour 2		hour 3		*GC* (\$)
	Output (MW)	Prices (\$/MWh)	Output (MW)	Prices (\$/MWh)	Output (MW)	Prices (\$/MWh)	*43,031* *+ 6.20 %*
1	460	13.90	645	13.90	560	13.90	23,145
2	340	**14.83**	340	14.83	340	**14.83**	15,126
3	-	-	165	**16.72**	-	-	2,760
4	1000	0	1000	0	1000	0	0
PC (\$)	26,693		35,957		28,176		**90,826** **- 12.65%**

Similar results are obtained when the fixed cost allocation Scheme 4 is used to determine the unit prices. These results are presented in Table 5.5 and Table 5.6 for the standard ED algorithm and the alternative ED algorithm, respectively.

Table 5.5: Solution of the price minimisation scheduling problem for the 4-unit system, using the cost allocation Scheme 4 and a standard ED algorithm

Unit	hour 1		hour 2		hour 3		*GC* ($)
	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	*40,549* *+ 0.07 %*
1	740	13.62	740	13.74	740	13.66	30,360
2	_	_	340	15.42	-	-	5,242
3	60	**18.40**	70	**18.50**	160	**15.92**	4,947
4	1000	0	1000	0	1000	0	0
PC ($)	33,125		39,776		30,255		**103,156** **- 2.12%**

Table 5.6: Solution of the price minimisation scheduling problem for the 4-unit system, using the cost allocation Scheme 4 and the alternative ED algorithm

Unit	hour 1		hour 2		hour 3		*GC* ($)
	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	Output (MW)	Prices ($/MWh)	*41,031* *+ 1.26 %*
1	460	14.00	645	13.85	560	13.87	23,145
2	340	**14.75**	340	14.93	340	**14.80**	15,126
3	-	-	165	**16.72**	-	-	2,760
4	1000	0	1000	0	1000	0	0
PC ($)	26,550		35,957		28,126		**90,633** **- 14.01 %**

5.4.5 Discussion of the Example

In a fair electricity market with perfect competition, generators and customers have intrinsic different objectives. Both parties seek to maximise their profits (pay-off), however, for generators this may represent higher prices of electricity, whereas for customers this may represent lower prices. It has been argued that the generation scheduling should optimise the customers' payments rather than the production costs [6-8]. This argument is supported by some examples in which a more expensive UC schedule may be cheaper to customers, or vice-versa, which has also been observed from the results presented in Section 3.8.

The application of the *price minimisation scheduling algorithm* to the 4-unit system has shown that it is possible to incorporate the customers' objective into the scheduling problem. When a standard ED algorithm is used, the customers' savings can go up to 12 % as shown in Table 5.4. The introduction of the alternative ED algorithm has further reduced the total customers' payments. Based on these findings, one may say that a better simulator of an electricity market can be achieved by simply replacing the generators' costs with the customers' payments in the objective function of the scheduling problem.

However, as discussed previously, the most important features of the market are equity and efficiency. Therefore, it is important to assess whether an electricity market in which the generation scheduling is based upon the minimisation of the customers' payments would be equitable and efficient. To perform this investigation an exercise has been proposed in which the bidding prices of Unit 3 were multiplied by a constant parameter and the behaviour of the market analysed. Table 5.7 presents the units' total output, the generators' total production costs (GC) and the total customers' payments (PC) for this simulation, when the alternative ED algorithm and the cost allocation Schemes 3 and 4 are used. The figures in bold character show that Unit 3 can increase its market share by increasing its bidding prices. This phenomenon is similar to "*the Wollenberg's Paradox*", presented in Appendix C. It affects the equity of the electricity market and hence the alternative ED algorithm is not suitable for the *price minimisation scheduling algorithm* presented in this thesis.

Table 5.8 presents the results of a similar simulation when the standard ED algorithm is used. As expected, no violations regarding the equity of the market have been observed.

Table 5.7: price minimisation scheduling algorithm with the alternative ED algorithm

Bid multiplier for Unit 3	***Scheme 3***			***Scheme 4***		
	Output (MWh)	*GC* ($)	*PC* ($)	Output (MWh)	*GC* ($)	*PC* ($)
0.50	395	37,461	83,872	**290**	38,076	83,621
0.55	395	33,784	83,872	**295**	38,297	83,630
0.60	395	38,106	83,872	**390**	38,128	83,630
0.65	395	38,429	83,872	**495**	38,058	83,816
0.70	395	38,751	83,872	495	38,452	83,816
0.75	**395**	39,074	83,872	495	38,846	83,816
0.80	**400**	39,388	83,878	495	39,240	83,816
0.85	**495**	39,634	84,002	495	39,634	83,816
0.90	**495**	40,549	86,374	495	40,028	85,877
0.95	**495**	40,944	88,454	165	40,893	88,835
1.00	165	41,031	90,826	165	41,031	90,633

Table 5.8: price minimisation scheduling algorithm with the standard ED algorithm

Bid multiplier for Unit 3	***Scheme 3***			***Scheme 4***		
	Output (MWh)	*GC* ($)	*PC* ($)	Output (MWh)	*GC* ($)	*PC* ($)
0.50	495	36,800	85,227	495	36,800	85,164
0.55	495	37,194	85,227	495	37,194	85,164
0.60	495	37,588	85,227	495	37,588	85,164
0.65	495	37,982	85,227	495	37,982	85,164
0.70	495	38,376	85,227	495	38,376	85,164
0.75	495	38,770	85,227	495	38,770	85,164
0.80	495	39,164	85,227	495	39,164	85,164
0.85	495	39,588	85,227	495	39,558	85,164
0.90	495	39,952	87,497	495	39,952	87,225
0.95	385	40,281	92,065	385	40,282	90,708
1.00	230	40,569	96,920	290	40,549	103,156

5.5 Application of the Price Minimisation Scheduling Algorithm

In the following sub-sections, the results of the application of the *price minimisation scheduling algorithm* to the 10-, 26- and 110-unit systems, which are described in Appendix B, are presented and discussed. The results presented in this section are compared to the results obtained by the *cost minimisation scheduling approach*, which are presented in Section 4.3.

5.5.1 10-Unit System

The 10-unit system has a total capacity of 3,125 MW, and peak load and minimum load of 2,000 MW and 1,420 MW, respectively. The complete data of this system are presented in Appendix B. Table 5.9 presents the total generators' production costs (GC) and the total customers' payments, or pool costs, (PC) resulting from the application of the *price minimisation scheduling algorithm* to the 10-unit system. The results are shown for cost allocation schemes 3 and 4, and for the standard ED algorithm.

By comparing the figures in Table 5.9 with those of Table 4.7, it can be seen that the pool cost can be reduced by the *price minimisation scheduling algorithm* at the expenses of an increase in the generators costs. The generators' costs associated with the solution of *cost minimisation scheduling problem* presented in Section 4.3 is 0.36 % cheaper than those of Table 5.9. It can also be seen that the cost allocation scheme 4 produces a solution that is more economically attractive for the customers than the solution produced by Scheme 3.

Table 5.9: Production costs and customers' payments for the 10-unit system for cost allocation Schemes 3 and 4

Scheme 3		***Scheme 4***	
GC ($)	PC ($)	GC ($)	PC ($)
79,972	96,247	79,972	96,013

5.5.2 26-Unit System

The data of the 26-unit system, which is derived from the IEEE-RTS, are given in Appendix B. The four different load profiles shown in Fig. B.3 were simulated in this case study. The total generators' production costs (GC) and the total customers' payments, or pool costs, (PC) obtained by the application of the *price minimisation scheduling algorithm* to the 26-unit system are presented in Table 5.10. The unit prices were calculated using the cost allocation schemes 3 and 4. Again, it can be seen that considerable savings for the customers can be introduced by the price optimisation approach, compared to the results presented in Table 4.8. These savings can go up to 7 % for Load level 2 when the Scheme 3 is used. However, the generators' production cost increases significantly, mainly because the *price minimisation algorithm* tends to commit more capacity than the *cost minimisation algorithm*, as shown in Fig. 5.3. In addition, it has been observed that the spot prices of the *price optimisation algorithm* are much less volatile than those of the *cost minimisation algorithm*, as illustrated in Fig. 5.4.

Table 5.10: Production costs and customers' payments for the 26-unit system for cost allocation Schemes 3 and 4

	Scheme 3		***Scheme 4***	
	GC (\$)	PC (\$)	GC (\$)	PC (\$)
Load level 1	725,664	1,840,722	725,664	1,866,919
Load level 2	589,741	1,193,759	586,836	1,247,962
Load level 3	600,823	1,184,067	599,265	1,200,783
Load level 4	763,394	2,100,140	763,394	2,107,940

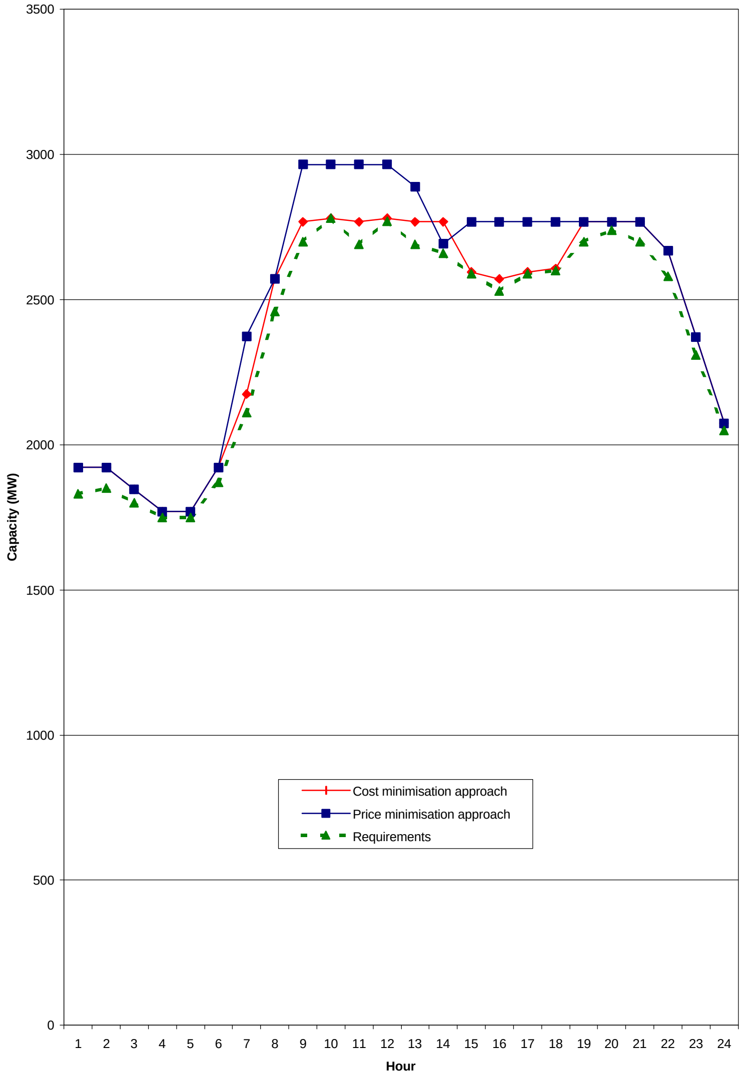

Fig. 5.3: Capacity requirements and capacities committed by the *price and cost minimisation algorithm* for the 26-unit system, Load level 2

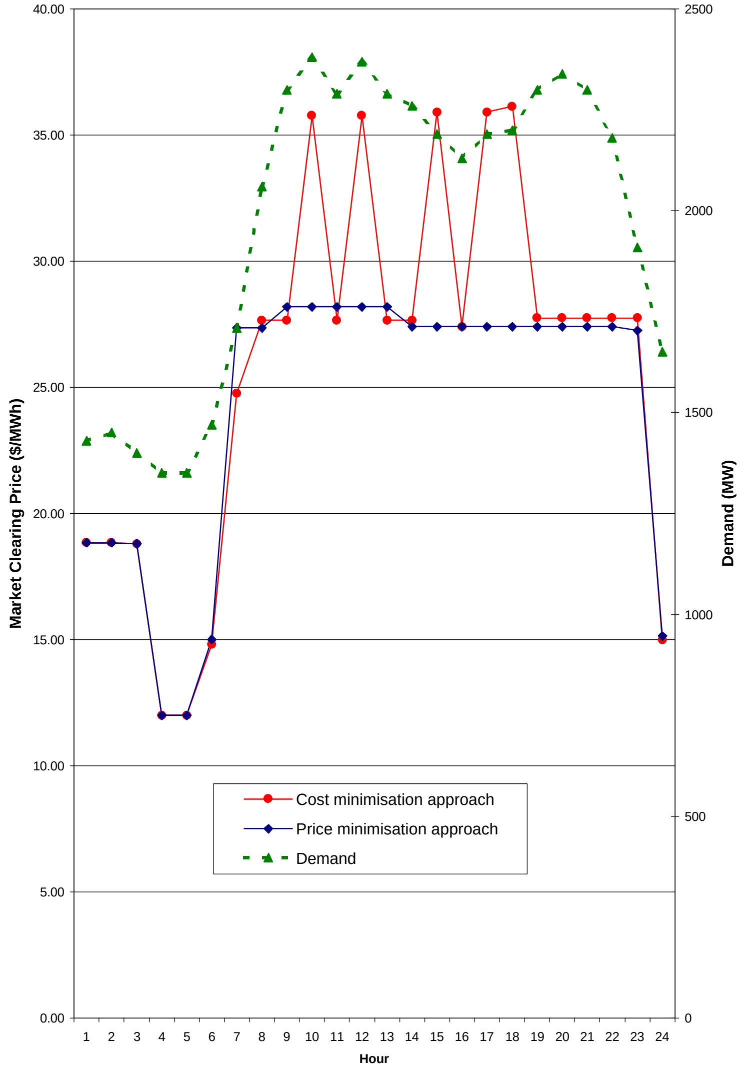

Fig. 5.4: Spot prices of electricity and demand for the 26-unit system Load level 2, obtained by the *price and cost minimisation algorithms*

5.5.3 110-Unit System

The 110-unit system has a total capacity of 20,502 MW, and peak load and minimum load of 16,500 MW and 9,300 MW, respectively. The complete data for this system is presented in Appendix B. Table 5.11 presents the total generators' production costs (GC) and the total customers' payments, or pool costs, (PC) resulting from the application of the *price minimisation scheduling algorithm* to the 110-unit system.

Similarly, compared to the results of the *cost minimisation approach* presented in Table 4.9, the *price minimisation approach* is more advantageous to the customers.

Table 5.11: Production costs and customers' payments for the 110-unit system for cost allocation Schemes 3 and 4

Scheme 3		***Scheme 4***	
GC ($)	*PC* ($)	*GC* ($)	*PC* ($)
3,755,780	6,007,996	3,755,609	6,101,035

5.6 Discussion

The results of the simulations performed in this chapter show that changing the objective of the generation scheduling algorithm from a minimisation of the generators' production "costs" to a minimisation of the customers' payments is feasible. Even though the solutions produced by the proposed algorithm are not optimal, they yield a substantial reduction in cost to the consumers and an accompanying increase in production costs. It should be noted, however, that the prices are calculated in such a way that the increase in production cost does not cause the producers to lose money, that is the generators are guaranteed to recover their bidding prices.

Nevertheless, in a real market with perfect competition, the "objective" is neither the minimisation of customer payments nor the minimisation of production costs for a fixed quantity. In such a market, a natural equilibrium establishes itself at the point where the marginal value to consumers is equal to the marginal cost to producers. This equilibrium maximises the social welfare, which is the sum of the consumers' surplus and the producers' profit. An electricity market based on a centralised pool is an artificial market and not a real one. The generators' bids do not necessarily reflect their actual costs and the optimisation that is being carried out does not maximise the social welfare or accurately reflect the natural behaviour of consumers. There is thus no sound theoretical justification for minimising "production costs" rather than consumer payments.

The proposed method is complex, computationally demanding and produces results that are even harder to interpret than those produced by the optimisation used in the EPEW. Its value is therefore mostly theoretical in the sense that it demonstrates that it is possible to combine the optimisation and pricing aspects. It shows that the main issue is not so much the type of optimisation that is performed but the concept of simulating the behaviour of a real market using an optimisation procedure.

5.7 Summary

This chapter has presented an alternative generation scheduling algorithm in which the objective is to minimise the total customers' payments. A forward DP algorithm is used to search for the UC schedule. The fixed cost allocation Schemes 3 and 4, as defined in Section 4.2.3 and Section 4.2.4, was used in the price computation algorithm. The implementation of the *price minimisation scheduling problem* is feasible. The algorithm to solve the problem, albeit not optimal, reduces the total payments by consumers while ensuring cost recovery to the generating companies. The proposed method is complex and computationally demanding but extremely useful to assess the theoretical implications of the introduction of the customers' interests in the generation scheduling problem.

CHAPTER 6

Assessing Generators' Bidding Strategies

6.1 Introduction

The investigation of generators' bidding strategies in competitive electricity markets has been the subject of previous publications. Some conceptual models and mathematical tools to understand a genuine commercially competitive electricity supply market have been described by David [9]. He assumed that producers bid against each other to supply the grid and that the bid prices reflect what part of the market each supplier hopes to win. This depends on production costs estimates, temporal considerations of system demand variation, unit commitment costs, and commercial considerations such as profit or economic utility maximisation and expectation of system behaviour. A supplier can use the theory and algorithm to bid different generating units taking into account the forecast competitor behaviour and the expected system requirements. Uncertainties in these factors can be included. Simple pairs of quantity-prices are offered and the revenue earned by each unit winning the auction is based on a "*pay as bid*" approach.

Lucas and Taylor [11, 12] used a model based on game theory [131, 132] to assess the generators' behaviour in the Electricity Pool of England and Wales (EPEW). They worked on the problem of finding the set of bids that maximises a generator's profit, assuming that the bids from the competitors and the demand of the system are known. They also considered the influence of contracts in the market for electricity and argue that, in the absence of contracts and market power, all generators tend to offer prices close to their marginal costs. Market power is characterised by the situation in which one or more generators are absolutely needed to satisfy the demand and hence there is a percentage of the electricity market where there is no competition. In such cases, a monopolistic situation prevails and the generators' profit cannot be limited by the market rules. Taylor presented detailed results of the investigation of the operation of the EPEW by relating the theoretical concepts of market behaviour to the practical experience of the EPEW [13].

A general optimisation-based framework for the analytic investigation of bidding in competitive power pools has been presented by Gross and Finlay [14]. The pool framework incorporates the constraints of the electric system and generating units, such as reserve requirements and minimum up and down times. The framework uses the Lagrangian relaxation technique to firstly solve the least-cost dispatch and commitment problem faced by the pool operator. Then the optimal bidding strategy for a bidder is formulated. The authors argue that, under conditions of perfect competition, and regardless of generation resources, costs and constraints, a generator maximises profits by bidding to supply generation and cost at maximum availability. In their framework generators receive their own start-up bidding price each time a unit is started up. This is slightly different from the EPEW, where generating units receive the start-up bidding price of the marginal unit, that is the unit that sets the market-clearing price.

Several other authors have addressed the strategic bidding problem in a wholesale electricity market [15-19]. In general, it is assumed that a generating company formulates its optimisation problem by forecasting its competitors' bidding curves based on historical data and heuristic knowledge of the competitor's structure. They all assumed a model in which the market participants offer energy price and quantity for every trading interval, the so-called simple bids. However, they do not discuss the generators' problem of converting the several parameters of their operational costs into simple quantity-price bids.

The main objective of this chapter is to further discuss the generators' bidding strategies in the light of the knowledge gained in the previous chapters on generation scheduling and auction pricing mechanisms assuming a complex bidding structure. A simplified model of a pool-based electricity market has been developed in this project to investigate some aspects that might influence the generators' bidding profile in a competitive environment. However, the aim of this chapter is not to perform a detailed analysis of the strategic bidding behaviour of the generating companies, but rather to discuss the requirements of a model to successfully represent the main aspects of the market operation.

6.2 Generators' Profit Optimisation Problem

The generators' profit optimisation problem can be modelled as a strategic game in which generators *play* against each other to maximise their own benefits. Game theory [131, 132] can then be used to model the problem. Game theory is a branch of economics in which *players* are involved in a *game* that requires taking *actions and decisions* based on *information* to optimise their *payoffs*. The *players,* or the decision-makers, are the generation companies of which *payoffs* are their economic benefits, that is their profits. A player's *actions and decisions* are the choices the player can make. A *strategy* is a rule that guides the players' choice of action. *Information* can be described as the players' knowledge about the value of different variables at a particular time (e.g., the knowledge of the system demand and reserve requirements). Another important element of a game is the *outcome*, which is defined as the set of relevant elements that the modeller selects from the values of actions, payoffs and other variables after the game is played out. The players, actions and outcomes are collectively referred as the rules of the game, and the modeller's objective is to use the rules of the game to determine the equilibrium [132]. The situation in which no player has an incentive to deviate from its strategy given that the other players do no deviate from theirs is known as a "*Nash equilibrium*", which is the most common equilibrium in a game.

Several other definitions may be required to fully describe a game. In general, games can be divided into *zero sum* games, in which the sum of payoffs of all players is constant, that is one player's gain is another's loss; or *non-zero sum* games, of which one player's decision can benefit other players. In addition, a game can be classified into a *co-operative* game, in which the players can make binding commitments to coordinate their strategies, or a *non co-operative* game, in which a player selects the strategy that is optimal against the actions of its competitors. Moreover, a game can be played once (*one-off* game) or repeatedly (*repeated* game), in which the history of the games can be used as additional information for the players. The generators' profit maximisation problem can be described as a non co-operative, repeated, non-zero sum game.

As mentioned previously, no attempts will be made to fully study the problem using the concepts of game theory, in this chapter. The objective of this chapter is rather to review some of the important aspects for the assessment of the generators' strategic bidding behaviour. In this regard, this discussion will be focused on the evaluation of the outcome of the game, which requires the execution of the scheduling program whenever the bidding prices of a unit are changed. This process is based on a *fixed target* idea where a unit changes its bids against a fixed set of bids from its competitors.

Having a tool to simulate the market operation and to assess the outcome of a game is extremely important because perfect competition is a theoretical concept, which is unlikely to be verified by the analytical speculation of the market operation or by very simple models. Hence the more detailed is the model, the better is its ability to assess the generators' bidding strategies in a real competitive electricity market. For example, some concern has been raised about the ability of generating companies to manipulate prices in the EPEW by playing with the different parameters of their bidding files [2-4].

The model of a pool-based competitive electricity market from the perspective of the centralised scheduling entity, as illustrated in Fig. 2.1, is rather different from the model as seen by an individual market participant. Fig. 6.1 illustrates the model of the market from the perspective of one generating company that attempts to optimise its profit. The following sections present some additional comments regarding the main blocks of the generators' profit optimisation problem.

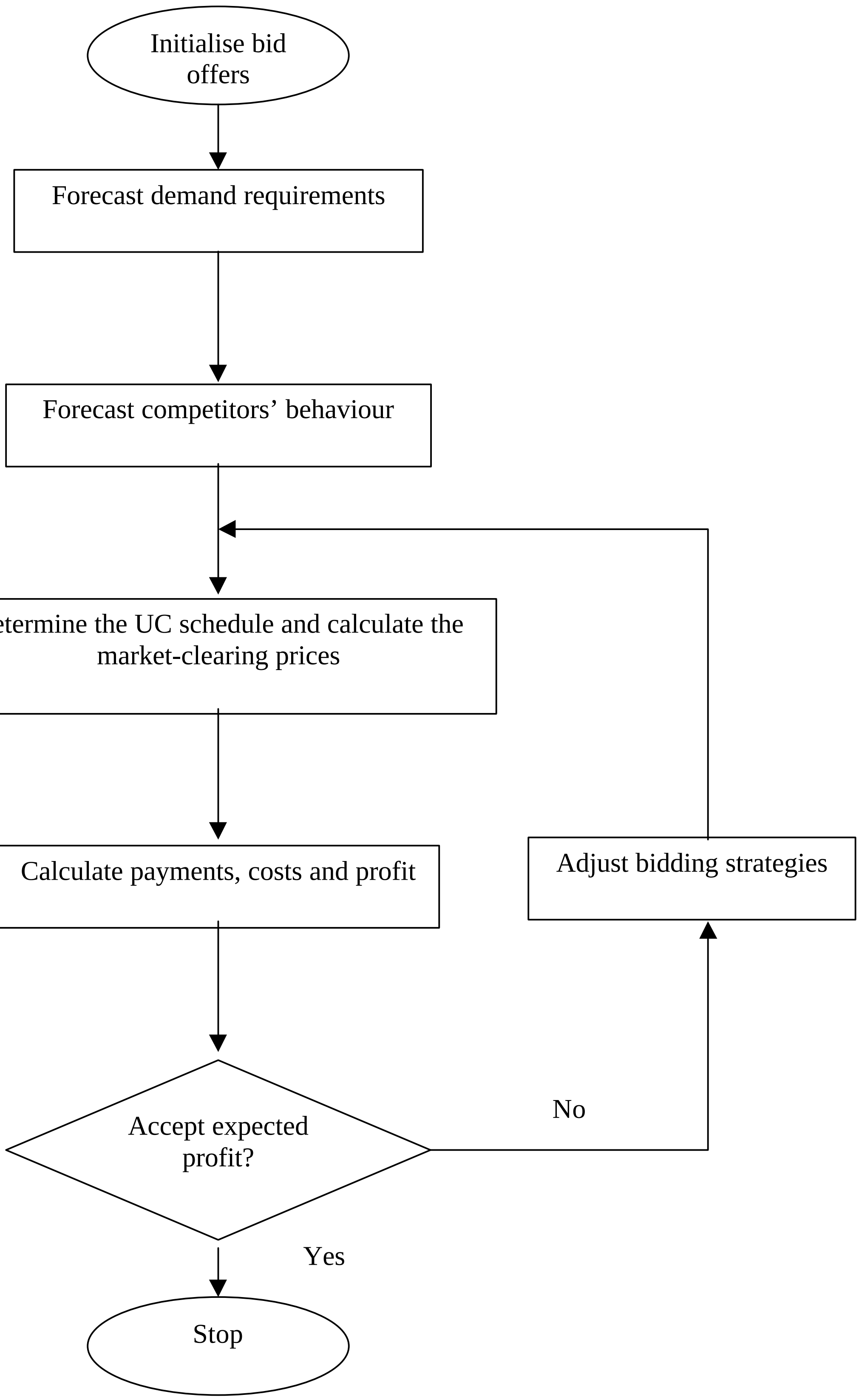

Fig. 6.1: Model of a pool-based electricity market from the perspective of an individual company

6.2.1 Demand Forecasting

Demand is assumed to be known and the capacity of the largest unit was adopted as the spinning reserve requirements for all periods. This is part of the generators' knowledge of the system. As demand is assumed non-responsive to prices, customers are not players in the game.

6.2.2 Bids Forecasting

When defining a bidding strategy, a generator assumes that its competitors will be using similar strategies. A major simplification of the model is to assume that the generators have a finite and discrete set of strategies to choose. Furthermore, a generator defines the strategies of its competitors in a deterministic way. The probabilistic assessment of these strategies may require the inclusion of uncertainties in the process. The stochastic analysis of the competitors' bidding behaviour could be performed by a Monte Carlo simulation. However, this is out of the scope of this research project.

6.2.3 Generation Scheduling and Price Computation

This is a major block in which the outcome of the game is determined. In a market with complex bidding parameters the least-cost dispatch and commitment problem is assessed through a UC algorithm. In this chapter, the *cost minimisation scheduling algorithm* is used to select the generating units. In the simulation of games presented in this chapter, the hybrid algorithm that combines the Lagrangian relaxation and Dynamic Programming techniques described in Section 3.6 has been used. The generators' payments are based upon the market-clearing prices. The revenue achieved by each generator is determined according to the pricing mechanisms described in Chapter 4, adopting the fixed cost allocation Scheme 2 (Table A/B scheme) to calculated the unit prices. The computation of revenue does not include payment for reserve, and it has been assumed that the loss of load probability (*lolp*) is negligible and hence the revenue does not include the capacity element described in Section 2.4.

6.2.4 Adjusting Bidding Strategies

It is assumed that the operational constraints reflect the actual characteristics of the generating units. The units' operational characteristics are kept constant and only their incremental, no-load and start-up bidding prices are adjusted. In the simulation of simple games, the above parameters are multiplied by an adjustable factor to determine the generators' bidding characteristics. A simple exercise is also proposed in which the bidding parameters are adjusted using some heuristic knowledge of the market operation.

6.2.5 Convergence

Theoretically, the process should be interrupted when the equilibrium is achieved, that is when no generator has incentive to change its bidding strategy given that the others do not change, that is the *"Nash equilibrium"*.

6.3 Simulation of Simple Games

Some tests were performed by adjusting the unit bidding prices and the results were analysed. The games were formulated by assuming that two generating companies are competing against each other while assuming that the other generators have no incentive to bid differently than their costs. In the games discussed in this section the generating units have different capacities and cost structures, and hence they are classified as asymmetric games.

6.3.1 10-Unit System

In this first game, it is assumed that one generator owns one single unit (U60) and the other generator also owns only one unit (U80). It is also assumed that the other generating units bid their true costs. The unit cost data and operational constraints used in the case study are summarised in Appendix B. The bidding parameters of Table B.3 are considered the true units' operational costs. Table 6.1 presents the outcome of the game between unit U60 and unit U80. The first column and first row of the table show the bidding strategies of units U60 and unit U80, respectively. That is, the bidding prices of unit U60 and unit U80 are determined by the simple product of their true cost parameters and the corresponding figures in the first column and first row of the table. In each cell, the figure on the bottom left corner represent the profit of unit U60 whereas the figure on top right corner represent the profit of U80. It can be seen that the profit of unit U80 cannot be limited by the competition and hence unit U80 can exercise market power. This situation would require the intervention of the market operator. The shaded area shows that the equilibrium of this game is achieved when unit U80 bids as high as possible and unit U60 bids 1.2 times its operational costs.

Table 6.1: Outcome of the game between U60 and U80 from the 10-unit system

U80 **U60**	**0.95**	**1.00**	**1.05**	**1.10**	**1.15**	**1.20**	**10.00**
0.95	50 0	46 -10	32 -11	32 -11	10 -35	13 -35	584 -35
1.00	50 0	44 0	32 0	32 0	29 0	39 0	584 0
1.05	50 0	44 0	32 7	32 7	29 7	39 7	584 15
1.10	50 0	44 0	32 15	32 15	29 15	39 15	584 30
1.15	50 0	44 0	52 0	32 22	29 22	39 22	584 45
1.20	50 0	44 0	52 0	60 0	30 30	39 30	**584** **60**
10.00	50 0	44 0	52 0	60 0	58 0	77 0	3,461 0

The reserve requirements of the 10-unit system is reduced from 550 MW to 450 MW to simulate a game in which neither unit U60 nor unit U80 is absolutely necessary to satisfy the demand and reserve requirements. Table 6.2 presents the outcome of this second game. It can be seen that there is no scope for the exercise of market power and that the optimal strategy for both players is to bid at 1.05 times their own operational costs.

Table 6.2: Outcome of the game between U60 and U80 from the 10-unit system with reduced reserve requirements

U80 / **U60**	**0.95**	**1.00**	**1.05**	**1.10**	**1.15**	**1.20**	**10.00**
0.95	-20 0	0 -11	16 -11	0 -11	0 -11	0 -11	0 -11
1.00	-20 0	0 0	16 0	0 0	0 0	0 0	0 0
1.05	-20 0	0 0	**16** **7**	0 0	0 0	0 0	0 7
1.10	-20 0	0 0	16 0	0 0	0 0	0 0	0 0
1.15	-20 0	0 0	16 0	0 0	0 0	0 0	0 0
1.20	-20 0	0 0	30 0	0 0	0 0	0 0	0 0
10.00	-20 0	0 0	30 0	0 0	0 0	0 0	0 0

6.3.2 26-Unit System

This section presents the results of a game between two generators of the 26-unit system, Load level 2. It is assumed that generating company G_1 owns a portfolio of three units (U100a, U100b and U100c) and that, units U197a, U197b and U197c belong to generating company G_2. It is again assumed that the other generators are bidding at their true costs. The bidding prices of G_1 and G_2 are determined by multiplying their true costs by the adjustable factor in the first column and first row of the table, respectively. It is assumed that the bidding parameters of Table B.5 are the true operational costs of the generating units.

Table 6.3 presents the outcome of the above game between generators G_1 and G_2. The figures on the top right corner of each cell represent the profit of the most expensive unit of the portfolio of G_2, that is unit U197c, for the corresponding pair of bidding strategies. Similarly, the figures on the bottom left corner are the profits of the most expensive unit of the portfolio of G_2, that is unit U100c. It can be observed that this game does not have an equilibrium point and hence it is not possible to define the generators' optimal bidding strategies by simply assessing the profits of their units individually.

Table 6.3: Outcome of the game between generators G_1 and G_2 from the 26-unit system, load level 2, in which the cost allocation Scheme 2 is used

U197c / **U100c**	**0.95**	**1.00**	**1.05**	**1.10**	**1.15**	**1.20**	**10.00**
0.95	0 / 829	0 / 1,971	0 / 2,427	0 / 2,221	0 / 3,504	0 / 3,450	0 / 107,566
1.00	-562 / 0	0 / 1,959	0 / 1,761	0 / 2,333	0 / 3,289	0 / 4,009	0 / 107,590
1.05	-562 / 0	0 / 1,965	0 / 2,538	0 / 2,407	0 / 3,449	0 / 4,044	0 / 107,613
1.10	-3,864 / 0	0 / 2,021	0 / 2,464	0 / 2,974	0 / 3,650	0 / 3,624	0 / 103,962
1.15	-664 / 0	0 / 2,404	0 / 2,688	0 / 3,116	0 / 3,509	0 / 4,018	0 / 107,659
1.20	-557 / 0	591 / 0	0 / 3,067	0 / 3,226	0 / 3,492	0 / 4,135	0 / 107,801
10.00	11,672 / 0	11,867 / 0	12,376 / 0	12,886 / 0	12,810 / 0	13,700 / 0	0 / 103,621

Table 6.4 presents the outcome of the game from the perspective of the generators' profits and not based upon the profits of their individual units. It can be observed that the equilibrium is achieved when both generators bid as high as they can and hence the generators' ability to exercise market power can be easily identified. By comparing the

cell in which the equilibrium is verified with the corresponding cell from Table 6.3, it can be seen that one of the units from the portfolio of generator G_1 (unit U197c) makes zero profit (in this case it was not called upon to generate). However, the profit made by the other two units (unit U197a and unit U197b) is high enough to offset this unit being out of the UC schedule.

Table 6.4: Outcome of the game between generators G_1 and G_2 from the 26-unit system, Load level 2

G_2 / G_1	**0.95**	**1.00**	**1.05**	**1.10**	**1.15**	**1.20**	**10.00**
0.95	1,689 4,518	2,246 7,033	5,091 9,297	5,708 8,746	9,445 12,776	11,134 12,808	377,475 355,216
1.00	526 5,554	3,107 7,792	3,381 7,056	5,886 8,950	8,148 11,593	11,562 14,392	377,475 355,309
1.05	2,441 6,506	2,718 7,366	1,170 9,497	6,027 9,196	8,567 12,130	10,792 14,108	377,475 355,402
1.10	3,790 3,905	3,362 8,050	4,679 8,989	6,512 10,534	9,847 13,279	11,177 13,229	381,248 344,728
1.15	-1,572 2,947	5,637 9,466	5,483 9,640	8,298 11,932	8,724 12,353	10,792 14,070	377,475 355,587
1.20	461 1,536	2,730 2,979	6,855 11,350	7,310 11,474	8,706 12,409	11,115 14,589	377,475 356,608
10.00	45,022 0	47,128 0	48,854 0	49,414 0	43,234 0	45,532 0	**383,548** **377,994**

6.4 Simulation of Complex Games

As mentioned previously, it has been reported that the generating companies in England and Wales increase their profits by "playing" with all parameters of their bidding files [2-4]. It has been argued that with a combination of first incremental price equal to zero, zero start-up prices, zero no-load prices and a very high second incremental price for the last few MW of its output, a generating unit can be scheduled to generate when the system requires a relatively small increment of energy. Hence the system-clearing price is set by the very high incremental price. It has also been reported that generators can combine high incremental prices with high no-load and start-up prices with close elbow points to increase the incidence of spike prices.

In this section, an attempt is made to define the optimum bidding strategy for unit U80 of the 10-unit system, of which the reserve requirements were reduced from 550 MW to 450 MW to eliminate the scope for the exercise of market power. The objective is to search for some heuristics to optimise the profit of unit U80 by adjusting its incremental, no-load and start-up prices in a more sophisticated way. No changes are made on the elbow points and on the other unit's operational parameters.

From Table 6.2 it can be seen that the highest profit that unit U80 can achieve is equal to \$ 16, when its bidding prices are multiplied by a constant equal to 1.05. Table 6.5 summarises the outcome of this combination of strategies presenting the total output power, total production cost, total revenue and total profit of all generating units. The last row of Table 6.5 shows the energy production, the total production cost and the total customers' payments for the whole scheduling horizon.

The search for the optimum set of bids were made in a trial-and-error basis, in which the bids were adjusted and the scheduling program run to determine the unit's profit. Some heuristic knowledge of the market operation has been used in an attempt to define an optimum bidding strategies by adjusting the bid parameters of unit U80. For example, as the scheduling process is continuous, the units' initial statuses play an important role in the search for the optimum bidding strategy. If a unit is running on the last period of the previous day, its start-up price is not so important in defining the unit commitment. On the other hand, the start-up bidding prices are decisive in the current scheduling day if the unit is not running on the last period of the preceding day. Moreover, the search was more effective when it started with zero no-load and start-up prices and the adjustments were made only in the incremental prices. After a few trials, it was found that the profit of unit U80 could be improved by the following set of bids: $inc_i^1 = inc_i^2$ = 2.28 \$/MWh; inc_i^3 = 2.60 \$/MWh, nl_i^1 = 28 \$/h and a_i = \$ 200.

Table 6.6 summarises the outcome of the combination of strategies, in which unit U80 bids as above and unit U60 bids at 1.05 its operational costs. One should notice that the implementation of the above bids slightly change the profits of all units, except unit U60, of which the profit does not change at all. In addition, it should be observed that the total customers' payments reduces 0.04 % while the total generators' production costs increase 0.01%. This is another example of the phenomenon discussed in previous chapters of this thesis, in which the minimisation of the customers' payments is not always obtained by minimising the generators' production costs.

Table 6.5: Units' output, cost, revenue and profit for the 10-unit system, with reduced reserve requirements, when unit U60 and unit U80 bid 1.05 times their own costs

Unit	Total Output	Total cost	Total revenue	Total profit	
	(MWh)	(\$)	(\$)	(\$)	(%)
U60	30	149	157	7	5.0
U80	120	324	340	16	5.0
U100	0	0	0	0	0
U120	1,144	2,799	3,433	634	22.7
U150	2,267	5,308	6,569	1,261	23.8
U280	1,859	4,581	5,536	955	20.9
U320	4,212	8,681	11,136	2,456	28.3
U445	9,187	18,463	24,627	6,164	33.4
U520	10,273	20,519	27,662	7,144	34.8
U550	9,676	18,603	25,412	6,809	36.6
	38,770	**79,449**	**104,872**		

Table 6.6: Units' output, cost, revenue and profit for the 10-unit system, with reduced reserve requirements, when unit U60 bids 1.05 times its own costs and unit U80 chose a complex bidding strategy

Unit	Total Output	Total cost	Total revenue	Total profit	
	(MWh)	($)	($)	($)	(%)
U60	30	149	157	7	5.0
U80	180	463	494	**32**	6.9
U100	0	0	0	0	0.0
U120	1,144	2,799	3,410	611	21.8
U150	2,267	5,308	6,529	1,221	23.0
U280	1,859	4,581	5,499	918	20.0
U320	4,212	8,681	11,087	2,407	27.7
U445	9,187	18,463	24,510	6,047	32.8
U520	10,240	20,439	27,437	6,999	34.2
U550	9,650	18,538	25,231	6,692	36.1
	38,770	**79,459**	**104,353**		

6.5 Discussion

Each generator uses some heuristic knowledge about the market behaviour to adjust its bidding prices in pursuit of the optimum profit. The initial results have shown that every generating unit tends to bid lower and lower until they reach their cost-plus-a-profit-margin limit. This is in agreement with the remarks found in some recent publications.

The situation in which market power can be exercised requires the intervention of the market regulator and is of no interest in this project. However, due to the complexity of the market structure, the exercise of market power cannot always be easily identified. The enormous number of possibilities to be investigated in a complex bidding structure prevented us from reaching any finding that could be generalised. Nevertheless, to proper assess the outcomes of the games it is important to design a model that can most closely simulate the operation of the real market.

6.6 Summary

One objective of this thesis was to develop a mathematical model of a competitive electricity market to investigate the generators' strategic bidding behaviour. This chapter has presented a simple optimisation approach for the investigation of the operation and bidding aspects of a competitive electricity power pool. In this approach generators are modelled as independent agents trying to maximise their profit. The solution of the least-cost dispatch and commitment problem is obtained by a hybrid UC algorithm that combines the Lagrangian relaxation and the Dynamic Programming technique. The market-clearing prices and generators' payments are determined based upon the rules of the EPEW.

In preparing its bid, each agent uses some knowledge of the pool operation and assumes that the other agents may be using the same strategy as at the previous iteration. Each agent passes on its bidding prices to the scheduling program, which establishes the global schedule. Each generator then analyses this schedule and tries to improve its bid for the next round in an iterative process.

Some tests have been performed where the bidding prices of a 10- and 26-unit system were adjusted and the results were analysed. It was assumed that no participant of the pool exercised market power. The results have shown that the generating units tend to offer prices close to their operational costs. However any model designed to fully represent the most important features of the electricity market is likely to be complex albeit essential for the assessment of the generators' bidding behaviour.

CHAPTER 7

Conclusions and Recommendations for Further Research

7.1 Conclusions

The introductory chapter of this thesis proposed a number of fundamental questions regarding the operation of competitive electricity markets. The subsequent chapters reported the investigation of key issues related to the equity and efficiency of the mathematical models currently used to simulate the market operation. This investigation is performed through the implementation of mathematical models and the development of computational algorithms designed to simulate the operation of an electricity market. This chapter summarises the findings that contribute to the formulation of possible answers to those questions and presents the main conclusions of this research project. It also suggests some topics for further investigation.

Research questions: are the solutions of the UC problem generated by the available optimisation techniques of satisfactory quality for the implementation of a competitive electricity market? How can the quality of the UC schedule be improved and how does this solution affect the electricity markets? Is the traditional UC problem suitable for simulating the operation of a competitive electricity market?

The numerical results have shown that the hybrid algorithm that combines the Lagrangian relaxation (LR) and the Dynamic Programming (DP) techniques is an efficient tool to solve the problem of scheduling thermal generating units in competitive electricity markets. The results show that a poor UC solution of a small system, obtained by a stand-alone LR-based scheduling algorithm can be considerably improved by the DP-based post processor. For large systems, in which the preliminary LR solution is of good quality, the improvement obtained by the post-processor can be small if not negligible. However, it has been shown that a minor improvement in the quality of a solution can result in considerable changes in the profits of the generating units and in the total customers' payments. Furthermore, it has been shown that the traditional UC problem is not adequate to simulate the market operation because the pricing rules differ from the criteria to select the units called upon to generate.

Research questions: What are the consequences of changes in the fixed cost allocation scheme to the prices of electricity? Would one scheme benefit generators and another benefit customers?

The implementation of different fixed cost allocation schemes resulted in significant variations in the electricity prices, generators' revenue and customers' payments. When the fixed costs are allocated in the actual period they are incurred, the volatility of the electricity prices is unacceptably high, mainly due to the concentration of the start-up costs in only one period. The Table A/B scheme is efficient and produces lower total customers' payments compared to the others. However, due to the nature of the *Willans line*, the prices of Table B periods can be higher than prices in Table A periods. This is in conflict with the theoretical purpose of this classification, which is aimed to encourage lower prices when there is plenty of spare capacity and higher prices when the system is stressed, thus encouraging demand side management. In addition, the Table A/B scheme requires side payments to guarantee the cost recovery of generating units that are scheduled to generate only during Table B periods. The other two schemes do not pose the drawbacks albeit they yield to total customers' payments slightly higher than the Table A/B scheme does, in some cases.

Research questions: Is the standard ED problem in the interest of the customers? Is it possible to formulate and solve an alternative ED to minimise the total customers' payments? Would this alternative ED algorithm be equitable in the context of an electricity market?

The implementation of an alternative economic dispatch (ED) algorithm, of which the objective function is the total customers' payments, tends to increase the production of generating units with higher bidding prices if this minimises the overall cost to consumers. This is not equitable because a generating unit can increase its market share by increasing its bidding price. Therefore, the ED problem should be formulated and solved using the standard approach, in which the objective function is the generators' production costs.

Research questions: What are the effects of incorporating the customers' interests in the scheduling problem? Is it possible to formulate such a scheduling problem and implement an efficient algorithm to solve it? Would this problem ensure the equity of the electricity market?

Formulating a generation scheduling problem in which the objective is to minimise the customers' payments rather than the generators' production "costs" is feasible. However, the algorithm to solve such an optimisation problem is complex, computationally demanding and produces results that are even harder to interpret than those produced by the optimisation used in the EPEW. The solutions produced by the proposed algorithm albeit not optimal yield a considerable reduction in costs to the consumers and an accompanying increase in production costs, while ensuring generation cost recovery. However, in a fair market with perfect competition, the "objective" is neither the minimisation of customers' payments nor the minimisation of production costs for a fixed quantity. The value of the *price minimisation scheduling algorithm* described in this thesis is to ratify that the equity and efficiency of an electricity market may be jeopardised by the type of optimisation used to simulate the market operation. For example, the traditional UC problem can be classified as a market in which (*i*) the generators' revenue is calculated on a *"pay as bid"* basis and (*ii*) their fixed costs are allocated in the periods where they are incurred. The use of the solution of the traditional UC problem affects the equity and efficiency of a market in which an alternative cost allocation scheme is used in the pricing computation (e.g., the Table A/B scheme in the EPEW) and the generators' payments are based on the market-clearing prices.

Research questions: Is it possible to predict the generators' bidding strategies in an electricity market of great complexity such as the EPEW? Can a model capable of simulating different bidding strategies be developed? Is it possible to assess their effects on market participants? Can the scope for "gaming" through the complex rules of the market be identified?

The assessment of the generators' bidding strategies in a pool-based electricity market is again a very complex task. The number of bidding parameters that are available is so large that it is difficult to properly simulate the "games" between market participants. Simple "games", in which some bidding parameters were kept constant and others were multiplied by an adjustable factor, were simulated. The results have shown that, regardless of some imperfections in the market model, generators tend to offer prices close to their true costs to optimise their profit. This is in agreement with previously published work.

The computational algorithms developed in this project were designed in FORTRAN 90 [133]. The computational program is dimensioned to deal with systems of up to 300 generating units over a scheduling horizon that can be divided into 48 half-hourly periods.

7.2 Recommendations for Further Research

As discussed in Section 3.7.1, some additional heuristics could be used when selecting the candidate units to determine the variable window size for the DP-based post-processor. The investigation of the set of heuristics to be augmented to the criteria used to determine the variable window size, described in Section 3.6.2, could further improve the efficiency of the hybrid LR-DP algorithm.

It has been argued that a complex bidding process reduces the transparency of the pricing setting mechanism, and even though it reduces the generators' risks associated with bidding their fixed costs, it may not lead to lower prices for electricity. Therefore, in some electricity markets, the generators' offer involves only pairs of quantity-price bids. These simple bids would certainly increase the transparency and simplicity of the market framework. Nevertheless, they would increase the risks associated with the amortisation of the generators' fixed costs due to uncertainties regarding the part of the market that they are likely to win. Assessing the risks associated with converting the generators' complex cost structure into simple quantity-price bids is an interesting issue to be addressed.

Assessing whether the introduction of simple bids would alleviate the problem of "gaming" and would increase the transparency and equity of an electricity market is an interesting issue for further investigation. The search for robust bidding strategies may require the introduction of uncertainties in the model.

Another interesting issue for further investigation is the study of the strategic bidding behaviour of generating companies in a electricity market of which the scheduling problem is formulated on the basis of the minimisation of the total customers' payments rather than the minimisation of the total generators' production costs.

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APPENDIX A

Linearisation of the Polynomial Price Functions

The original polynomial price functions were replaced by piece-wise linear functions, where the two elbow points (e_i^1 and e_i^2) were obtained by dividing the range between the minimum stable generation (P_i^{min}) and the maximum capacity (P_i^{max}) of each unit into three equal segments. The incremental prices (inc_i^1, inc_i^2, inc_i^3) are such that the prices at P_i^{min}, e_i^1, e_i^2, and P_i^{max} are equal to those obtained with the polynomial functions. This procedure can yield the generation of negative no-load prices, as illustrated in Fig. A.1. This is the case of units U320 and U520 in the 10-unit system, and a few other units in the 110-unit system. Obviously, those negative figures have no physical meaning. As they do not affect the concept and results discussed in this thesis, no special treatment will be given to them.

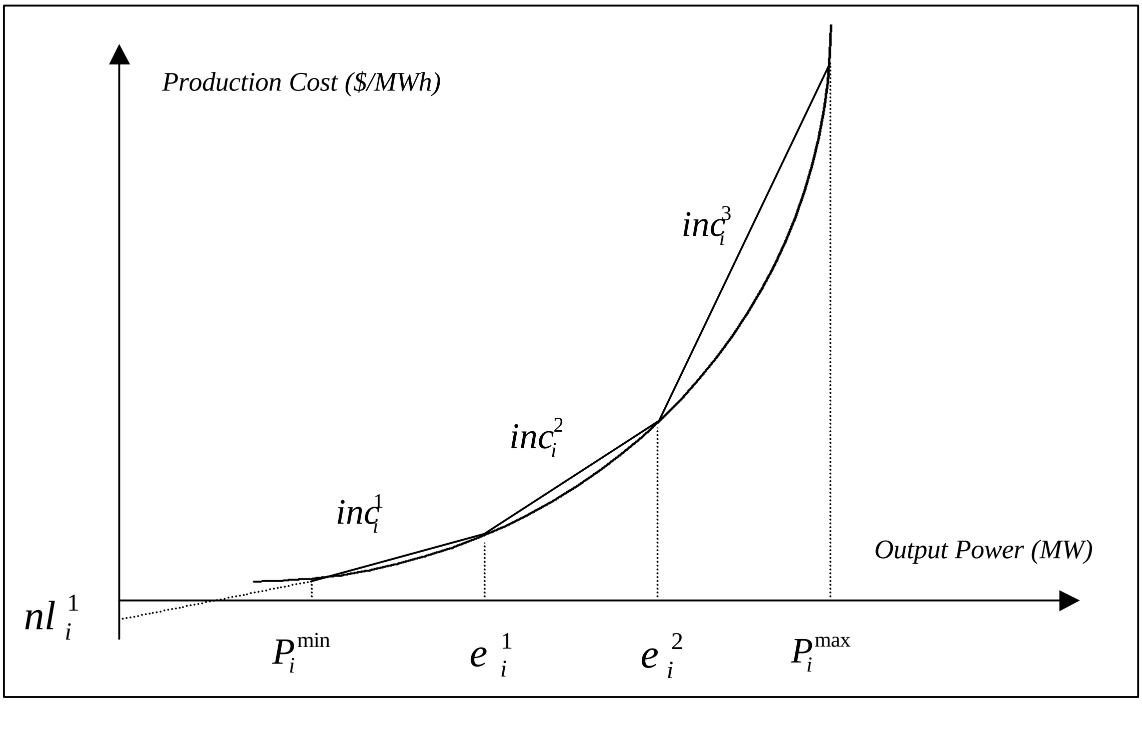

Fig. A.1: Linearisation of the polynomial price functions

APPENDIX B

Data for the Case Studies

B.1 4-Unit System

The data for this test system were obtained from [8] and is summarised in Table B.1 and Fig. B.1.

Table B.1: Bidding prices and operational characteristics of the 4-unit system

Unit	$P_i^{\min}$ (MW)	e_i^1 (MW)	e_i^2 (MW)	$P_i^{\max}$ (MW)	nl_i^1 ($/h)	inc_i^1 ($/MWh)	inc_i^2 ($/MWh)	inc_i^3 ($/MWh)	a_i ($)	b_i ($)	t_i (h)	T_i^{up} (h)	T_i^{down} (h)	X_i^0 (h)
U740	130			740	300	13.0			600	0	1	1	1	-1
U340	45			340	250	13.8			300	0	1	1	1	-1
U165	15			165	200	14.3			200	0	1	1	1	-1
U1000	100			1000	0	0.0			0	0	1	1	1	-1
				2245										

Period	Demand (MW)
1	1800
2	2150
3	1900

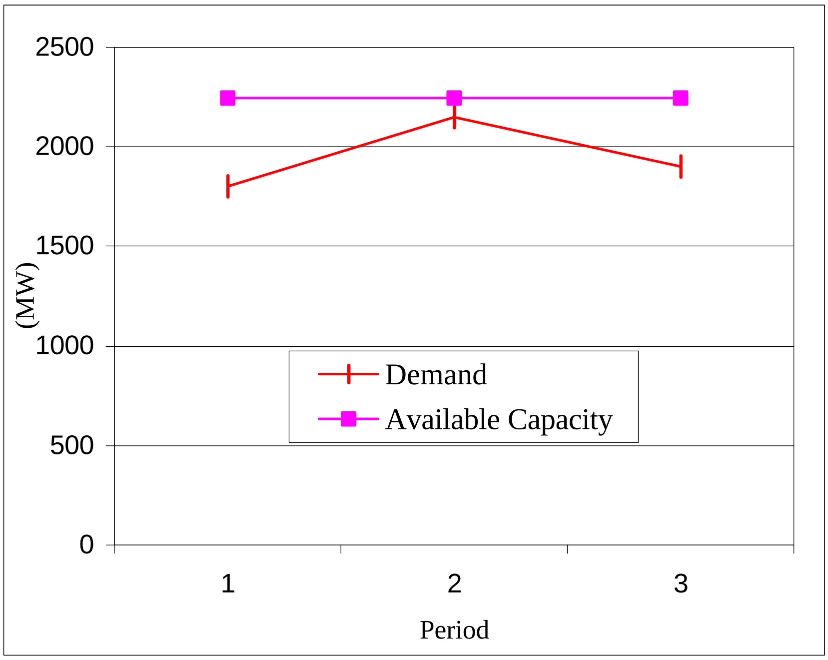

Fig. B.1: Demand requirements of the 4-unit system

B.2 10-Unit System

The data for this test system were obtained from [84]. Table B.2 presents the parameters of the original polynomial cost functions, and Table B.3 shows the parameters of the piece-wise linear cost functions for the 10-unit system.

Table B.2: Original polynomial fuel cost functions for the 10-unit system

Unit	a ($/MWh^2)	b ($/MWh)	c ($/h)
U60	0.00510	2.2034	15
U80	0.00396	1.9161	20
U100	0.00393	1.8518	40
U120	0.00382	1.6966	32
U150	0.00212	1.8015	29
U280	0.00261	1.5354	72
U320	0.00289	1.2643	49
U445	0.00148	1.2136	82
U520	0.00127	1.1954	105
U550	0.00135	1.1285	100

Table B.3: Bidding prices and operational characteristics of the 10-unit system

Unit	$P_i^{\min}$	e_i^1	e_i^2	$P_i^{\max}$	nl_i^1	inc_i^1	inc_i^2	inc_i^3	a_i	b_i	t_i	T_i^{up}	T_i^{down}	X_i^0
	(MW)	(MW)	(MW)	(MW)	($/h)	($/MWh)	($/MWh)	($/MWh)	($)	($)	(h)	(h)	(h)	(h)
U60	15	30	45	60	12.70	2.4329	2.5859	2.7389	50.98	0	1	1	1	1
U80	20	40	60	80	21.83	2.1537	2.3121	2.4705	60.09	0	1	1	1	1
U100	30	53.3	76.7	100	33.71	2.1793	2.3627	2.5461	67.88	0	1	1	1	1
U120	25	56.7	88.3	120	26.59	2.0086	2.2505	2.4924	55.92	0	1	1	1	1
U150	50	83.3	117	150	20.17	2.0842	2.2255	2.3668	67.32	0	1	1	1	1
U280	75	143	212	280	43.94	2.1053	2.4620	2.8187	95.97	0	1	1	1	1
U320	120	187	253	320	-15.74	2.1506	2.5359	2.9212	101.94	0	1	1	1	1
U445	125	232	338	445	39.14	1.7415	2.0572	2.3729	114.05	0	1	1	1	1
U520	250	340	430	520	-2.95	1.9447	2.1733	2.4019	134.08	0	1	1	1	1
U550	100	250	400	550	66.25	1.6010	2.0060	2.4110	141.58	0	1	1	1	1
				2,625										

Period	Demand (MW)
1	2000
2	1980
3	1940
4	1900
5	1840
6	1870
7	1820
8	1700
9	1510
10	1410
11	1320
12	1260
13	1200
14	1160
15	1140
16	1160
17	1260
18	1380
19	1560
20	1700
21	1820
22	1900
23	1950
24	1990

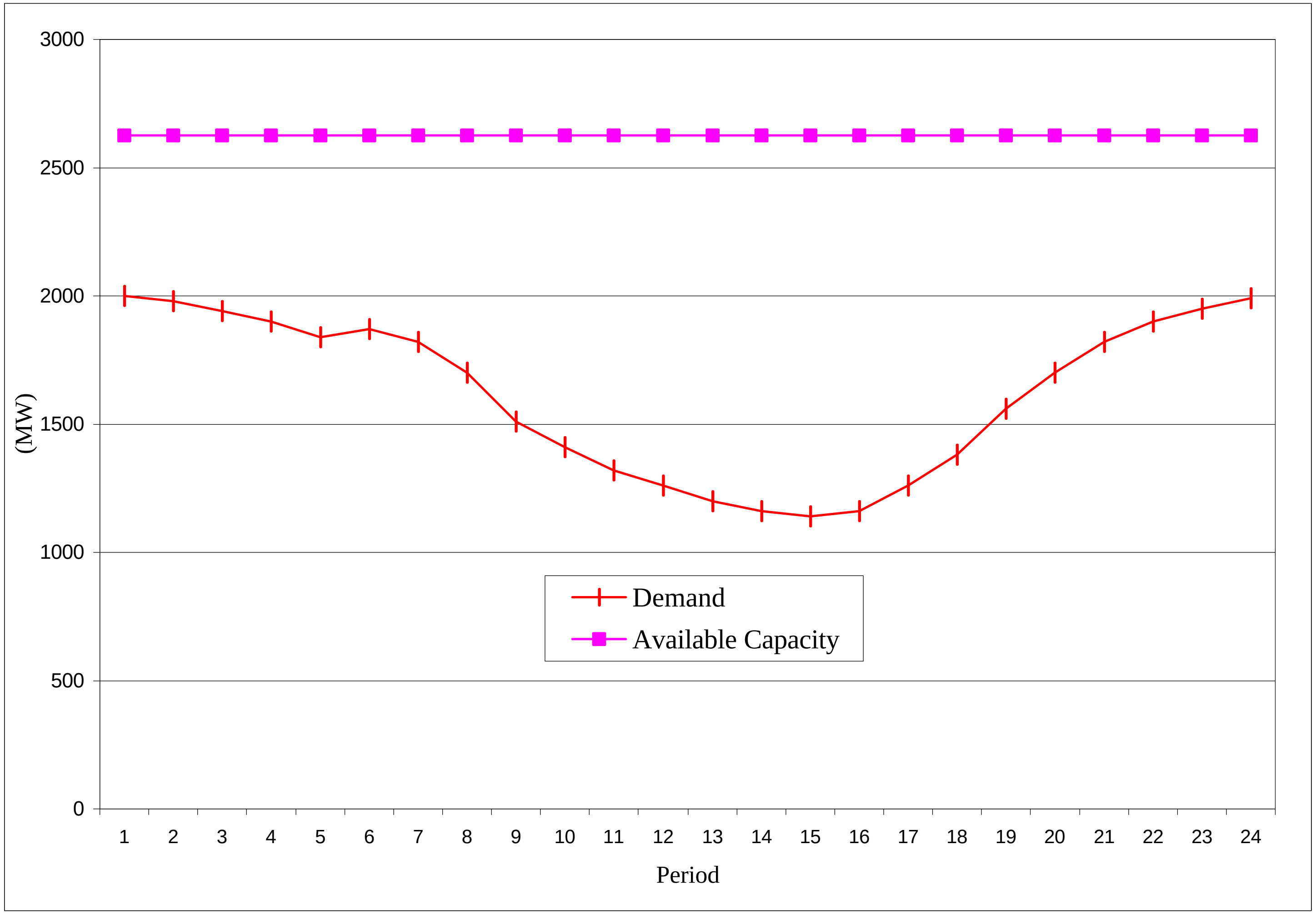

Fig. B.2: Demand requirements of the 10-unit system

B.3 26-Unit System

This test system derived from the IEEE-RTS [128], and the data can also be found in [108, 119]. Table B.4 presents the parameters of the original polynomial cost functions, and Table B.5 shows the parameters of the piece-wise linear cost functions for the 26-unit system. Fig. B.3 presents the four different load levels considered for this system.

Table B.4: Original polynomial fuel cost functions for the 26-unit system

Unit	a ($/MWh^2)	b ($/MWh)	c ($/h)
U12a	0.02533	25.5472	24.3891
U12b	0.02649	25.6753	24.4110
U12c	0.02801	25.8027	24.6382
U12d	0.02842	25.9318	24.7605
U12e	0.02855	26.0611	24.8882
U20a	0.01199	37.5510	117.7551
U20b	0.01261	37.6637	118.1083
U20c	0.01359	37.7770	118.4576
U20d	0.01433	37.8896	118.8206
U76a	0.00876	13.3272	81.1364
U76b	0.00895	13.3538	81.2980
U76c	0.00910	13.3805	81.4641
U76d	0.00932	13.4073	81.6259
U100a	0.00623	18.0000	217.8952
U100b	0.00612	18.1000	218.3350
U100c	0.00598	18.2000	218.7752
U155a	0.00463	10.6940	142.7348
U155b	0.00473	10.7154	143.0288
U155c	0.00481	10.7367	143.3179
U155d	0.00487	10.7583	143.5972
U197a	0.00259	23.0000	259.1310
U197b	0.00260	23.1000	259.6490
U197c	0.00263	23.2000	260.1760
U350	0.00153	10.8616	177.0575
U400a	0.00194	7.4921	310.0021
U400b	0.00195	7.5031	311.9102

Table B.5: Bidding prices and operational characteristics of the 26-unit system

Unit	$P_i^{\min}$ (MW)	e_i^1 (MW)	e_i^2 (MW)	$P_i^{\max}$ (MW)	nl_i^1 ($/h)	inc_i^1 ($/MWh)	inc_i^2 ($/MWh)	inc_i^3 ($/MWh)	a_i ($)	b_i ($)	t_i (h)	T_i^{up} (h)	T_i^{down} (h)	X_i^0 (h)
U12a	2.40	5.60	8.80	12.00	24.0487	25.7498	25.9119	26.0741	0	0	1	0	0	-1
U12b	2.40	5.60	8.80	12.00	24.0550	25.8872	26.0568	26.2263	0	0	1	0	0	-1
U12c	2.40	5.60	8.80	12.00	24.2617	26.0268	26.2060	26.3853	0	0	1	0	0	-1
U12d	2.40	5.60	8.80	12.00	24.3785	26.1592	26.3411	26.5229	0	0	1	0	0	-1
U12e	2.40	5.60	8.80	12.00	24.5045	26.2895	26.4722	26.6549	0	0	1	0	0	-1
U20a	4.00	9.33	14.67	20.00	117.3074	37.7109	37.8388	37.9667	20	20	2	0	0	-1
U20b	4.00	9.33	14.67	20.00	117.6375	37.8318	37.9663	38.1009	20	20	2	0	0	-1
U20c	4.00	9.33	14.67	20.00	117.9503	37.9582	38.1032	38.2481	20	20	2	0	0	-1
U20d	4.00	9.33	14.67	20.00	118.2856	38.0807	38.2335	38.3864	20	20	2	0	0	-1
U76a	15.20	35.47	55.73	76.00	76.4139	13.7710	14.1261	14.4812	50	50	3	3	2	3
U76b	15.20	35.47	55.73	76.00	76.4731	13.8073	14.1700	14.5328	50	50	3	3	2	3
U76c	15.20	35.47	55.73	76.00	76.5583	13.8416	14.2104	14.5793	50	50	3	3	2	3
U76d	15.20	35.47	55.73	76.00	76.6015	13.8795	14.2573	14.6351	50	50	3	3	2	3
U100a	25.00	50.00	75.00	100.00	210.1077	18.4673	18.7787	19.0903	70	70	4	4	2	-3
U100b	25.00	50.00	75.00	100.00	210.6851	18.5590	18.8650	19.1710	70	70	4	4	2	-3
U100c	25.00	50.00	75.00	100.00	211.3002	18.6485	18.9475	19.2465	70	70	4	4	2	-3
U155a	54.25	87.83	121.42	155.00	120.6731	11.3518	11.6628	11.9738	150	150	6	5	3	5
U155b	54.25	87.83	121.42	155.00	120.4906	11.3875	11.7052	12.0229	150	150	6	5	3	5
U155c	54.25	87.83	121.42	155.00	120.3985	11.4201	11.7432	12.0663	150	150	6	5	3	5
U155d	54.25	87.83	121.42	155.00	120.3918	11.4502	11.7773	12.1045	150	150	6	5	3	5
U197a	68.95	111.63	154.32	197.00	239.1956	23.4677	23.6888	23.9099	200	200	8	5	4	-4
U197b	68.95	111.63	154.32	197.00	239.6819	23.5695	23.7915	24.0134	200	200	8	5	4	-4
U197c	68.95	111.63	154.32	197.00	239.9326	23.6749	23.8994	24.1240	200	200	8	5	4	-4
U350a	140.00	210.00	280.00	350.00	132.0757	11.3971	11.6113	11.8255	300	200	8	8	5	10
U400a	100.00	200.00	300.00	400.00	271.2020	8.0741	8.4621	8.8501	500	500	8	8	5	10
U400b	100.00	200.00	300.00	400.00	272.9100	8.0881	8.4781	8.8681	500	500	10	8	5	10
				3,105.00										

Period	Load level1 (MW)	Load level2 (MW)	Load level3 (MW)	Load level4 (MW)
1	1700	1430	1400	2223.0
2	1730	1450	1430	2052.0
3	1690	1400	1390	1938.0
4	1700	1350	1400	1881.0
5	1750	1350	1450	1824.0
6	1850	1470	1550	1825.5
7	2000	1710	1700	1881.0
8	2430	2060	2130	1995.0
9	2540	2300	2240	2280.0
10	2600	2380	2300	2508.0
11	2670	2290	2370	2565.0
12	2590	2370	2290	2593.0
13	2590	2290	2290	2565.0
14	2550	2260	2250	2508.0
15	2620	2190	2320	2479.5
16	2650	2130	2350	2479.5
17	2550	2190	2250	2593.5
18	2530	2200	2230	2850.0
19	2500	2300	2200	2821.5
20	2550	2340	2250	2764.5
21	2600	2300	2300	2679.0
22	2480	2180	2180	2622.0
23	2200	1910	1900	2479.5
24	1840	1650	1540	2308.5

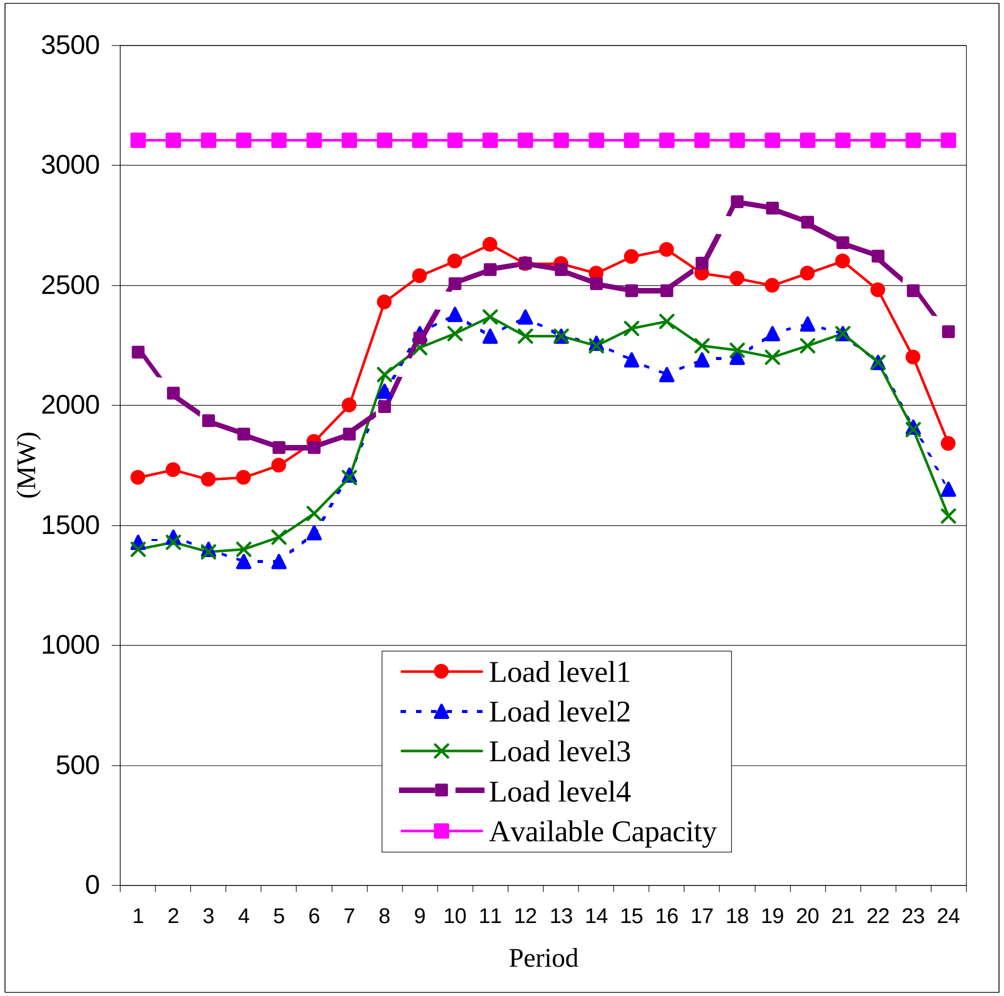

Fig. B.3: Demand requirements of the 26-unit system

B.4 110-Unit System

The data for test system were collected from [118]. Table B.6 presents the parameters of the original polynomial cost functions. The parameters of the piece-wise linear cost functions for this 110-unit test system is presented in Table B.7.

Table B.6: Original polynomial fuel cost functions for the 110-unit system

Unit	a ($/MWh^2)	b ($/MWh)	c ($/h)
U12a	0.0253	25.547	24.389
U12b	0.0265	25.675	24.411
U12c	0.0280	25.803	24.638
U12d	0.0284	25.932	24.76
U12e	0.0286	26.061	24.888
U15a	0.0353	26.547	26.389
U15b	0.0365	26.675	25.411
U20a	0.0050	13.500	50.000
U20b	0.0120	37.551	117.755
U20c	0.0126	37.664	118.108
U20d	0.0136	37.777	118.458
U20e	0.0143	37.890	118.821
U22a	0.0380	26.803	25.638
U22b	0.0384	26.932	25.76
U32a	0.0353	26.547	34.389
U32b	0.0365	26.675	34.411
U35	0.0021	14.400	90.000
U40a	0.0011	13.400	80.000
U40b	0.0070	14.500	60.000
U50a	0.0051	23.000	60.000
U50b	0.0031	9.407	23.626
U52a	0.0380	26.803	34.638
U52b	0.0384	26.932	34.761
U52c	0.0386	17.061	34.888
U55a	0.0061	24.000	70.000
U55b	0.0033	14.300	80.000
U60a	0.0210	15.300	65.000
U60b	0.0320	38.551	127.755
U60c	0.0326	36.664	128.108
U60d	0.0236	38.777	128.458
U60e	0.0243	38.890	128.821
U70	0.0023	13.300	70.000

Unit	a ($/MWh^2)	b ($/MWh)	c ($/h)
U76a	0.0088	13.327	81.136
U76b	0.0089	13.354	81.298
U76c	0.0091	13.080	81.464
U76d	0.0093	13.407	81.626
U80a	0.0078	13.200	200.000
U80b	0.0088	14.200	210.000
U80c	0.0230	16.000	82.000
U90	0.0099	11.380	32.464
U96a	0.0098	14.327	82.136
U96b	0.0099	14.354	82.298
U100a	0.0062	18.000	217.895
U100b	0.0061	18.100	218.335
U100c	0.0060	18.200	218.775
U100d	0.0039	12.500	220.000
U100e	0.0034	12.900	115.000
U100f	0.0092	14.380	82.464
U100g	0.0094	14.407	82.626
U100h	0.0034	13.900	125.000
U100I	0.0043	13.600	400.000
U100j	0.0240	20.200	86.000
U120a	0.0067	12.800	150.000
U120b	0.0072	19.000	218.895
U120c	0.0071	19.100	219.335
U120d	0.0070	19.200	219.775
U120e	0.0049	13.500	230.000
U120f	0.0350	20.200	84.000
U140	0.0066	13.700	50.000
U150a	0.0340	25.600	75.000
U150b	0.0350	26.000	68.000
U155a	0.0046	10.694	142.735
U155b	0.0047	10.715	143.029
U155c	0.0048	10.737	143.318
U155d	0.0049	10.758	143.597
U180	0.0056	12.700	40.000
U185a	0.0066	11.694	143.735
U185b	0.0057	11.715	144.029
U185c	0.0058	11.737	144.318
U185d	0.0059	11.758	144.597
U197a	0.0026	23.000	259.131
U197b	0.0026	23.100	259.649
U197c	0.0026	23.200	260.176
U197d	0.0036	24.000	269.131
U197e	0.0036	24.100	269.649
U197f	0.0036	24.200	270.176

Unit	a ($/MWh^2)	b ($/MWh)	c ($/h)
U200a	0.0026	12.200	240.000
U200b	0.0036	13.200	250.000
U200c	0.0022	13.400	150.000
U200d	0.0260	27.000	72.000
U220a	0.0023	12.600	300.000
U220b	0.0037	13.800	160.000
U250a	0.0012	12.400	140.000
U250b	0.0055	12.354	42.298
U280	0.0370	30.500	56.000
U300	0.0054	13.327	52.136
U320	0.0280	25.800	69.000
U325	0.0048	11.300	130.000
U350	0.0015	10.862	177.057
U360a	0.0038	10.300	120.000
U360b	0.0025	11.862	187.057
U400a	0.0019	7.492	210.002
U400b	0.0019	7.503	211.910
U400c	0.0043	9.900	90.000
U400c	0.0029	8.492	320.002
U400d	0.0030	8.503	321.910
U440a	0.0012	7.400	250.000
U440b	0.0053	8.900	80.000
U440c	0.0022	8.400	260.000
U450	0.0024	14.000	220.000
U500a	0.0014	12.000	210.000
U500b	0.0013	12.100	180.000
U500c	0.0055	7.600	110.000
U520	0.0390	32.500	67.000
U560	0.0045	6.600	100.000
U600a	0.0023	13.100	190.000
U600b	0.0032	7.500	170.000
U660	0.0022	6.500	160.000
U700a	0.0067	6.200	130.000
U700b	0.0077	7.200	140.000

Table B.7: Bidding prices and operational characteristics of the 110-unit system

Unit	P_i^{min} (MW)	e_i^1 (MW)	e_i^2 (MW)	P_i^{max} (MW)	nl_i^1 ($/h)	inc_i^1 ($/MWh)	inc_i^2 ($/MWh)	inc_i^3 ($/MWh)	a_i ($)	b_i ($)	t_i (h)	Sd_i ($)	T_i^{up} (h)	T_i^{down} (h)	X_i^0 (h)
U12a	2.40	5.60	8.80	12.00	24.0500	25.7494	25.9113	26.0732	0	0	1	0	0	0	-1
U12b	2.40	5.60	8.80	12.00	24.0500	25.8870	26.0566	26.2262	0	0	1	0	0	0	-1
U12c	2.40	5.60	8.80	12.00	24.2600	26.0270	26.2062	26.3854	0	0	1	0	0	0	-1
U12d	2.40	5.60	8.80	12.00	24.3800	26.1592	26.3410	26.5227	0	0	1	0	0	0	-1
U12e	2.40	5.60	8.80	12.00	24.5000	26.2898	26.4728	26.6559	0	0	1	0	0	0	-1
U15a	3.60	7.40	11.20	15.00	25.4500	26.9353	27.2036	27.4719	0	0	1	0	0	0	-1
U15b	3.60	7.40	11.20	15.00	24.4400	27.0765	27.3539	27.6313	0	0	1	0	0	0	-1
U20a	5.00	10.00	15.00	20.00	49.7500	13.5750	13.6250	13.6750	10	15	1	0	1	1	-2
U20b	4.00	9.30	14.70	20.00	117.3100	37.7110	37.8390	37.9670	20	20	2	0	0	0	-1
U20c	4.00	9.30	14.70	20.00	117.6400	37.8320	37.9664	38.1008	20	20	2	0	0	0	-1
U20d	4.00	9.30	14.70	20.00	117.9500	37.9583	38.1034	38.2485	20	20	2	0	0	0	-1
U20e	4.00	9.30	14.70	20.00	118.2900	38.0807	38.2332	38.3857	20	20	2	0	0	0	-1
U22a	4.40	10.30	16.10	22.00	23.9200	27.3603	27.8062	28.2521	0	0	1	0	0	0	-1
U22b	4.40	10.30	16.10	22.00	24.0300	27.4952	27.9458	28.3963	0	0	1	0	0	0	-1
U32a	5.40	14.30	23.10	32.00	31.6700	27.2412	27.8672	28.4932	0	0	1	0	0	0	-1
U32b	5.40	14.30	23.10	32.00	31.6000	27.3928	28.0401	28.6874	0	0	1	0	0	0	-1
U35	10.00	18.30	26.70	35.00	89.6200	14.4595	14.4945	14.5295	20	30	1	0	0	0	-1
U40a	10.00	20.00	30.00	40.00	79.7800	13.4330	13.4550	13.4770	10	20	1	0	1	1	-1
U40b	12.00	21.30	30.70	40.00	58.2100	14.7333	14.8640	14.9947	40	25	1	0	1	1	-2
U50a	12.00	24.70	37.30	50.00	22.7100	9.5207	9.5992	9.6777	68	30	2	0	1	1	-1
U50b	10.00	23.30	36.70	50.00	58.8100	23.1700	23.3060	23.4420	25	10	1	0	2	1	-3
U52a	8.40	22.90	37.50	52.00	27.4500	18.2705	19.3924	20.5144	0	0	1	0	1	1	-1

Unit	$P_i^{\min}$ (MW)	e_i^1 (MW)	e_i^2 (MW)	$P_i^{\max}$ (MW)	nl_i^1 ($/h)	inc_i^1 ($/MWh)	inc_i^2 ($/MWh)	inc_i^3 ($/MWh)	a_i ($)	b_i ($)	t_i (h)	Sd_i ($)	T_i^{up} (h)	T_i^{down} (h)	X_i^0 (h)
U52b	8.40	22.90	37.50	52.00	27.3200	27.9937	29.0982	30.2027	0	0	1	0	1	1	-1
U52c	8.40	22.90	37.50	52.00	27.3600	28.1352	29.2514	30.3675	0	0	1	0	1	1	-1
U55a	20.00	31.70	43.30	55.00	77.9100	14.4705	14.5475	14.6245	60	400	1	0	1	1	-2
U55b	10.00	25.00	40.00	55.00	68.4800	24.2135	24.3965	24.5795	35	20	1	0	1	1	-3
U60a	10.00	26.70	43.30	60.00	59.4000	16.0700	16.7700	17.4700	20	85	5	15	1	3	-1
U60b	12.00	28.00	44.00	60.00	117.1500	37.9680	39.0112	40.0544	30	30	2	0	1	2	-1
U60c	12.00	28.00	44.00	60.00	120.5300	39.7210	40.4762	41.2314	30	30	2	0	1	2	-1
U60d	12.00	28.00	44.00	60.00	117.0000	39.8310	40.8550	41.8790	30	30	2	0	1	2	-1
U60e	12.00	28.00	44.00	60.00	120.6600	39.8620	40.6396	41.4172	30	30	2	0	1	2	-1
U70	20.00	36.70	53.30	70.00	68.3100	13.4303	13.5070	13.5837	50	300	1	0	1	1	-2
U76a	15.20	35.50	55.70	76.00	76.5600	13.5411	13.9099	14.2788	50	50	3	0	2	3	3
U76b	15.20	35.50	55.70	76.00	76.3900	13.7729	14.1296	14.4863	50	50	3	0	2	3	3
U76c	15.20	35.50	55.70	76.00	76.5000	13.8049	14.1657	14.5264	50	50	3	0	2	3	3
U76d	15.20	35.50	55.70	76.00	76.6100	13.8782	14.2552	14.6321	50	50	3	0	2	3	3
U80a	20.00	40.00	60.00	80.00	193.7600	13.6680	13.9800	14.2920	40	30	2	0	2	3	-4
U80b	20.00	40.00	60.00	80.00	202.9600	14.7280	15.0800	15.4320	50	40	2	0	2	2	-4
U80c	10.00	33.30	56.70	80.00	74.3300	16.9967	18.0700	19.1433	20	101	5	25	1	3	-1
U90	30.00	50.00	70.00	90.00	17.6100	12.1720	12.5680	12.9640	60	90	2	0	2	2	-1
U96a	25.20	48.80	72.40	96.00	70.0800	15.0522	15.5148	15.9773	60	60	3	0	2	3	3
U96b	25.20	48.80	72.40	96.00	70.1200	15.0866	15.5539	16.0212	60	60	3	0	2	3	3
U100a	25.00	50.00	75.00	100.00	215.1300	12.7925	12.9875	13.1825	10	60	2	0	3	2	-2
U100b	25.00	50.00	75.00	100.00	110.7500	13.1550	13.3250	13.4950	10	150	2	0	2	2	-1
U100c	40.00	60.00	80.00	100.00	389.6800	14.0300	14.2020	14.3740	160	40	3	0	3	2	-1
U100d	20.00	46.70	73.30	100.00	121.8300	14.1267	14.3080	14.4893	20	160	2	0	3	2	-1

Unit	$P_i^{\min}$ (MW)	e_i^1 (MW)	e_i^2 (MW)	$P_i^{\max}$ (MW)	nl_i^1 ($/h)	inc_i^1 ($/MWh)	inc_i^2 ($/MWh)	inc_i^3 ($/MWh)	a_i ($)	b_i ($)	t_i (h)	Sd_i ($)	T_i^{up} (h)	T_i^{down} (h)	X_i^0 (h)
U100e	35.00	56.70	78.30	100.00	64.2200	15.2233	15.6220	16.0207	60	60	3	0	3	3	3
U100f	35.00	56.70	78.30	100.00	63.9800	15.2687	15.6760	16.0833	60	60	3	0	3	3	3
U100g	25.00	50.00	75.00	100.00	210.1500	18.4650	18.7750	19.0850	70	70	4	0	2	4	-3
U100h	25.00	50.00	75.00	100.00	210.7100	18.5575	18.8625	19.1675	70	70	4	0	2	4	-3
U100i	25.00	50.00	75.00	100.00	211.2700	18.6500	18.9500	19.2500	70	70	4	0	2	4	-3
U100j	20.00	46.70	73.30	100.00	63.6000	21.8000	23.0800	24.3600	22	114	5	40	2	4	1
U120a	20.00	53.30	86.70	120.00	142.8500	13.2913	13.7380	14.1847	15	120	3	0	2	4	-3
U120e	20.00	53.30	86.70	120.00	224.7700	13.8593	14.1860	14.5127	20	70	2	0	3	3	-2
U120b	45.00	70.00	95.00	120.00	196.2200	19.8280	20.1880	20.5480	80	80	4	0	3	4	-3
U120c	45.00	70.00	95.00	120.00	196.9700	19.9165	20.2715	20.6265	80	80	4	0	3	4	-3
U120d	45.00	70.00	95.00	120.00	197.7200	20.0050	20.3550	20.7050	80	80	4	0	3	4	-3
U120f	20.00	53.30	86.70	120.00	46.6700	22.7667	25.1000	27.4333	10	84	5	32	2	4	5
U140	30.00	66.70	103.30	140.00	36.8000	14.3380	14.8220	15.3060	60	90	3	0	3	3	-4
U150a	30.00	70.00	110.00	150.00	-5.5000	29.5000	32.3000	35.1000	45	282	11	49	2	4	3
U150b	40.00	76.70	113.30	150.00	-29.2700	29.5667	32.0600	34.5533	18	113	5	29	3	5	-7
U155a	54.30	87.90	121.40	155.00	120.7900	11.3480	11.6568	11.9656	150	150	6	0	3	5	5
U155b	54.30	87.90	121.40	155.00	120.6000	11.3832	11.6987	12.0142	150	150	6	0	3	5	5
U155c	54.30	87.90	121.40	155.00	120.4200	11.4194	11.7416	12.0639	150	150	6	0	3	5	5
U155d	54.30	87.90	121.40	155.00	120.2200	11.4546	11.7836	12.1125	150	150	6	0	3	5	5
U180	40.00	86.70	133.30	180.00	20.5900	13.4093	13.9320	14.4547	50	80	3	0	3	4	-5
U185a	54.30	97.90	141.40	185.00	113.7400	12.5823	13.0790	13.5757	160	160	6	0	4	5	5
U185b	54.30	97.90	141.40	185.00	113.5000	12.6196	13.1249	13.6303	160	160	6	0	4	5	5
U185c	54.30	97.90	141.40	185.00	113.2400	12.6558	13.1699	13.6840	160	160	6	0	4	5	5
U185d	54.30	97.90	141.40	185.00	108.6600	12.6983	13.2734	13.8485	160	160	6	0	4	5	5
U197a	68.90	111.60	154.30	197.00	239.1400	23.4693	23.6913	23.9134	200	200	8	0	4	5	-4

Unit	P_i^{min} (MW)	e_i^1 (MW)	e_i^2 (MW)	P_i^{max} (MW)	nl_i^1 ($/h)	inc_i^1 ($/MWh)	inc_i^2 ($/MWh)	inc_i^3 ($/MWh)	a_i ($)	b_i ($)	t_i (h)	Sd_i ($)	T_i^{up} (h)	T_i^{down} (h)	X_i^0 (h)
U197b	68.90	111.60	154.30	197.00	239.6600	23.5693	23.7913	24.0134	200	200	8	0	4	5	-4
U197c	68.90	111.60	154.30	197.00	240.1800	23.6693	23.8913	24.1134	200	200	8	0	4	5	-4
U197d	70.00	112.30	154.70	197.00	240.8200	24.6564	24.9612	25.2660	210	210	8	0	4	5	-4
U197e	70.00	112.30	154.70	197.00	241.3400	24.7564	25.0612	25.3660	210	210	8	0	4	5	-4
U197f	70.00	112.30	154.70	197.00	241.8700	24.8564	25.1612	25.4660	210	210	8	0	4	5	-4
U200a	50.00	100.00	150.00	200.00	227.0000	12.5900	12.8500	13.1100	40	300	3	0	4	4	1
U200b	50.00	100.00	150.00	200.00	139.0000	13.7300	13.9500	14.1700	60	30	3	0	4	4	1
U200c	50.00	100.00	150.00	200.00	232.0000	13.7400	14.1000	14.4600	50	400	3	0	4	4	1
U200d	20.00	80.00	140.00	200.00	30.4000	29.6000	32.7200	35.8400	26	227	9	62	5	5	-3
U220a	50.00	106.70	163.30	220.00	287.7300	12.9603	13.2210	13.4817	150	50	3	0	4	5	-1
U220b	40.00	100.00	160.00	220.00	145.2000	14.3180	14.7620	15.2060	25	130	3	0	2	3	-3
U250a	75.00	133.30	191.70	250.00	128.0000	12.6500	12.7900	12.9300	50	20	3	0	4	4	-1
U250b	50.00	116.70	183.30	250.00	10.2100	13.2707	14.0040	14.7373	65	70	2	0	3	3	-1
U280	40.00	120.00	200.00	280.00	-121.6000	36.4200	42.3400	48.2600	27	176	6	42	2	5	3
U300	60.00	140.00	220.00	300.00	6.7800	14.4070	15.2710	16.1350	40	60	3	0	4	4	-1
U320	40.00	133.30	226.70	320.00	-80.3300	30.6533	35.8800	41.1067	38	187	7	70	5	5	-6
U325	80.00	161.70	243.30	325.00	67.9200	12.4600	13.2440	14.0280	300	45	4	0	4	4	-2
U350	140.00	210.00	280.00	350.00	132.9600	11.3870	11.5970	11.8070	300	200	8	0	5	8	10
U360a	110.00	193.30	276.70	360.00	39.1900	11.4527	12.0860	12.7193	200	35	4	0	4	5	-2
U360b	150.00	220.00	290.00	360.00	104.5600	12.7870	13.1370	13.4870	210	210	8	0	5	8	10
U400a	100.00	200.00	300.00	400.00	172.0000	8.0620	8.4420	8.8220	500	500	10	0	5	8	10
U400b	100.00	200.00	300.00	400.00	173.9100	8.0730	8.4530	8.8330	500	500	10	0	5	8	10

Unit	P_i^{min} (MW)	e_i^1 (MW)	e_i^2 (MW)	P_i^{max} (MW)	nl_i^1 ($/h)	inc_i^1 ($/MWh)	inc_i^2 ($/MWh)	inc_i^3 ($/MWh)	a_i ($)	b_i ($)	t_i (h)	Sd_i ($)	T_i^{up} (h)	T_i^{down} (h)	X_i^0 (h)
U400c	160.00	240.00	320.00	400.00	208.6400	9.6520	10.1160	10.5800	510	510	10	0	6	8	9
U400d	160.00	240.00	320.00	400.00	206.7100	9.7030	10.1830	10.6630	510	510	10	0	6	8	9
U400e	130.00	220.00	310.00	400.00	-32.9800	11.4050	12.1790	12.9530	400	30	5	0	8	8	3
U440a	120.00	226.70	333.30	440.00	217.3600	7.8160	8.0720	8.3280	450	30	4	0	8	7	2
U440b	100.00	213.30	326.70	440.00	213.0700	9.0893	9.5880	10.0867	460	40	4	0	6	6	2
U440c	120.00	226.70	333.30	440.00	-64.1600	10.7373	11.8680	12.9987	500	40	5	0	5	6	3
U450	160.00	256.70	353.30	450.00	121.4400	15.0000	15.4640	15.9280	600	900	4	0	5	6	5
U500a	100.00	233.30	366.70	500.00	-18.3300	9.4333	10.9000	12.3667	310	55	5	0	8	8	-6
U500b	140.00	260.00	380.00	500.00	159.0400	12.5600	12.8960	13.2320	500	800	4	0	5	8	5
U500c	140.00	260.00	380.00	500.00	132.6800	12.6200	12.9320	13.2440	250	800	4	0	7	8	-2
U520	50.00	206.70	363.30	520.00	-336.0000	42.5100	54.7300	66.9500	34	267	11	75	7	7	-5
U560	160.00	293.30	426.70	560.00	-111.2000	8.6400	9.8400	11.0400	300	45	5	0	8	8	-6
U600a	100.00	266.70	433.30	600.00	84.6700	8.6733	9.7400	10.8067	410	60	6	0	9	8	4
U600b	150.00	300.00	450.00	600.00	86.5000	14.1350	14.8250	15.5150	350	900	4	0	7	8	-2
U660	150.00	320.00	490.00	660.00	54.4000	7.5340	8.2820	9.0300	400	50	6	0	9	9	4
U700a	200.00	366.70	533.30	700.00	-361.3300	9.9967	12.2300	14.4633	650	70	8	0	12	12	4
U700b	200.00	366.70	533.30	700.00	-424.6700	11.5633	14.1300	16.6967	660	80	8	0	12	12	4
				20,502.00											

Period	Demand (MW)
1	11600
2	10900
3	9500
4	9300
5	10500
6	11200
7	12500
8	12900
9	13500
10	14500
11	14600
12	14000
13	13200
14	13000
15	14500
16	14600
17	14000
18	14700
19	15600
20	16200
21	16500
22	15000
23	14300
24	13500

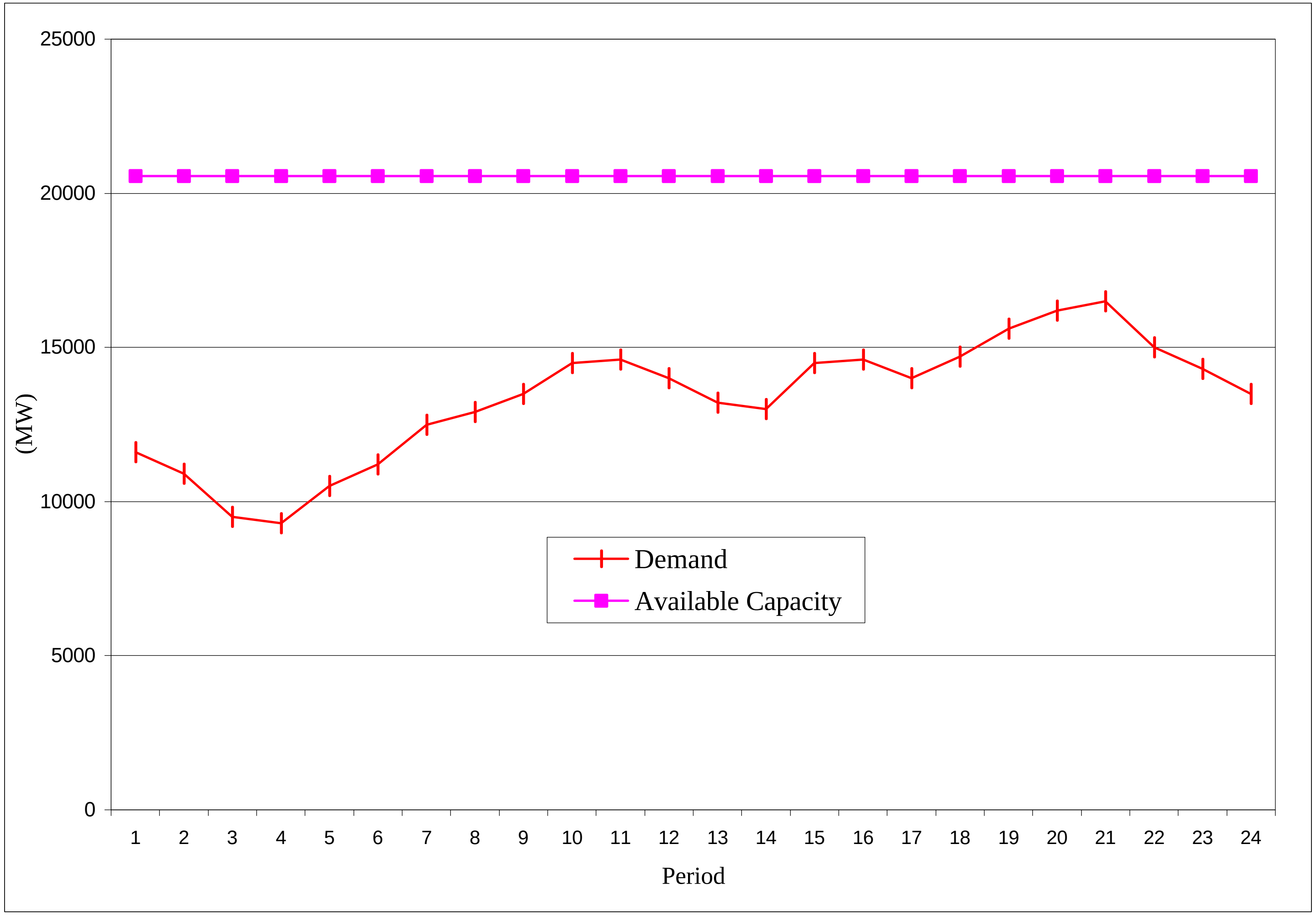

Fig. B.4: Demand requirements of the 110-unit system

APPENDIX C

The Wollenberg's Paradox

A power exchange obtains bid prices for energy from generating units and uses a unit commitment (UC) calculation to allocate energy purchases over a 24-hour period. One of the energy supply companies bidding into this exchange suspects that the UC calculation is not being fair in this allocation and obtains a court to conduct the following experiment.

The data for one 24-hour period is collected by the power exchange. The UC is calculated and the results are printed out. The entire data is duplicated and the UC is recalculated with a single change allowed, this being a slight reduction in the price for energy from one of the units owned by the company questioning the original results. All other data in this second UC calculation is identical to the first. The results are printed out.

Upon comparing the results of the first and second UC calculations, the company found that with a lower price for its energy it receives less business, i.e., it was called on to supply less energy.

This paradox was presented at the IEEE Power Engineering Society 1996 Summer Meeting in Denver CO at the Monday afternoon session "Unit Commitment in a Deregulated Environment". The idea for this paradox comes from the data in the paper "Equity and Efficiency of Unit Commitment in Competitive Electricity Markets" by R. B. Johnson, S. S. Oren, and A. J. Svoboda, Power Conference March 15, 1996, University of California, Berkeley.

The conclusion of the discussion stated that it is not incumbent upon Wollenberg to prove that the paradox can happen, but rather it is incumbent upon those who so use UC to prove that it does not.

APPENDIX D

Unit Commitment Schedule of the 110-Unit System

Table D.1 presents the solution of the UC problem obtained by the hybrid LR-DP algorithm. The table shows only the units that have been scheduled to generate for at least one period.

Table D.1: LR-DP schedule of the 110-unit system (1=on-line)

Unit	***statuses from hour 0 to hour 24***																								
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*
U15b	0																					1			
U20a	0	1																		1	1	1			
U22a	0	1																				1			
U22b	0	1																				1			
U32a	0	1																				1			
U32b	0	1																				1			
U35	0	1																				1			
U40a	0	1																		1	1	1	1		
U40b	0	1																		1	1	1			
U50a	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U52a	0	1																				1			
U52b	0	1																			1	1			
U52c	0	1																			1	1			
U55a	0	1																		1	1	1			
U55b	0																					1			
U60a	0																			1	1	1			
U70	0	1	1							1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U76a	1	1	1								1	1				1	1	1	1	1	1	1	1	1	
U76b	1	1	1								1	1				1	1	1	1	1	1	1	1	1	
U76c	1	1	1								1	1				1	1	1	1	1	1	1	1		
U76d	1	1	1								1	1				1	1	1	1	1	1	1	1		
U80a	0	1	1								1	1				1	1	1	1	1	1	1	1		
U80b	0	1	1					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U80c	0																			1	1	1			
U90	0		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U96a	1	1	1																	1	1	1			
U96b	1	1	1																	1	1	1			
U100a	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U100b	0		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U100c	0							1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
U100d	0		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U100e	1	1	1																1	1	1	1			
U100f	1	1	1																	1	1	1			

Unit	***statuses from hour 0 to hour 24***																								
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*
U100g	0																			1	1	1			
U100h	0																			1	1	1			
U100i	0																			1	1	1			
U100j	1	1																			1	1			
U120a	0		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U120e	0		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U120b	0																				1	1	1		
U120c	0																				1	1	1		
U120d	0																				1	1	1		
U120f	1	1																			1	1			
U140	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
U150b	1	1																							
U155a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U155b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U155c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U155d	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U180	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U185a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U185b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U185c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U185d	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U200a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U200b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U200c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U220a	0					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U220b	0	1	1				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U250a	0				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U250b	0			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U280	1	1																							
U300	0					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U325	0			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U350	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U360a	0				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U360b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U400a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U400b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U400c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U400d	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U400e	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U440a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U440b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U440c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U450	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
U500a	0			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U500b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U500c	0							1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U560	0			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U600a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U600b	0							1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U660	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U700a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
U700b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1