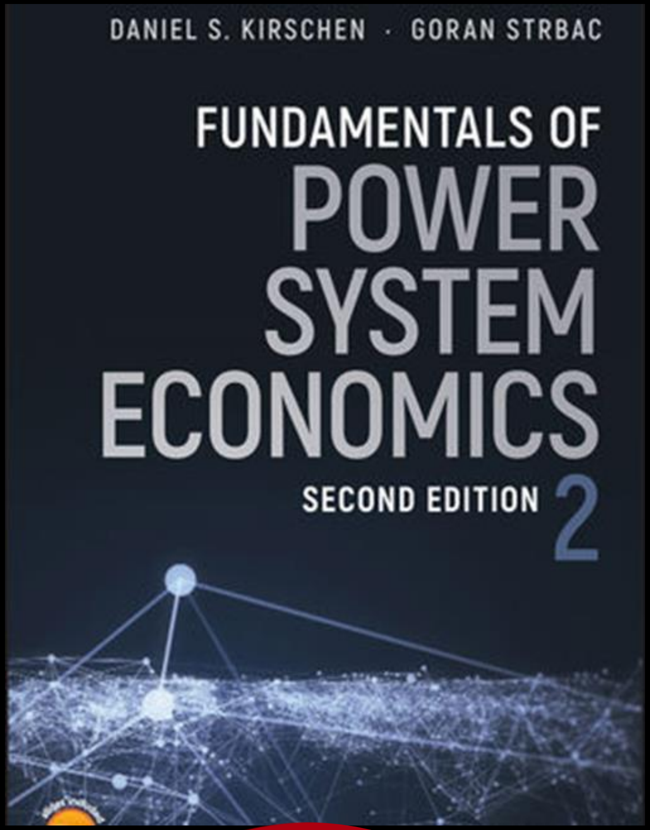
How do we keep the lights on?

Dr. Abeer Almaimouni

Disclaimer

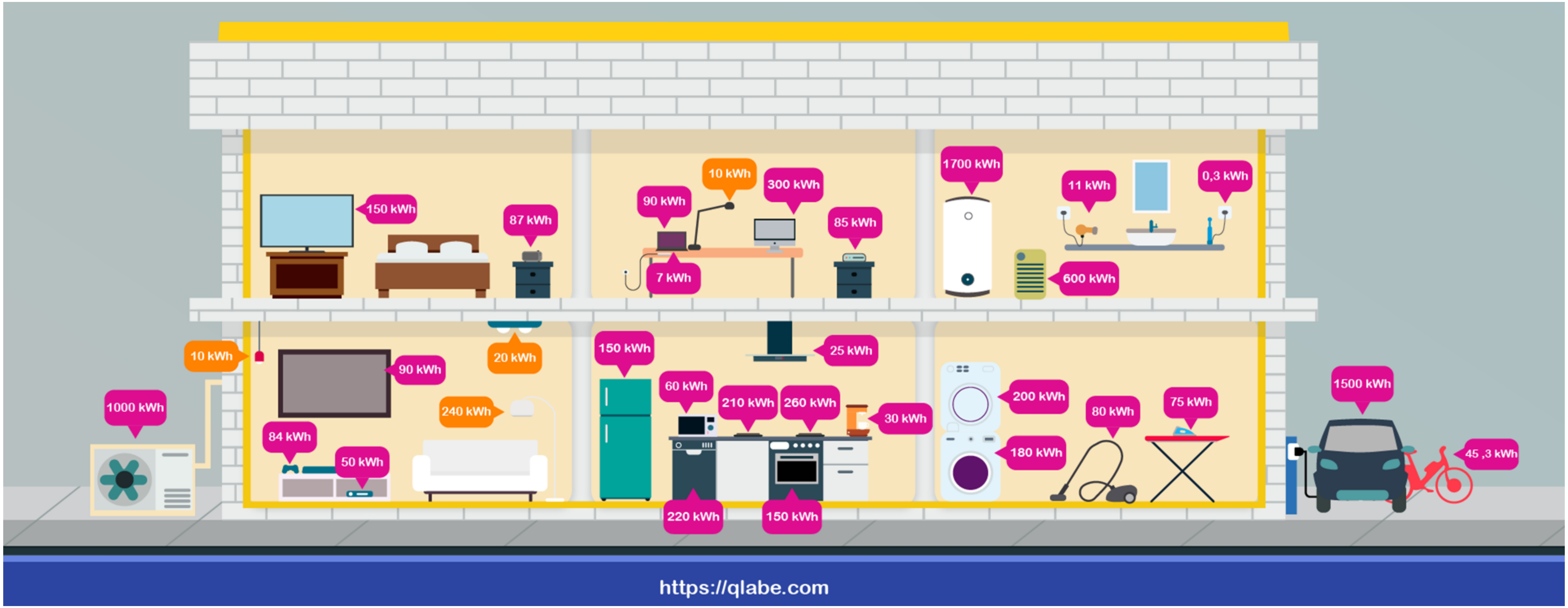
These slides were designed by Dr. Abeer Almaimouni and were used while teaching the course of Power System Economics at Kuwait University. Unless it is specified on individual slides, the content, graphs, and numerical examples were based on the book of Prof. Daniel Kirschen and Goran Strbac, Fundamentals of Power System Economics, or the slides that Prof. Kirschen used while teaching the course at the University of Washington. In many places, a direct quotation was made from either his slides or book.



Key terms

Electric demand	Load Duration Curve
Residential, industrial, and commercial load	Base load , Intermediate load , Peak load
Load profile	Economic dispatch
Electricity supply	Unit commitment
Generation	Power flow
Transmission	Optimal power flow
Distribution	DC power flow
Conventional generators	Fixed cost
Renewable resources	Variable cost
Operational flexibility	Unit constraints
	System Constraints

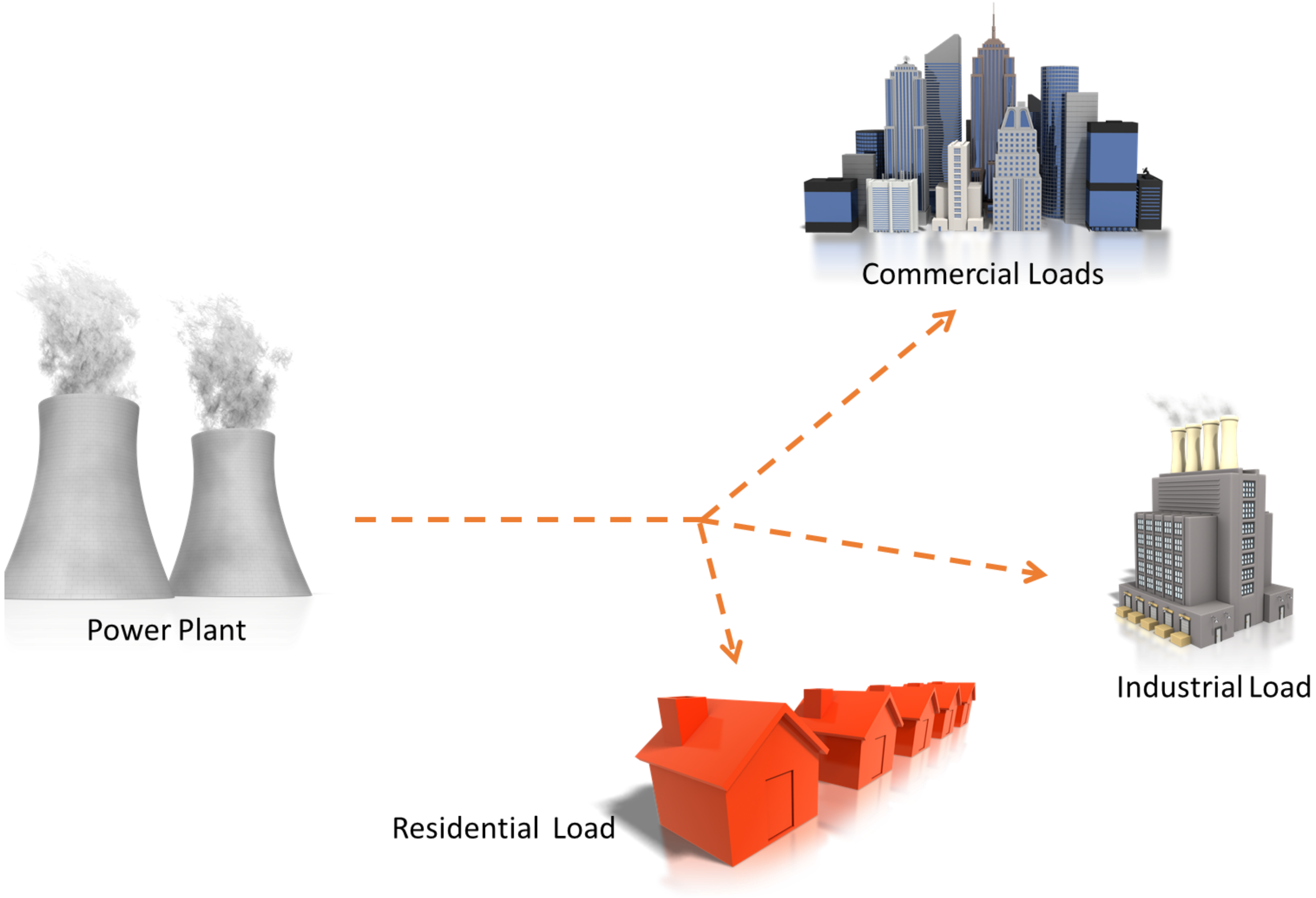
Electricity Demand



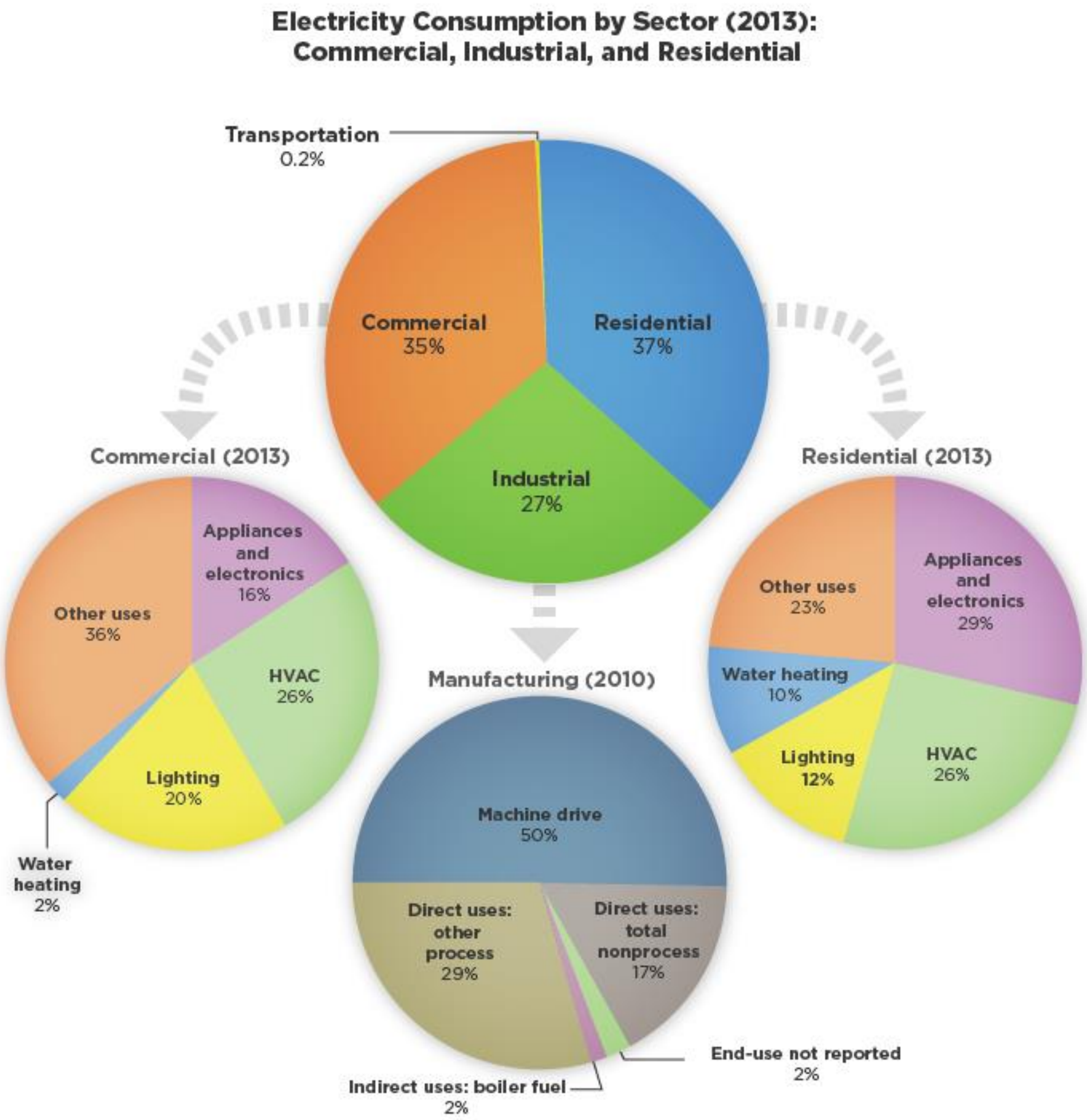
Residential Load

Residential Load

Types of Electric Loads

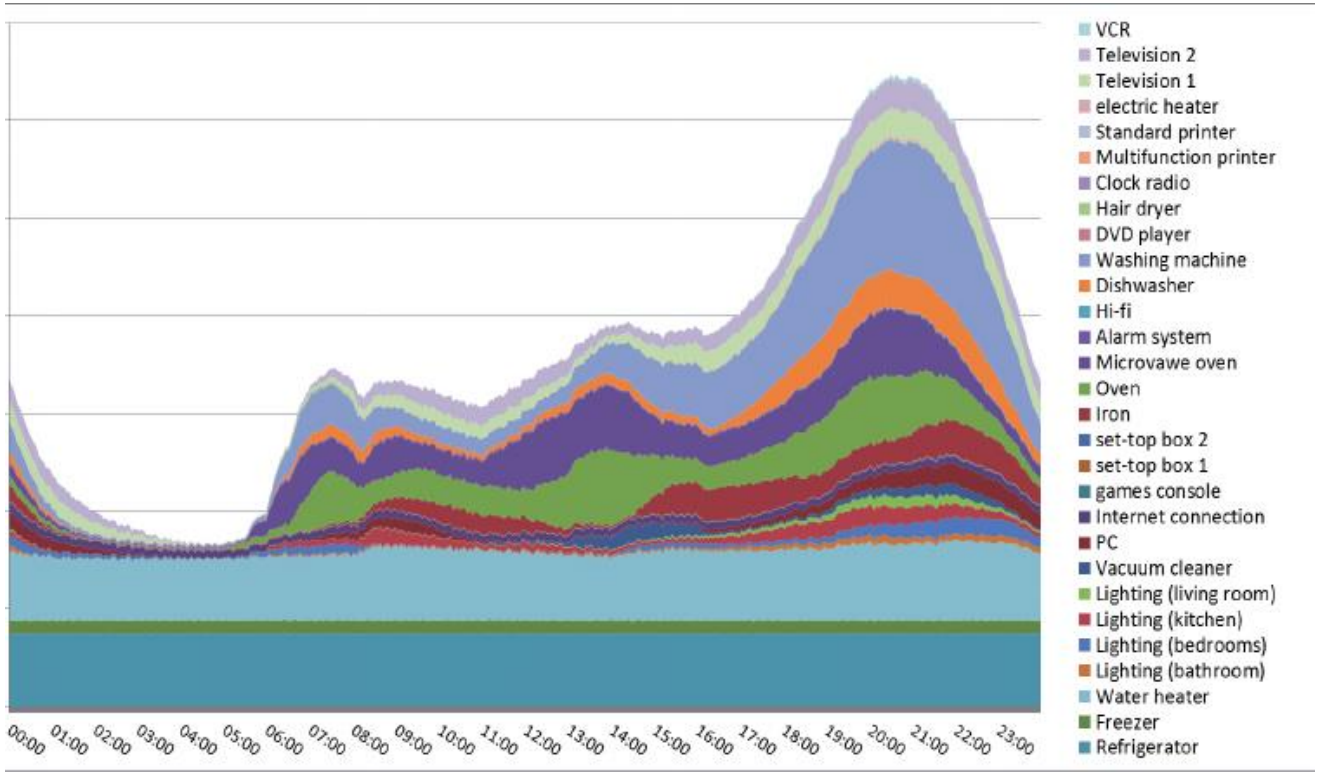


Electricity Consumption by Sector



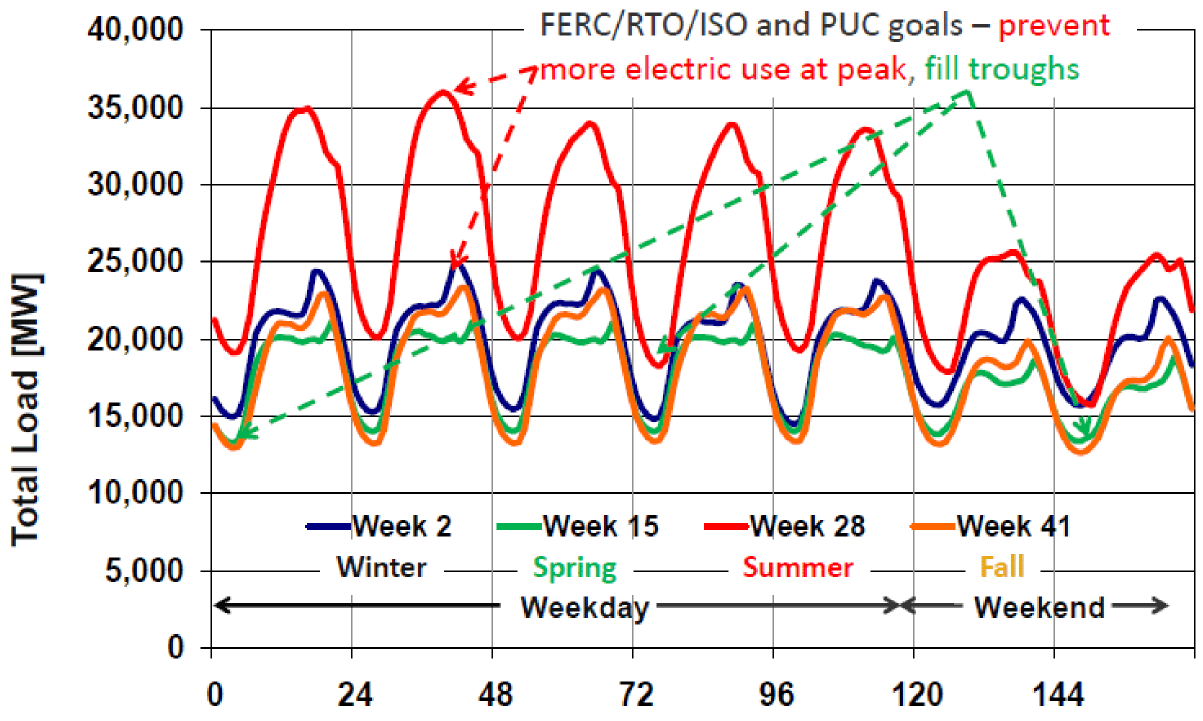
Source: Environment Protection Agency

A Sample of a Daily Load Profile



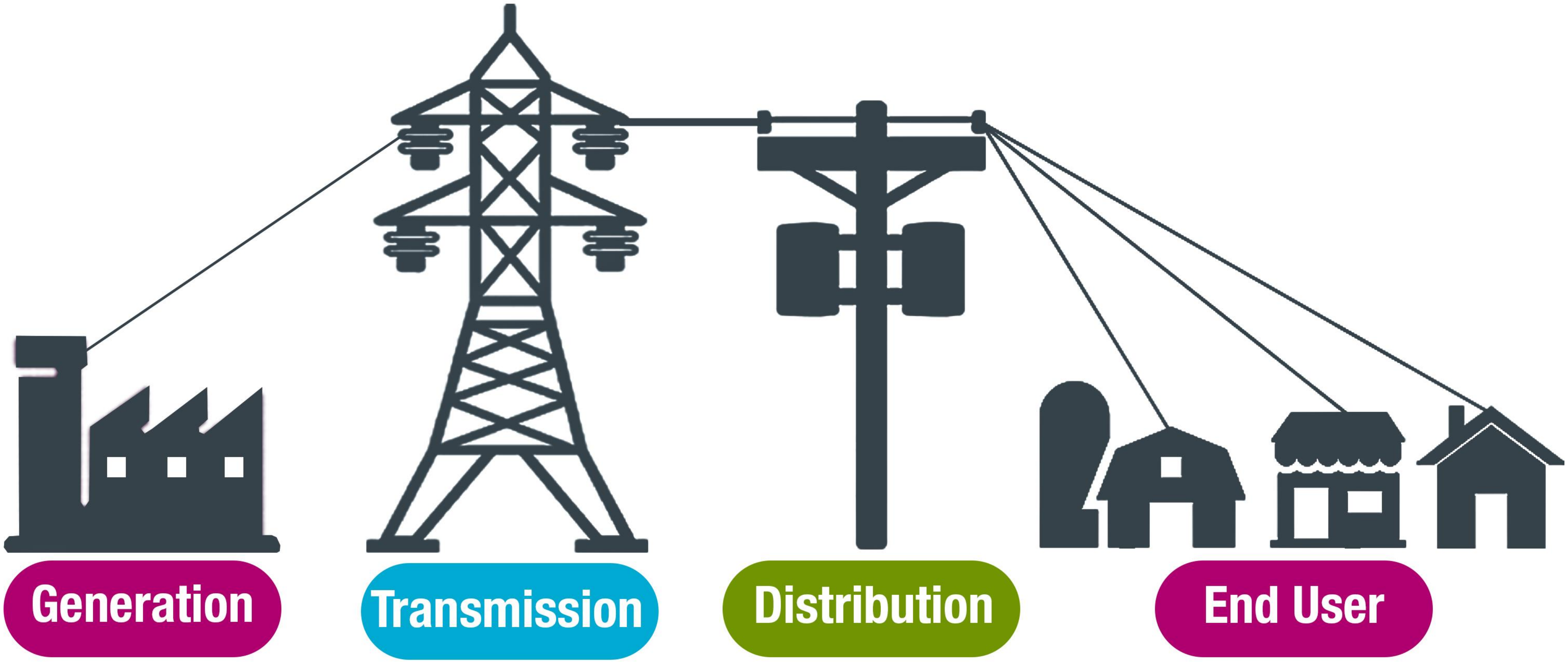
Maggiore, Simone. (2013). Evaluation of the effects of a tariff change on the Italian residential customers subject to a mandatory time-of-use tariff.

Representative Weekly Load profile



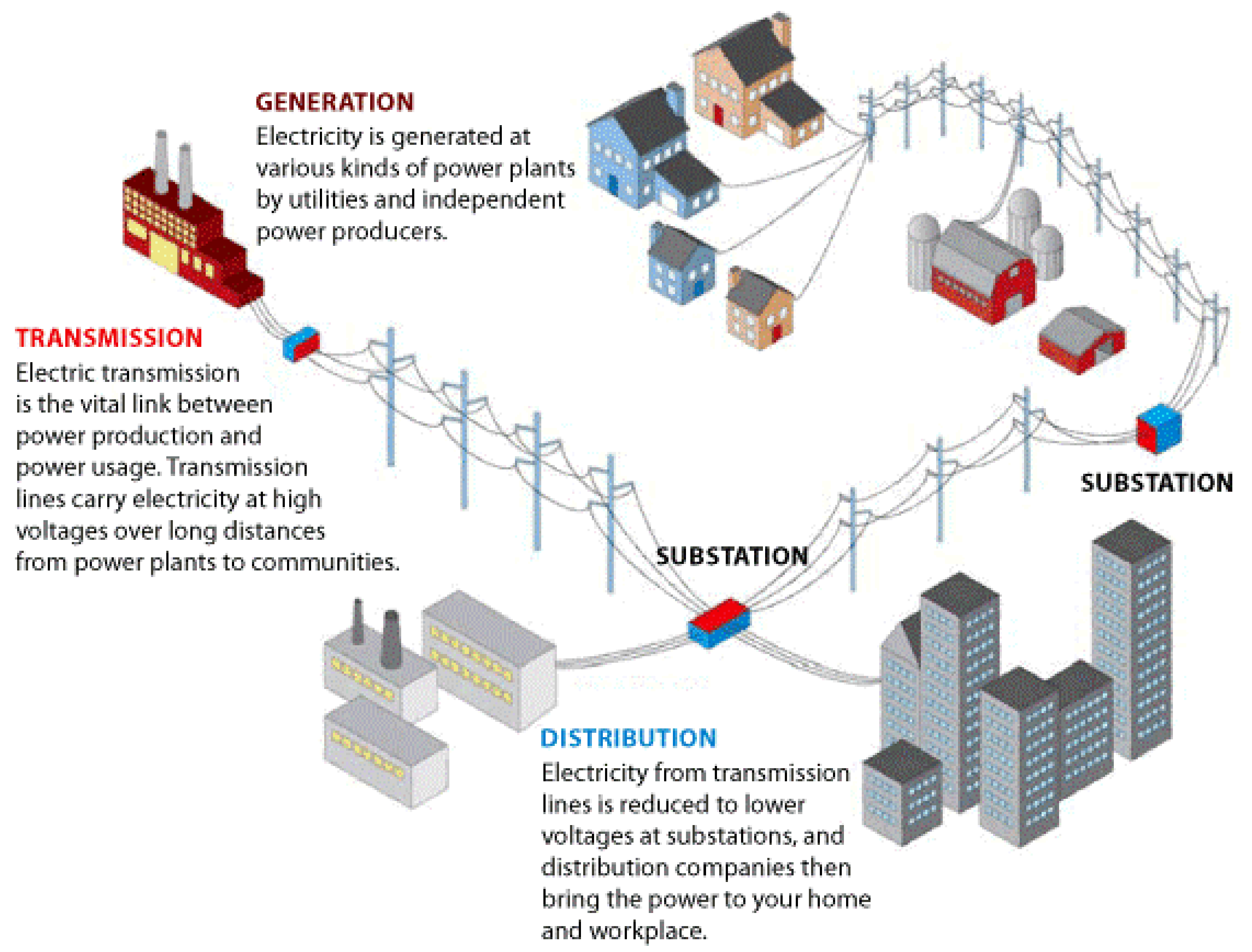
Santini, Danilo & Zhou, Yan & Vyas, Anant. (2012). An Analysis of Car and SUV Daytime Parking for Potential Opportunity Charging of Plug-in Electric Powertrains. World Electric Vehicle Journal. 5. 652-666. 10.3390/wevj5030652.

Electricity Supply



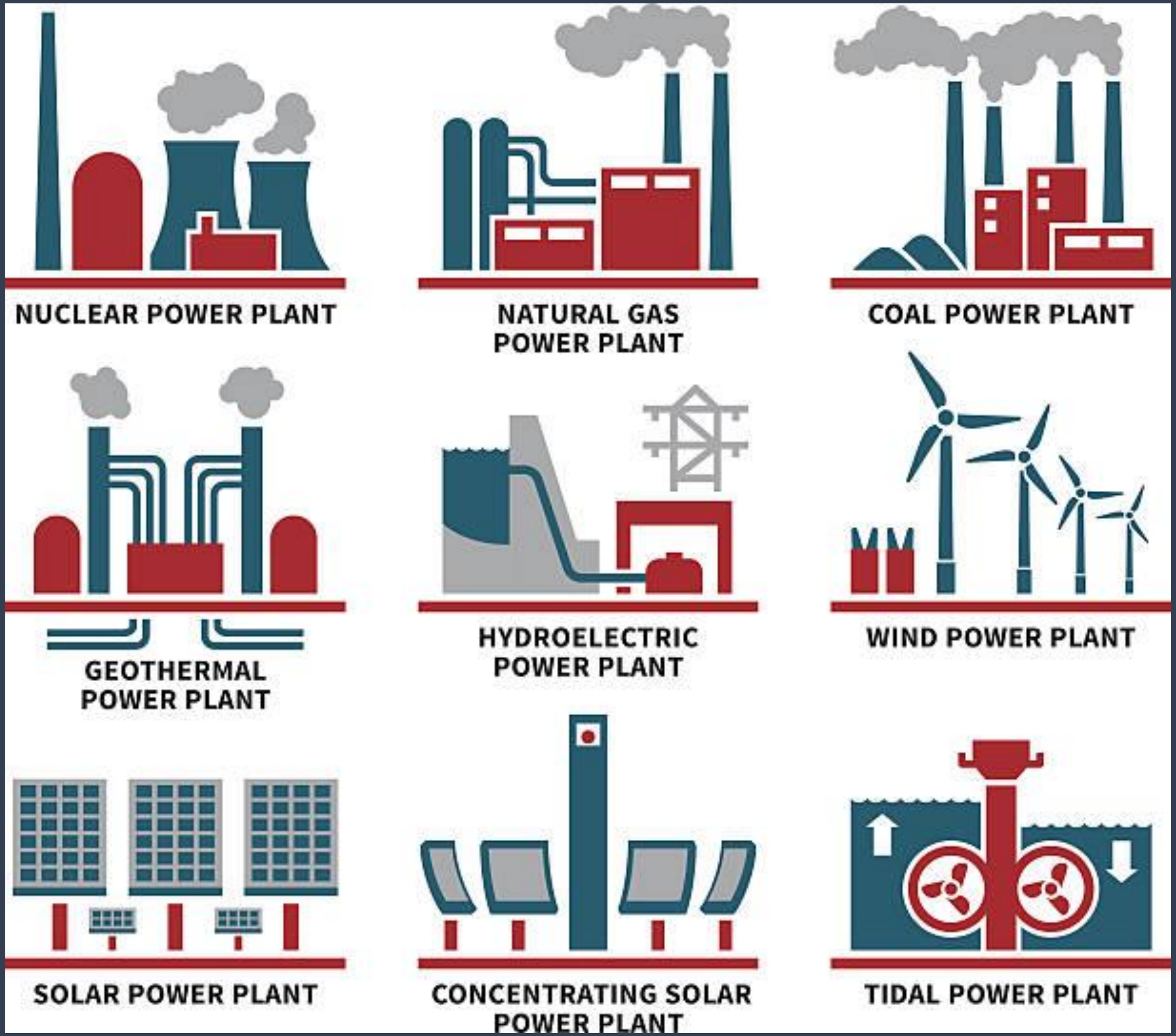
The Electric Utility Network

Source: Utilities Consumer Advocate (UCA)



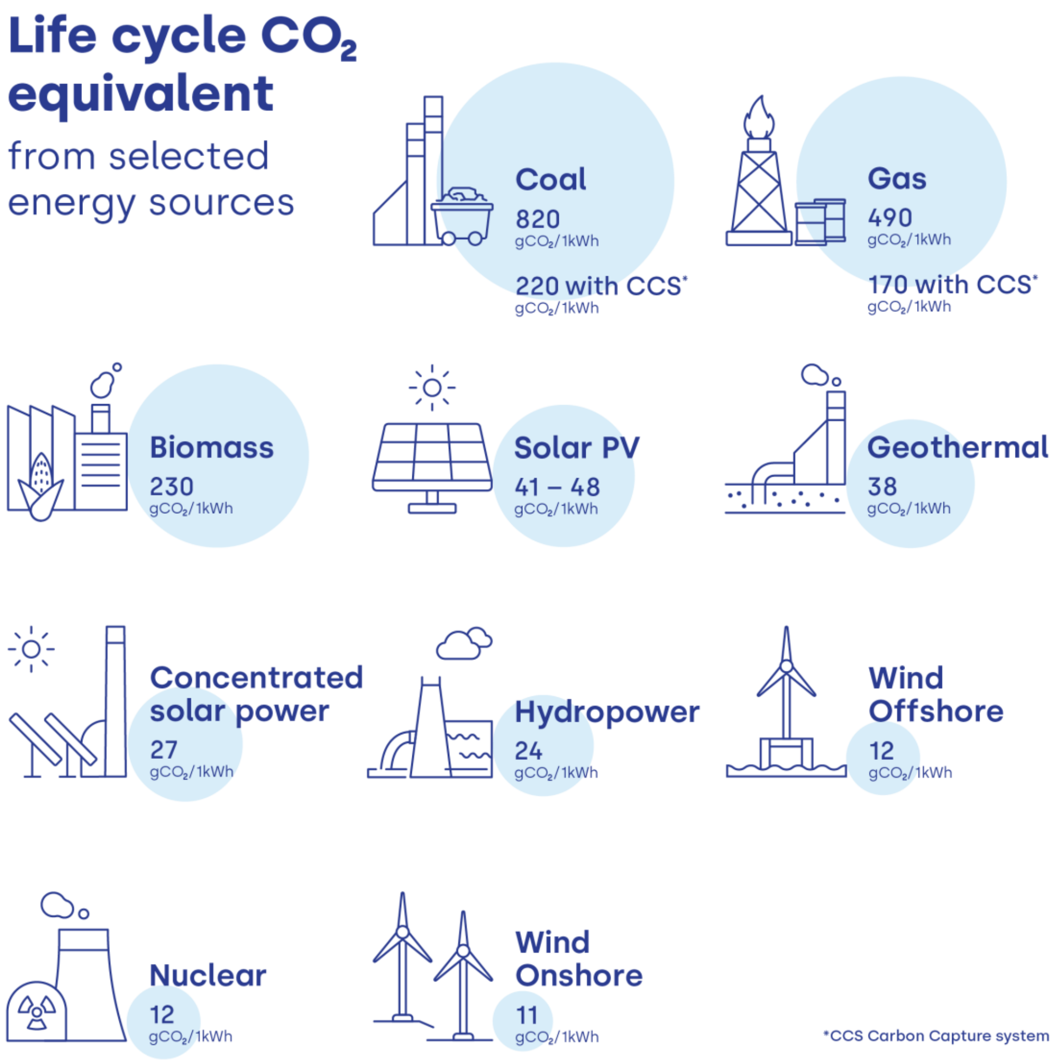
http://www.incontext.indiana.edu/

Different Technologies to Generate Electricity



https://www.freevector.com/

Some of these technologies are environment-friendly and some of them are not as they produce greenhouse gases, chiefly CO_2



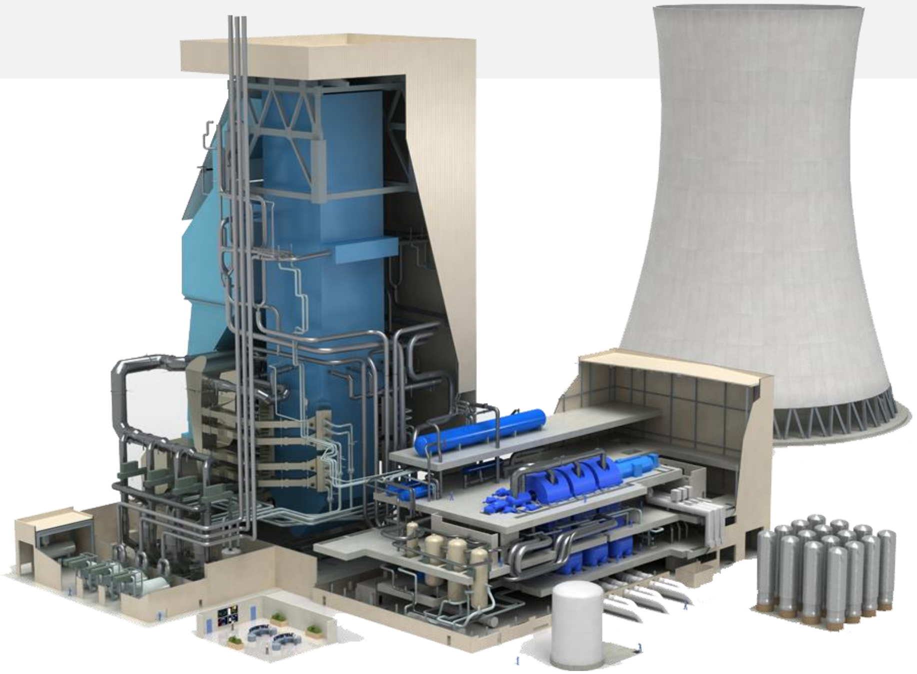
https://ee-ip.org/

Some of these technologies are environment-friendly and some of them are not as they produce greenhouse gases, chiefly CO_2

https://letters2president.org/letters/1166

The main technologies that are used to generate electricity are Thermal Power Plants

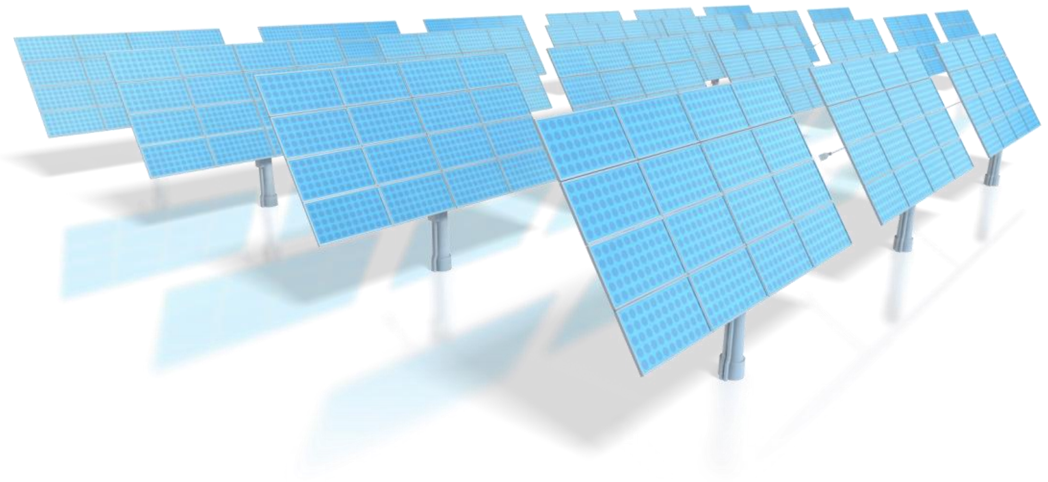
Fossil-fuel power plant (coal, natural gas or oil)
Nuclear Power Plant



Fossil-Fuel Power Plant

Nuclear Power Plant

Renewable Resources

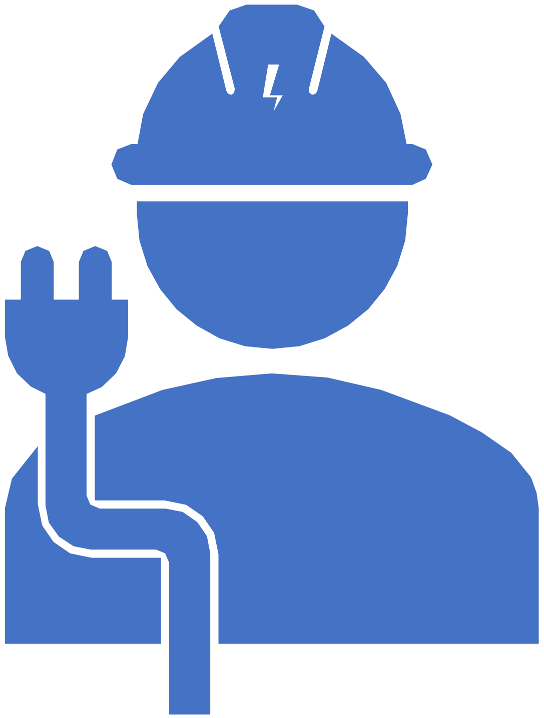


Solar Panels

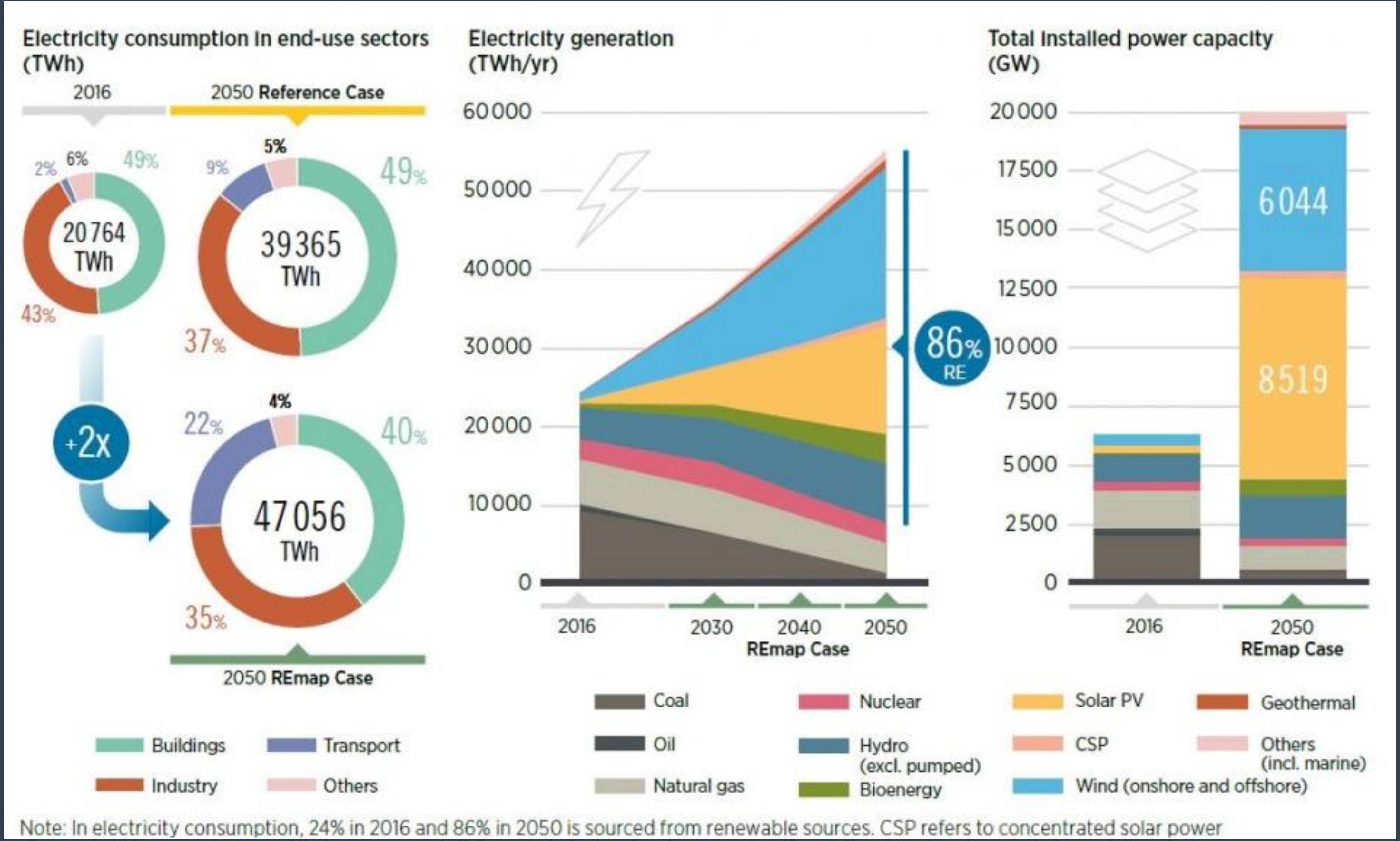
Wind Turbines

Utilities usually own a fleet that consists of a mix of different generation technologies

How does the utility decide which units to dispatch to meet the demand of electricity?

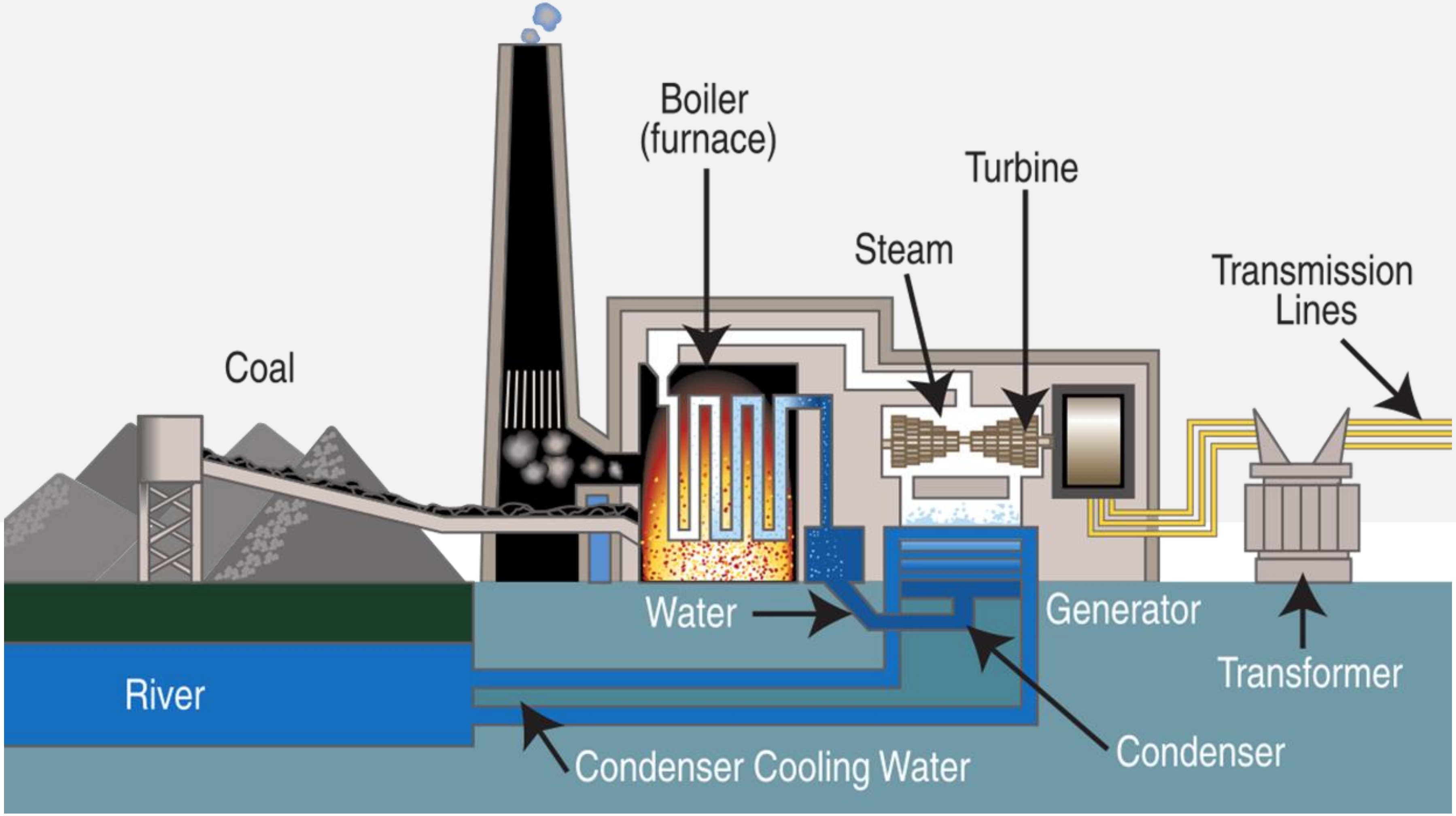


International Renewable Energy Agency

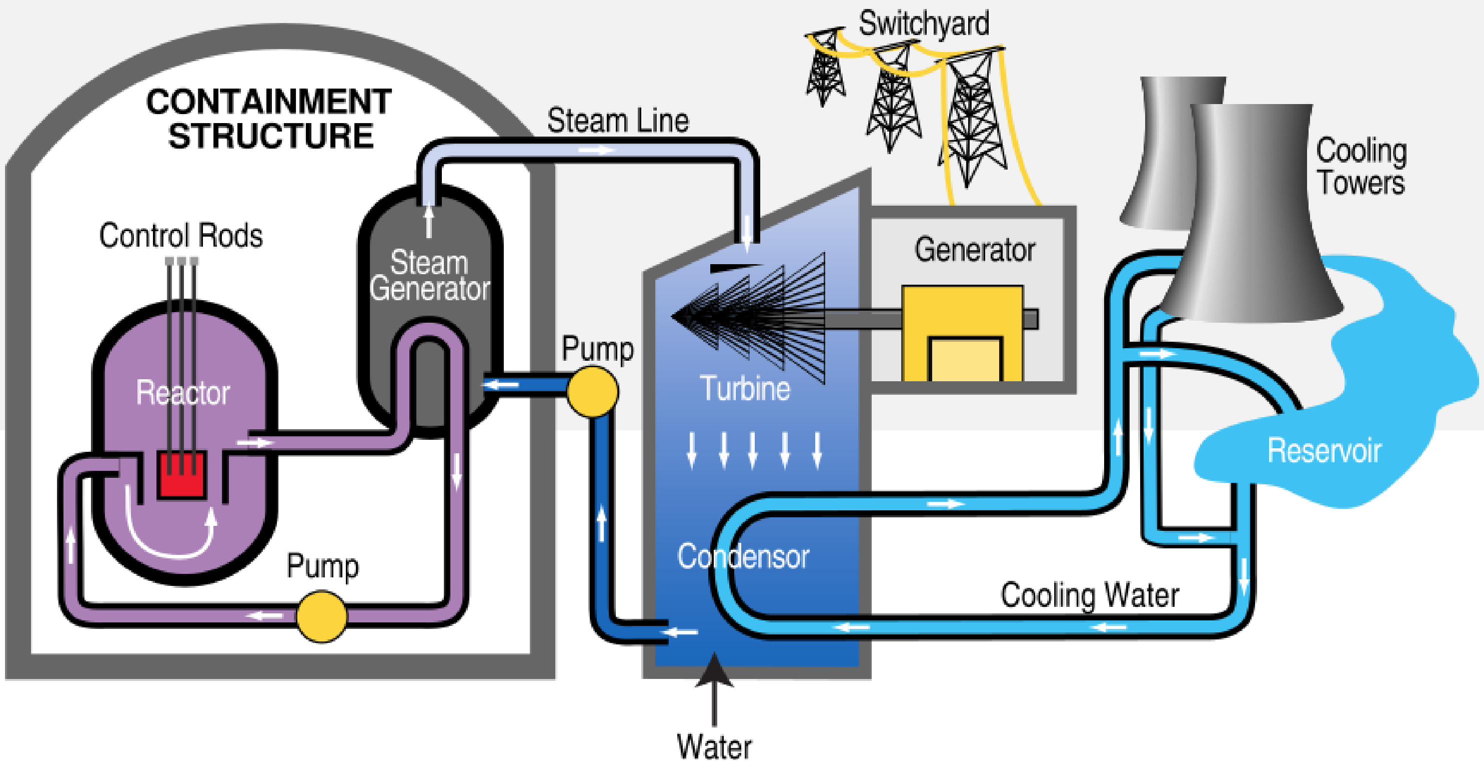


IRENA's Global Energy Transformation Report

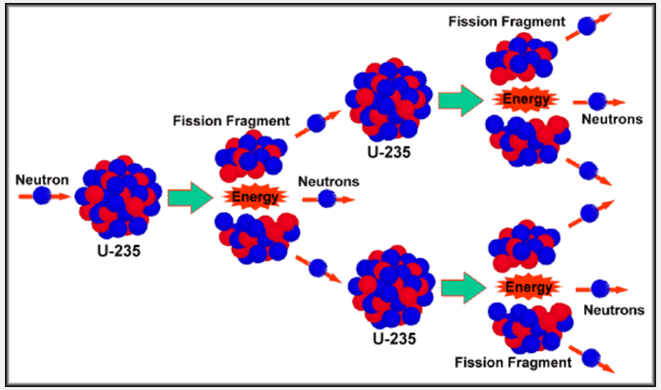
How do fossil-fuel plants work?



http://butane.chem.uiuc.edu/



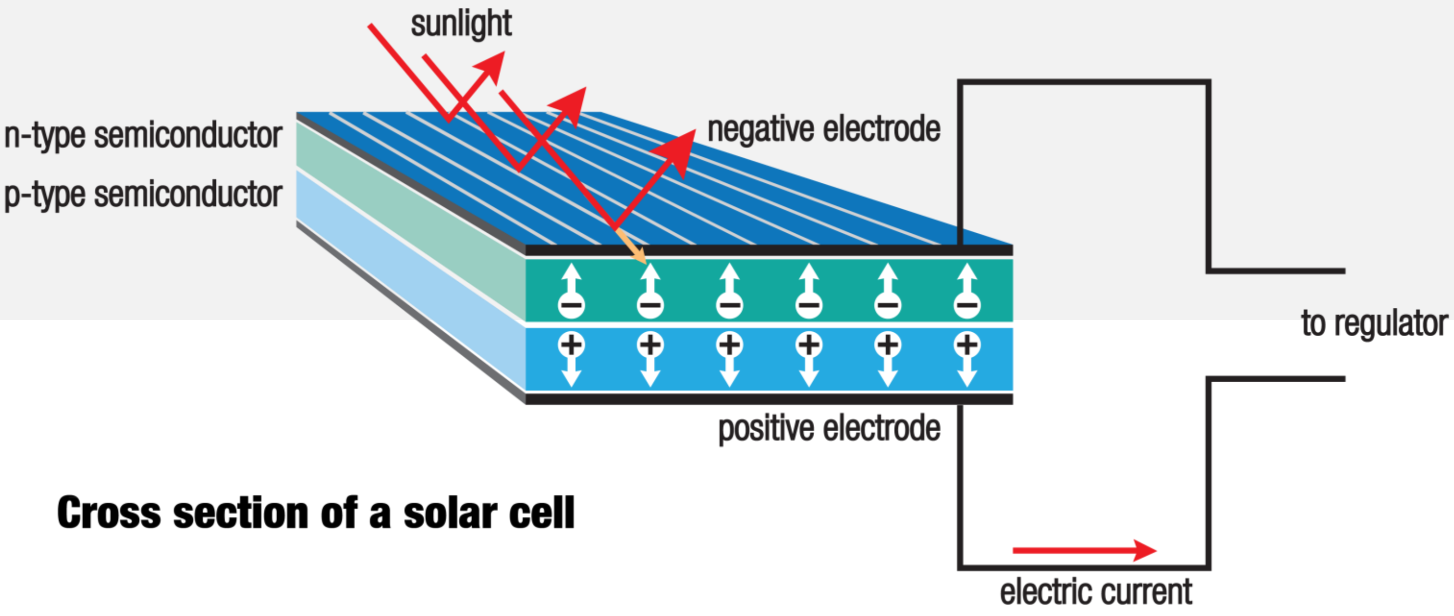
https://khwarizmi.org/event/harnessing-nuclear-energy/



nuclearpower3rdpro/home/how-nuclear-energy-works-1

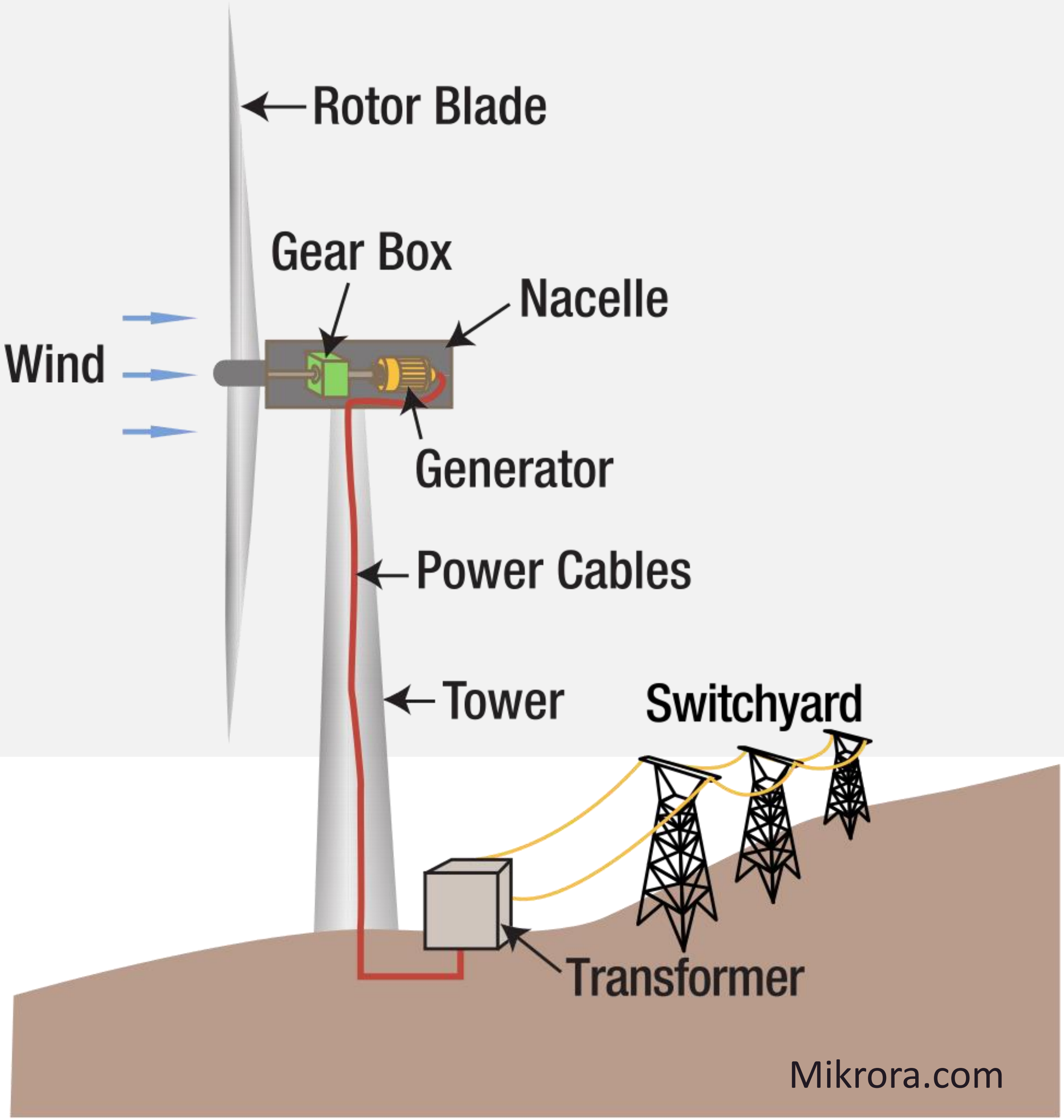
How do nuclear plants work?

How do solar panels work?

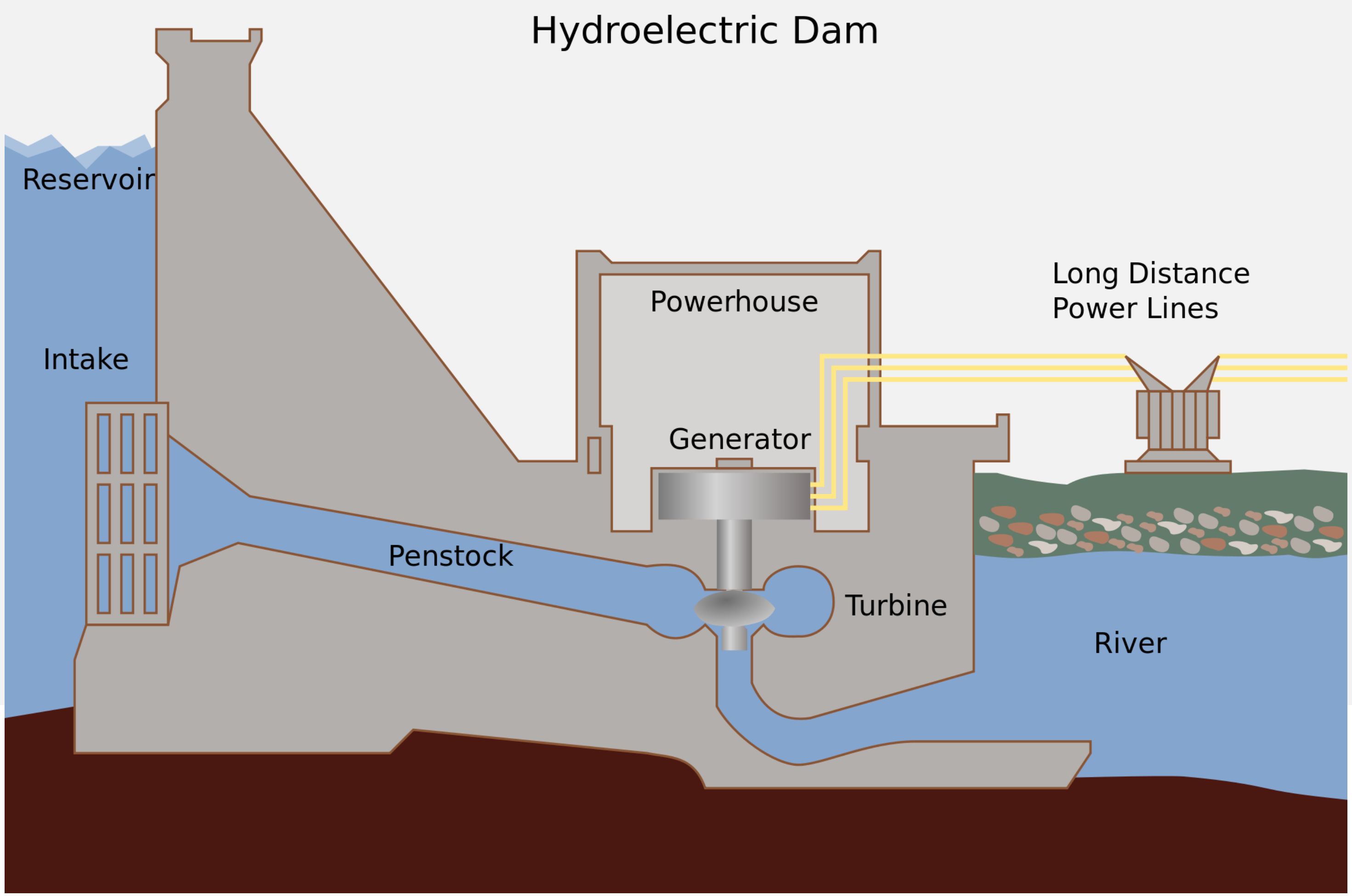


Cross section of a solar cell

https://solarmagazine.com/solar-panels/



How do wind turbines work?



thinglink.com

How hydroplant work?

The Power System Economic Problems

What is the cheapest way to meet the demand reliably?

Economic Dispatch (ED)

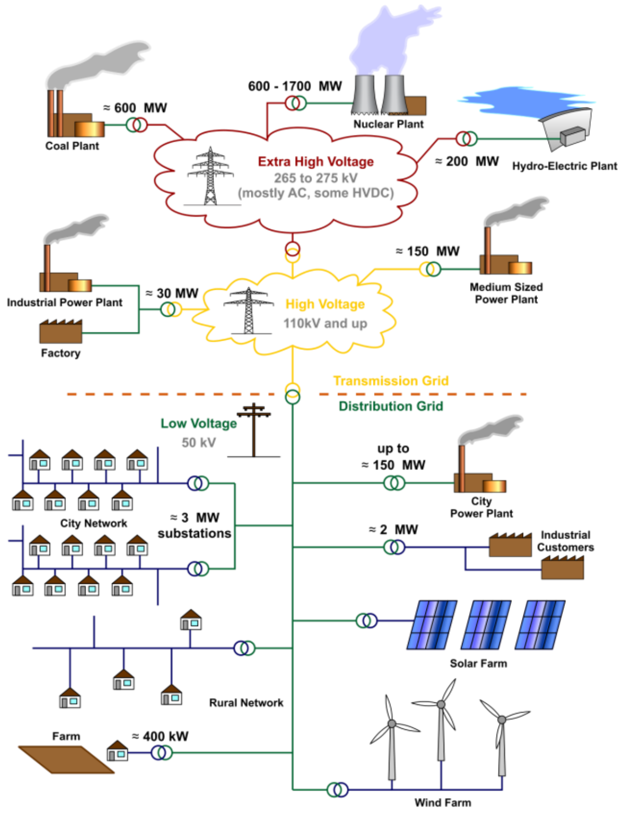
Unit Commitment (UC)

Optimal Power Flow (OPF)

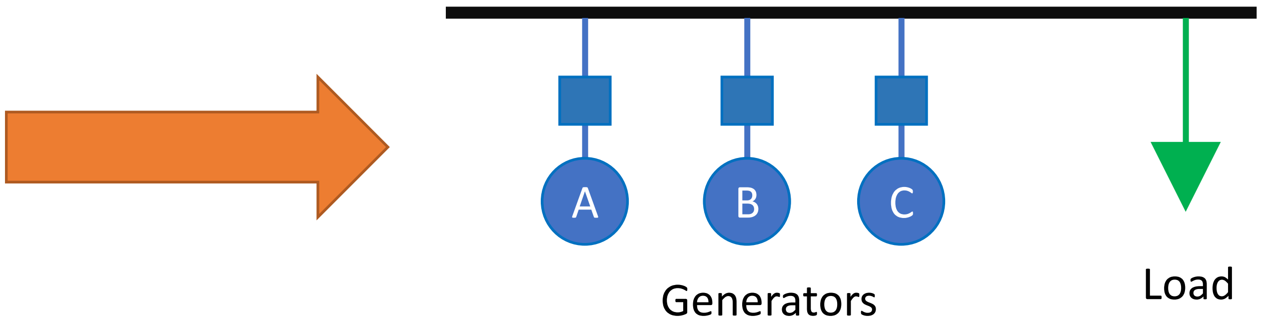
http://www.steamenginerevolution.com/

Economic Dispatch

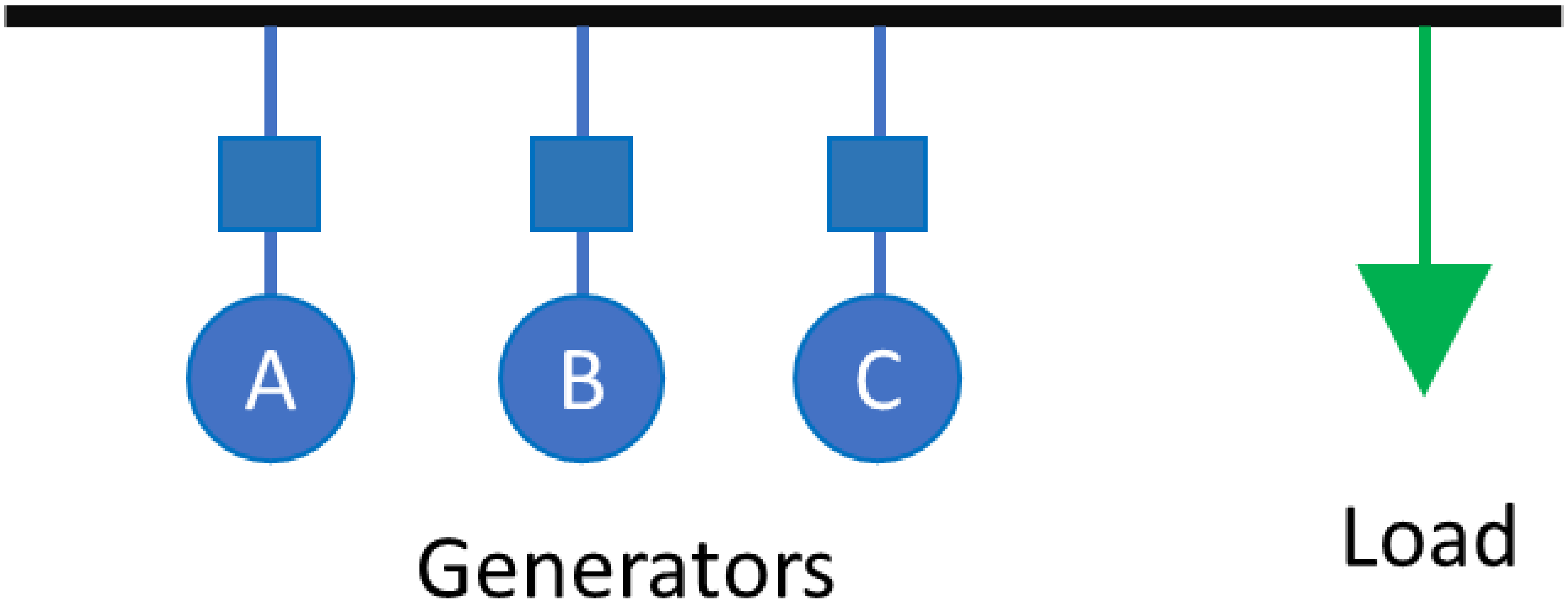
Assumption: Single Bus System



https://en.wikipedia.org/wiki/Electrical_grid



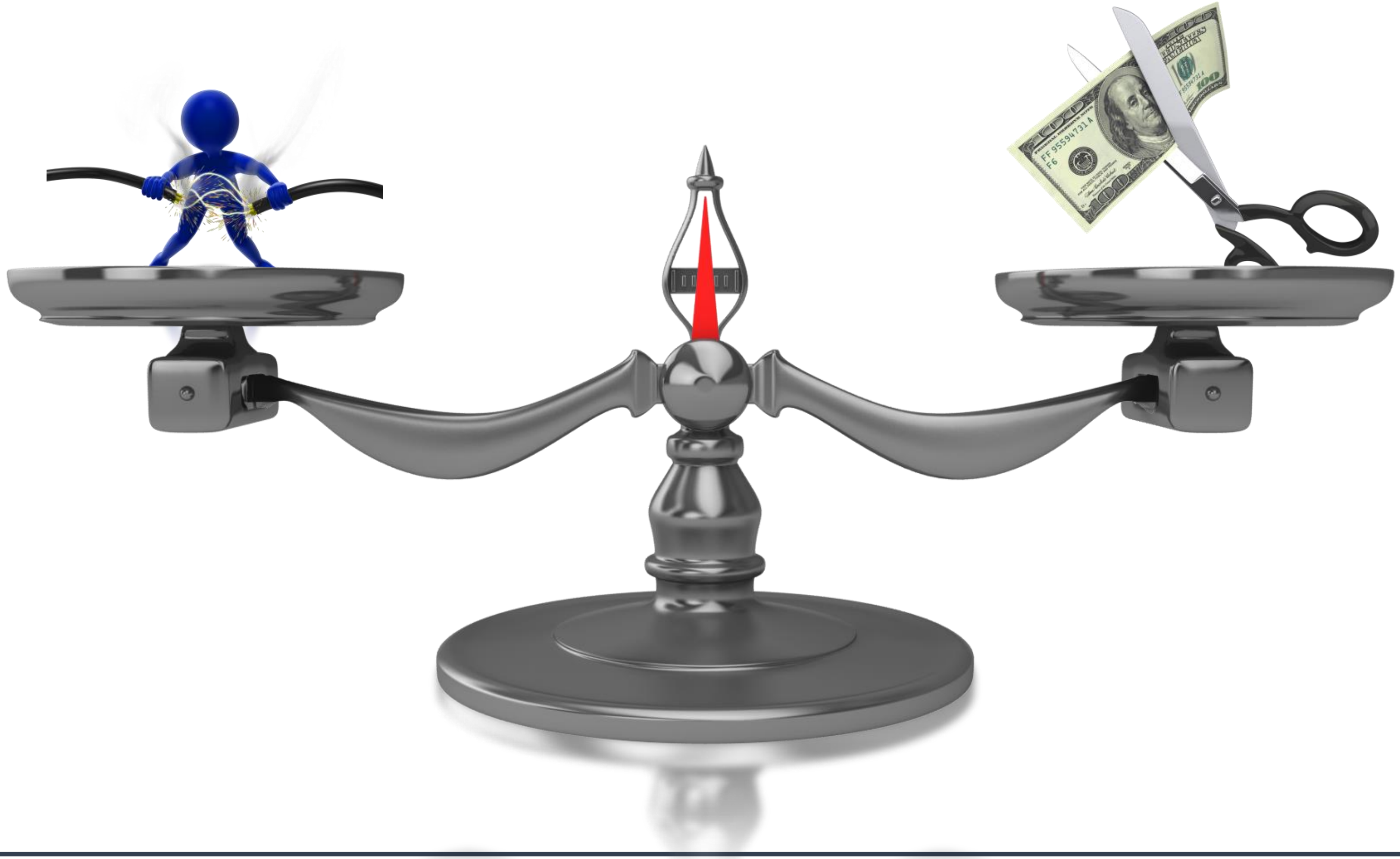
Ignore the limits of the network



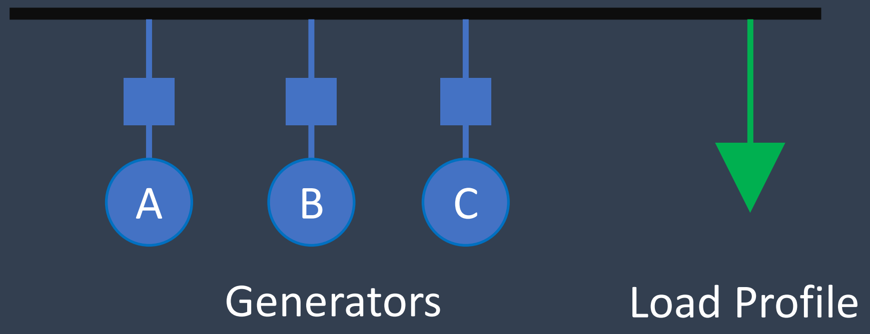
Economic Dispatch: Problem Definition

- Given: load and a set of units that are on-line
- Consider the limits of the generating units
- How much should each unit generate to meet this load at minimum cost?

Balancing the Greed and the Fear

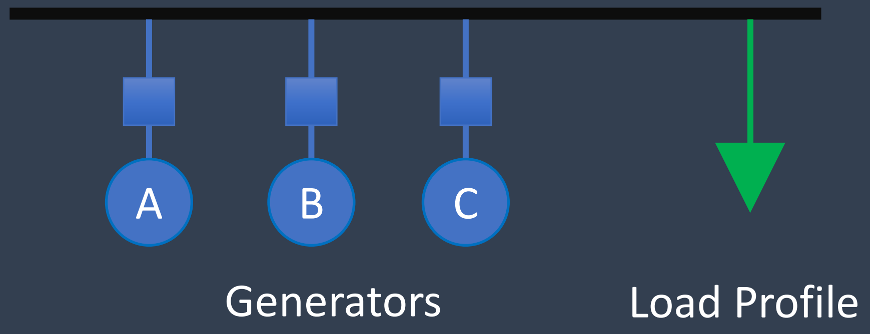


Economic Dispatch: Problem Definition



- Objective:
  - Minimum cost to meet the demand
- Such that:
  - The total load = total generation
  - The output of each unit does not exceed its upper and lower limits.

Economic Dispatch: Mathematical Formulation



- Objective: Minimize

  $$C = C(P_A) + C(P_B) + C(P_C)$$

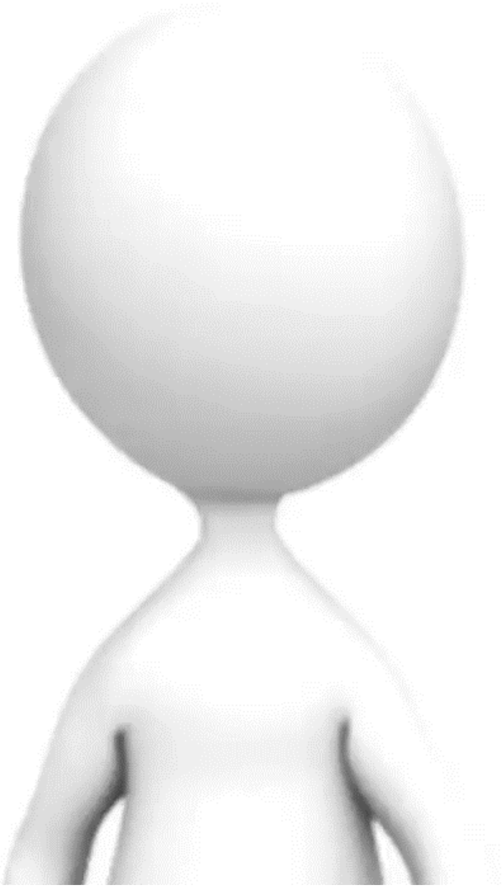
  Such that:

  Load/generation balance:

  $$P_A + P_B + P_C = L$$

  - Units constraints:

  $$P_A^{min} \leq P_A \leq P_A^{max}$$
  $$P_B^{min} \leq P_B \leq P_B^{max}$$
  $$P_C^{min} \leq P_C \leq P_C^{max}$$



What are types of costs of generation?

Fixed cost:

- Capital cost.
- Maintenance and Operation (M&O)
- Personal
- Land

Variable Cost:

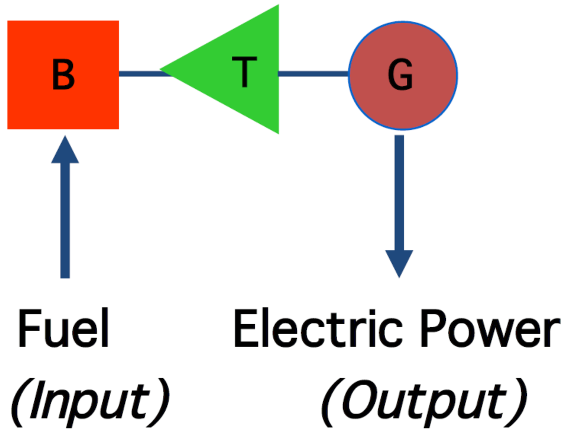
- Fuel Cost
- Start-up cost
- Maintenance Cost
- CO_2 tax

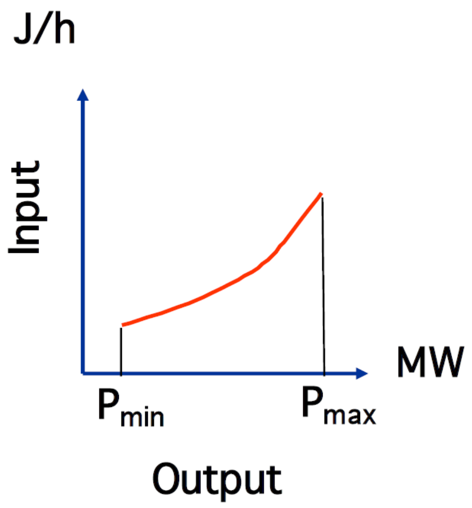
What are the types of costs to be considered in the economic dispatch problem?

Fixed cost:
- Capital cost
- Maintenance and Operation (M&O)
- Personal
- Land

Variable Cost:
- Fuel Cost ✓
- Start-up cost
- Maintenance Cost ✓
- CO_2 tax ✓

Calculating the Running Cost of Thermal Plant

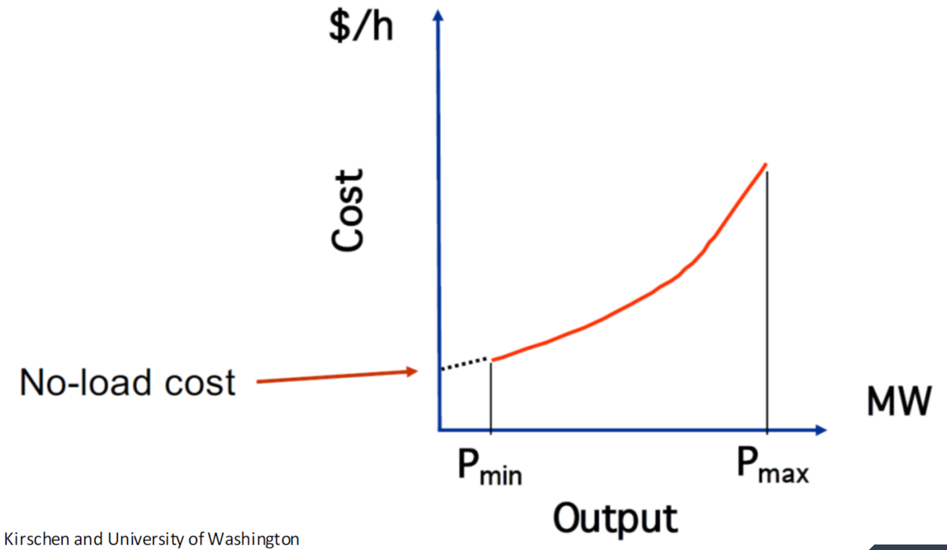




- Thermal generating units
- Consider the running costs only
- Input / Output curve
  - Fuel vs. electric power
- Fuel consumption measured by its energy content
- Upper and lower limit on output of the generating unit

Cost Curve

- Multiply fuel input by fuel cost
- No-load cost
  - Cost of keeping the unit running if it could produce zero MW

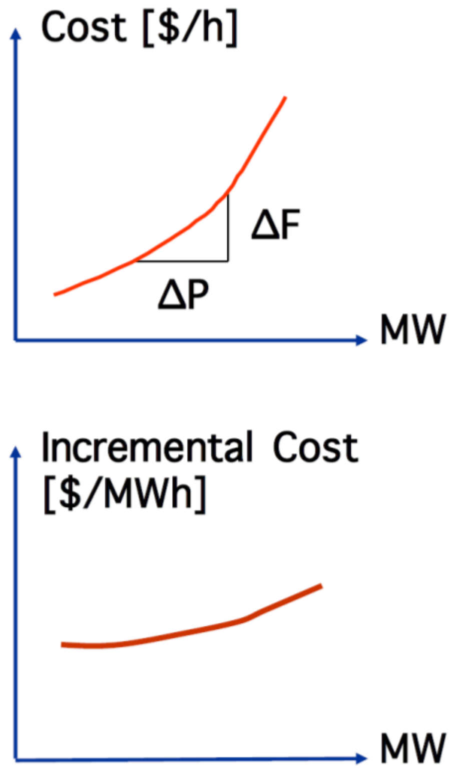


Incremental Cost Curve

- Incremental cost curve

$$\frac{\Delta FuelCost}{\Delta Power} \; vs \; Power$$

- Derivative of the cost curve
- In \$/MWh
- Cost of the next MWh



Practical Economic Dispatch

Example

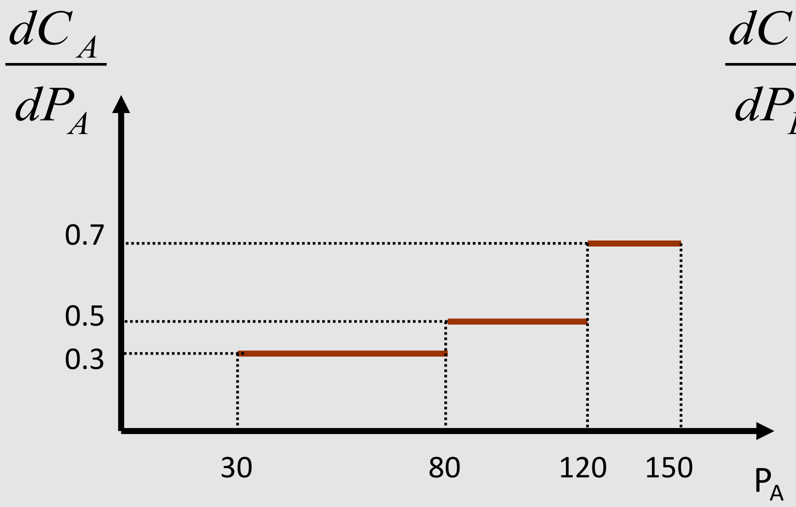
P_A = 120MW
P_B = 90 MW
λ = 0.6

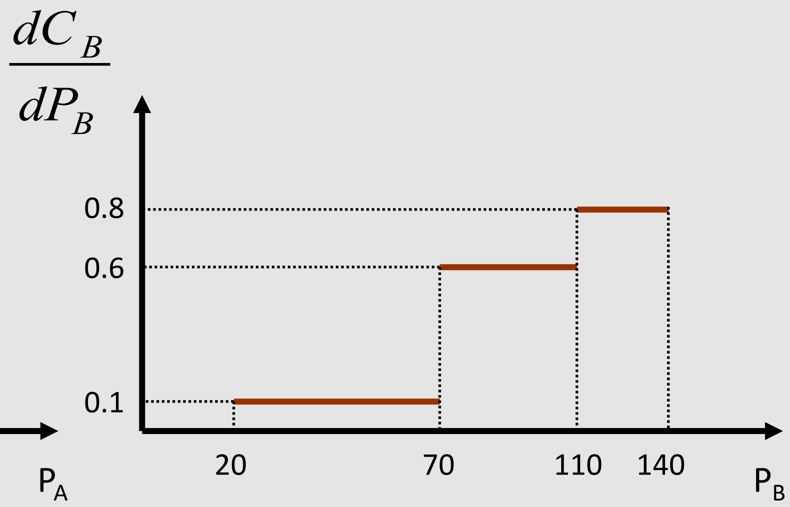
If P_{load} = 210 MW, and the systems consists of two generating units (A and B).

What is the optimal economic dispatch, if the incremental or marginal cost curves are as shown below?

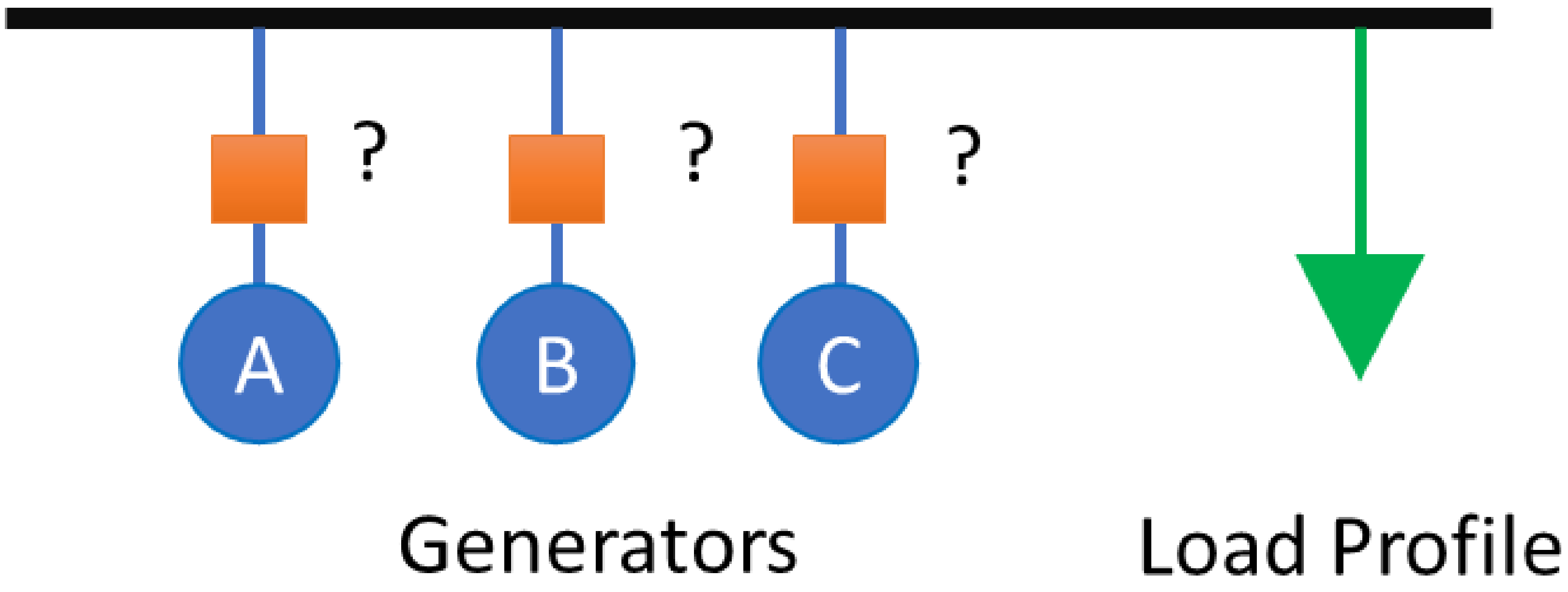
Only consider the minimum and maximum generation constraints and Ignore the other constraints.

Unit	$P_{Segment}$	P_{total}	Lambda
A&B min	20+30 = 50	50	
B	70-20=50	100	0.1
A	80-30=50	150	0.3
A	120-80=40	190	0.5
B	110-70=40	230	0.6
A	150-120=30	260	0.7
B	140-110=30	290	0.8





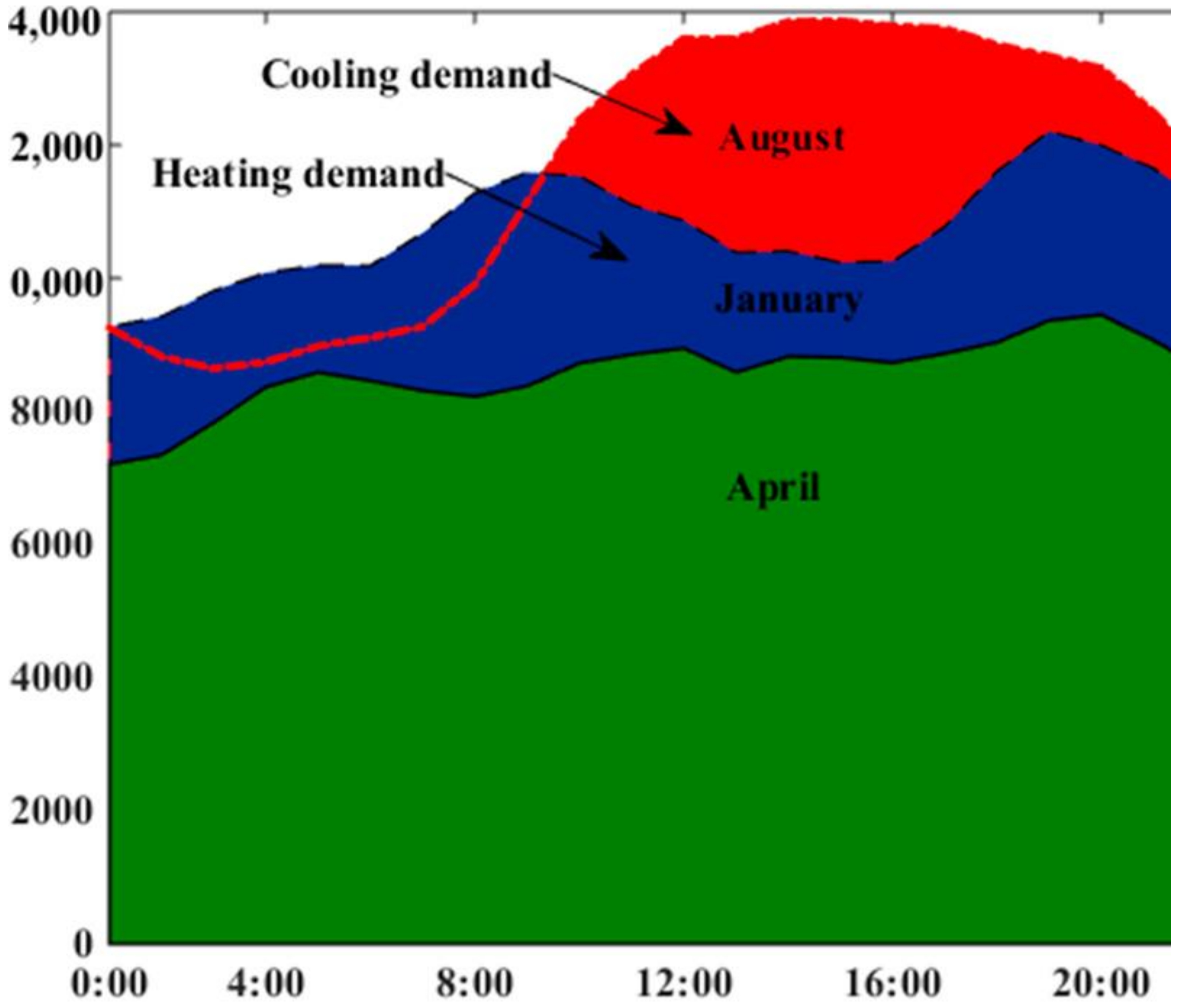
Unit Commitment



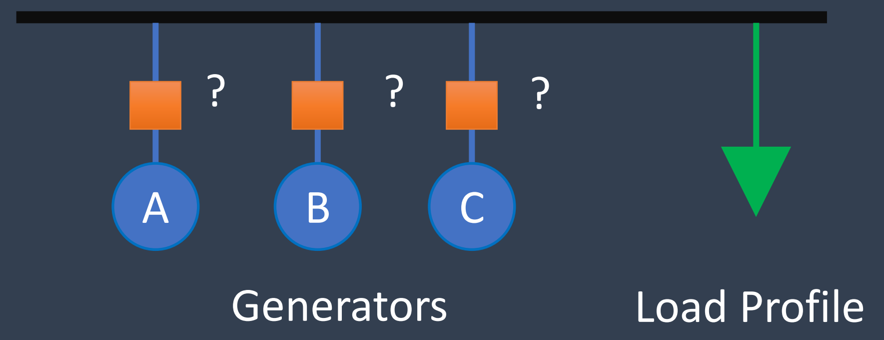
Unit Commitment: Problem Definition

- Given: Load profile (e.g. values of the load for each hour of a day)
- Given: Set of available units
- Consider the limits of the generating units
- When should each unit be started, stopped and how much should it generate to meet the load at minimum cost?

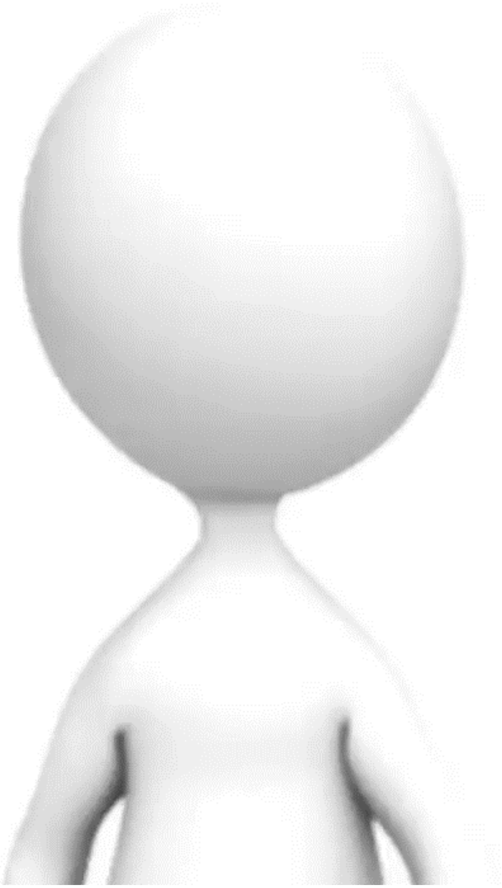
Examples of Daily Load Profile



Unit Commitment: Problem Definition



- Objective:
  - Minimum cost to meet the demand considering the load profile
- Such that:
  - The total load = total generation
  - The unit's constraints are not violated.
  - The system’s constraints are not violated.



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- Personal
- Land

Variable Cost:

- Fuel Cost
- Start-up cost
- Maintenance Cost
- CO_2 tax

What are the types of costs to be considered in the unit commitment problem?

Fixed cost:
- Capital cost
- Maintenance and Operation (M&O)
- Personal
- Land

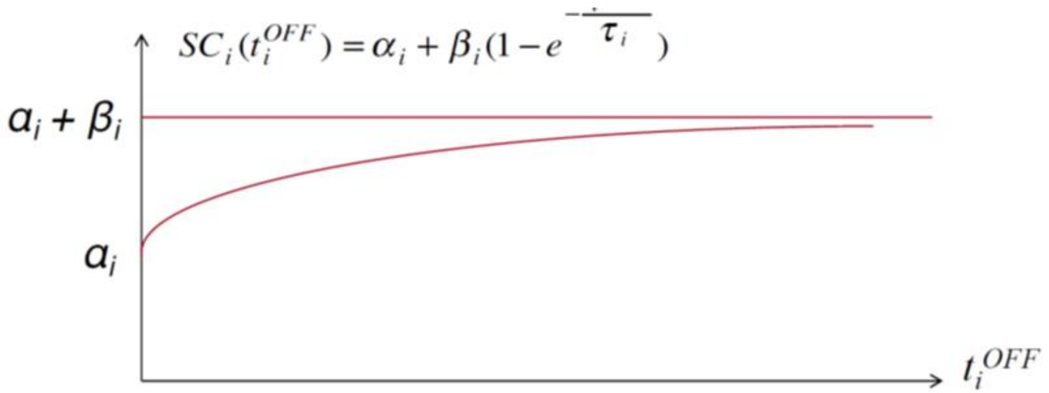
Variable Cost:
- Fuel Cost
- Start-up cost
- Maintenance Cost
- CO_2 tax

Calculating Start-Up Cost

- Thermal generating units:
  - Coal, gas, oil, biomass
- These units must be "warmed up" before they can be brought on-line
- Warming up a unit requires fuel and therefore costs money
- Incurred only during the first period when the unit is turned on

Calculating Start-Up Cost

- Start-up cost depends on the time the unit has been off:
  - If the unit has not been off for a long time, the water will still be warm
  - Hot start vs. cold start



Notations

$u(i,t):$ Status of unit *i* at period *t*

$u(i,t)=1:$ Unit *i* is on during period *t*

$u(i,t)=0:$ Unit *i* is off during period *t*

$p(i,t):$ Power produced by unit *i* during period *t*

Unit Constraints

- Maximum generating capacity
- Minimum stable generation
- Minimum “up time”
- Minimum “down time”
- Minimum ramp up rate
- Minimum ramp down rate

Unit Constraints

- Minimum up time
  - Once a unit is running it may not be shut down immediately:

$$\text{If } u(i,t)=1 \text{ and } t_i^{up} < t_i^{up,\min} \text{ then } u(i,t+1)=1$$

- Minimum down time
  - Once a unit is shut down, it may not be started immediately

$$\text{If } u(i,t)=0 \text{ and } t_i^{down} < t_i^{down,\min} \text{ then } u(i,t+1)=0$$

Unit Constraints

- Maximum ramp rates
  - To avoid damaging the turbine, the electrical output of a unit cannot change by more than a certain amount over a period of time:

    Maximum ramp up rate constraint:

    $$p(i,t+1) - p(i,t) \le \Delta P_i^{up,\max}$$

    Maximum ramp down rate constraint:

    $$p(i,t) - p(i,t+1) \le \Delta P_i^{down,\max}$$

System Constraints

- Constraints that affect more than one unit
  - Load/generation balance
  - Reserve generation capacity
  - Emission constraints
  - Network constraints

System Constraints

Load/Generation Balance Constraint

Total Generation=Total Demand

$$\sum_{i=1}^{N} u(i,t)p(i,t) = L(t)$$

where:

N: set of available units

System Constraints

Reserve Capacity Constraint

- The total capacity of the available units is equal or larger than the sum of the total load and the reserve

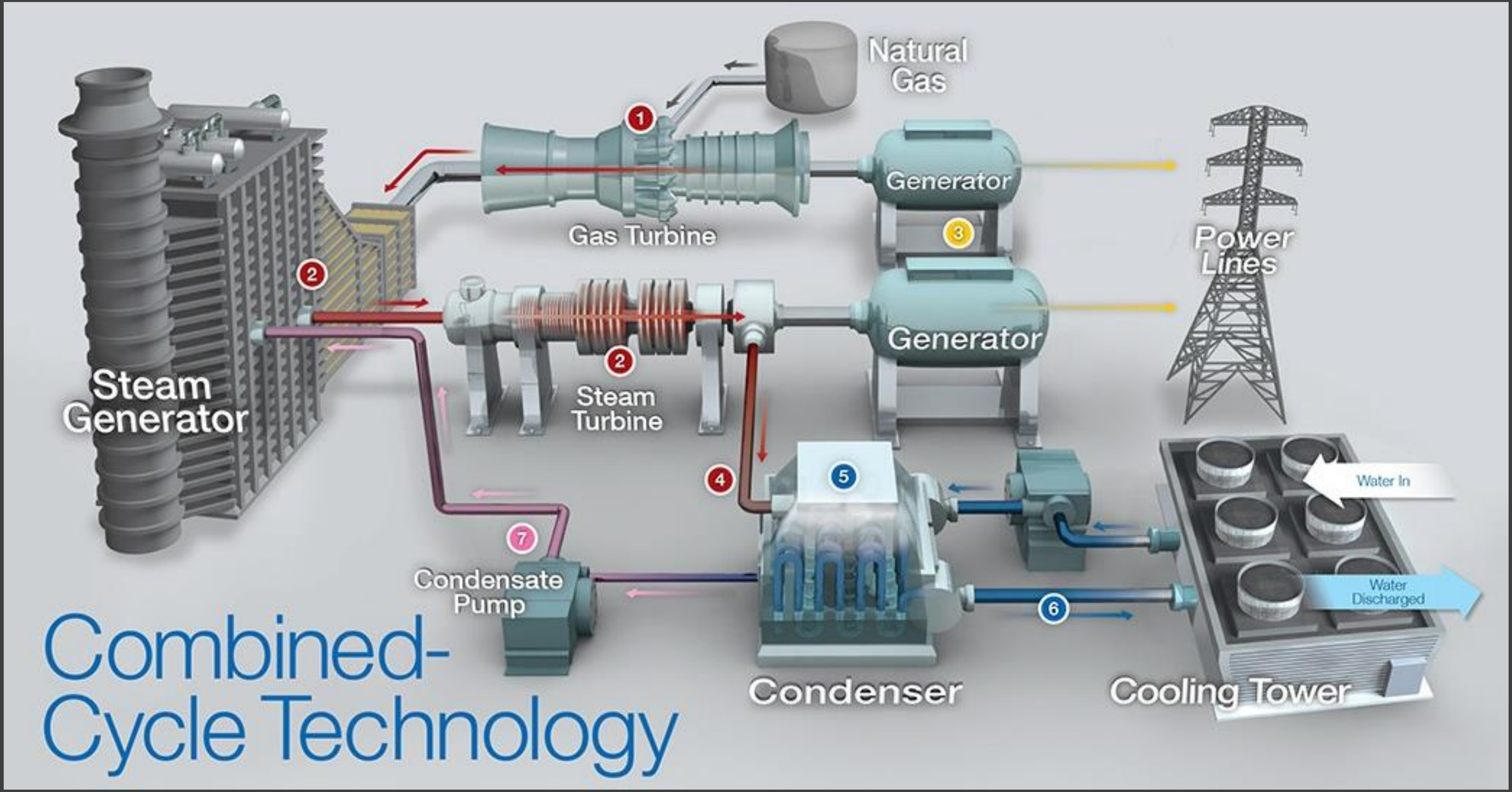
$$\sum_{i=1}^{N} u(i,t)P_i^{max} \geq L(t) + R(t)$$

where:

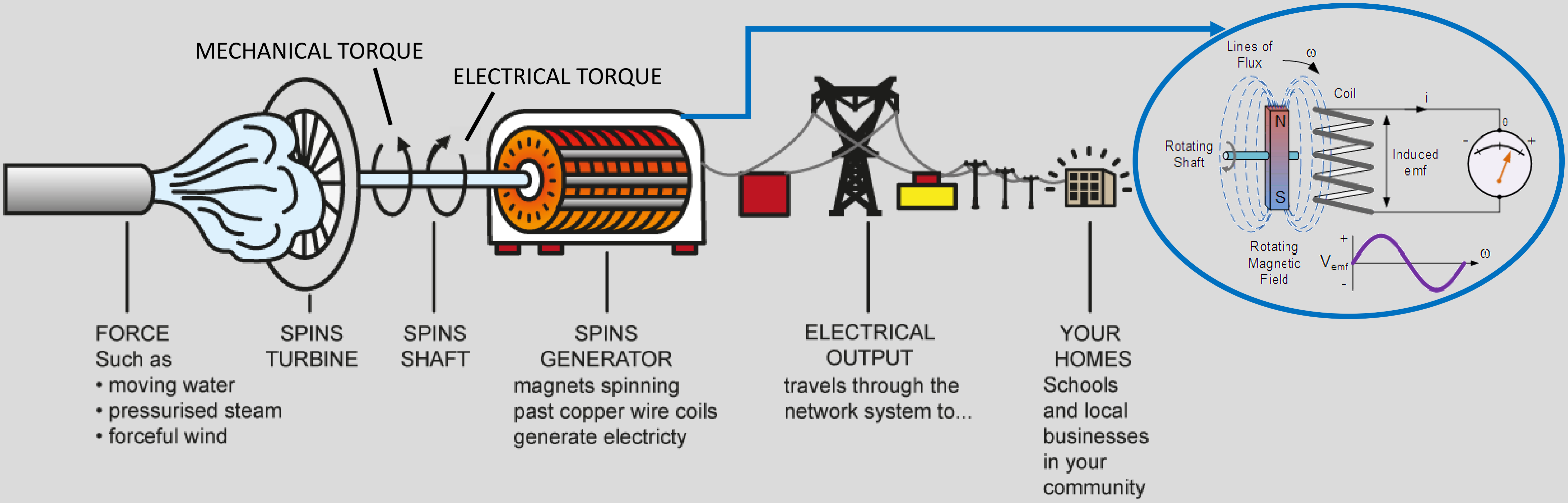
R(t): Reserve requirement at time t

Why Reserve is Needed?

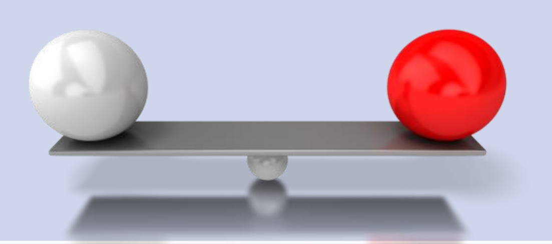
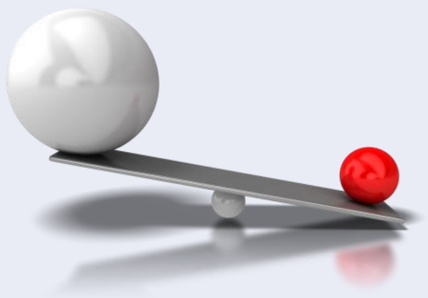
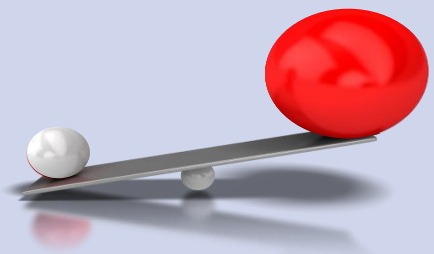
- Unanticipated loss of a generating unit or an interconnection causes unacceptable change in frequency if not corrected
- Need to increase or lower production from other units to keep frequency drop within acceptable limits
- Rapid increase in production only possible if committed units are not all operating at their maximum capacity


Natural Gas
1
Gas Turbine
Generator
3
Power Lines
2
Steam Generator
2
Steam Turbine
Generator
4
5
Water In
7
Condensate Pump
6
Water Discharged
Condenser
Cooling Tower
Combined-Cycle Technology

All Generators Must Run at the Same Electric Frequency



Why Reserve is Needed?

Upholding the delicate balance between the generation and the consumption	**Cause/effect/Required action**
Mechanical Power Electric Power 	The Steady state condition
	Frequency is not changing
	No action is required
Mechanical Power Electric Power 	A fault in the lines that causes a sudden drop in the demand of electricity.
	Increase in the frequency.
	Negative reserve: decrease output when generation > load
Mechanical Power Electric Power 	Loss of one of the generating units.
	Decrease in the frequency
	Positive reserve Increase output when generation < load

Type of Reserve?

- Spinning reserve
  - Primary
    - Quick response for a short time
  - Secondary
    - Slower response for a longer time
- Tertiary reserve
  - Replace primary and secondary reserve to protect against another outage
  - Provided by units that can start quickly (e.g. gas turbines)
  - Also called scheduled or off-line reserve
- Reserve must be spread around the network
  - Must be able to deploy reserve even if the network is congested

How to Determine the Needed Reserve?

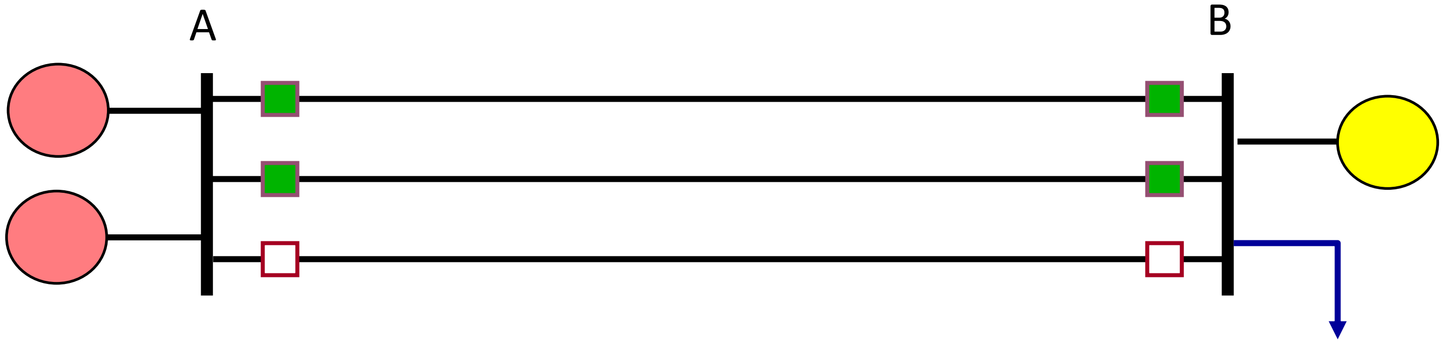
- Protect the system against "credible outages"
- Deterministic criteria:
  - Capacity of largest unit or interconnection
  - Percentage of peak load
- Probabilistic criteria:
  - Takes into account the number and size of the committed units as well as their outage rate

Cost of Reserve?

- Reserve has a cost even when it is not called
  - More units scheduled than required
    - Units not operated at their maximum efficiency
    - Extra start up costs
  - Must build units capable of rapid response
  - Cost of reserve proportionally larger in small systems
    - Important driver for the creation of interconnections between systems

System Constraints

Network Constraint



Cheap generators
May be "constrained off"

More expensive generator
May be "constrained on"

System Constraints

Network Constraint

- Transmission network may have an effect on the commitment of units
  - Some units must run to provide voltage support
  - The output of some units may be limited because their output would exceed the transmission capacity of the network

System Constraints

Emission Constraint

- Amount of pollutants that generating units can emit may be limited
- Pollutants:
  - SO_2, NOx
- Various forms:
  - Limit on each plant at each hour
  - Limit on plant over a year
  - Limit on a group of plants over a year
  - Etc…

Notation

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$u(i,t)$=0	Unit *i* is off during period *t*
$p(i,t)$:	Power produced by unit *i* during period *t*

Solving the Unit Commitment Problem

- Decision variables:
  - Status of each unit at each period:

$$u(i,t) \in \{0,1\} \quad \forall \; i,t$$

  - Output of each unit at each period:

$$p(i,t) \in \left\{0, \left[P_i^{\min}; P_i^{\max}\right]\right\} \quad \forall \; i,t$$

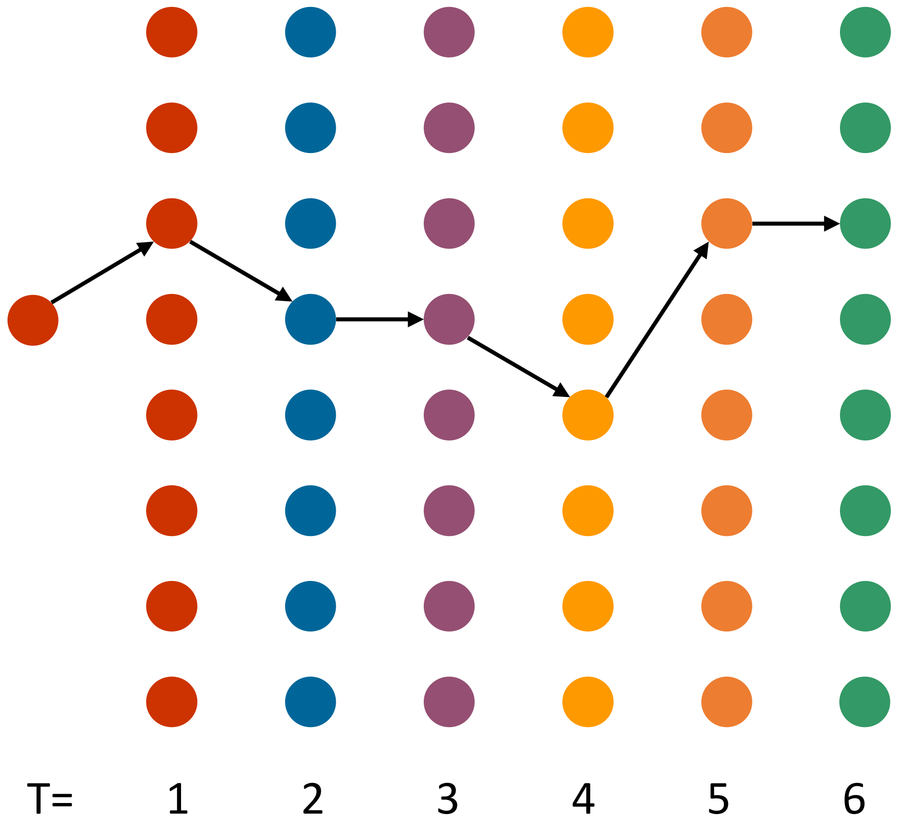
- Combination of integer and continuous variables

How many combinations are there?

- 111
- 110
- 101
- 100
- 011
- 010
- 001
- 000

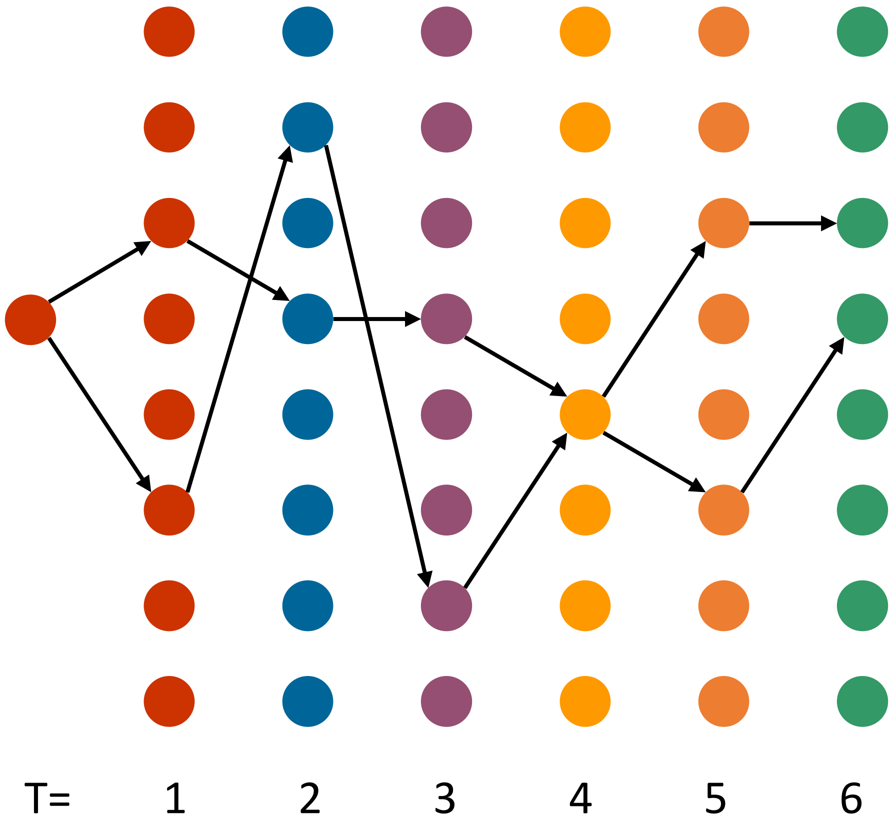
Examples
- 3 units: 8 possible states
- N units: 2^N possible states

How many solutions are there anyway?



- Optimization over a time horizon divided into intervals
- A solution is a path linking one combination at each interval
- How many such paths are there?

How many solutions are there anyway?



Optimization over a time horizon divided into intervals

A solution is a path linking one combination at each interval

How many such path are there?

Answer:

$$\left(2^N\right)\left(2^N\right)\cdots\left(2^N\right)=\left(2^N\right)^T$$

The Curse of Dimensionality

- Example: 5 units, 24 hours

$$\left(2^{N}\right)^{T} = \left(2^{5}\right)^{24} = 6.2\,10^{35} \text{ combinations}$$

- Processing 10^9 combinations/second, this would take 1.9 10^{19} years to solve
- There are 100’s of units in large power systems...
- Many of these combinations do not satisfy the constraints

How Can We Beat the Curse?

Brute force approach won't work!

- Need to be smart
- Try only a small subset of all combinations
- Can't guarantee optimality of the solution
- Try to get as close as possible within a reasonable amount of time

Optimization with Integer Variables

- Continuous variables
  - Can follow the gradients or use Linear Programming (LP)
  - Any value within the feasible set is OK

- Discrete variables
  - There is no gradient
  - Can only take a finite number of values
  - Problem is not convex
  - Must try combinations of discrete values

Main Solution Techniques

- Characteristics of a good technique
  - Solution close to the optimum
  - Reasonable computing time
  - Ability to model constraints

- Priority list / heuristic approach
- Dynamic programming
- Lagrangian relaxation ← State of the art
- Mixed Integer Programming

Can we at least get an idea of what the solution will look like?

Let's try to solve a simple example

A Simple Example

- Unit 1:
  - $P_{Min} = 250$ MW, $P_{Max} = 600$ MW
  - $C_1 = 510.0 + 7.9\ P_1 + 0.00172\ P_1^2$ $/h
- Unit 2:
  - $P_{Min} = 200$ MW, $P_{Max} = 400$ MW
  - $C_2 = 310.0 + 7.85\ P_2 + 0.00194\ P_2^2$ $/h
- Unit 3:
  - $P_{Min} = 150$ MW, $P_{Max} = 500$ MW
  - $C_3 = 78.0 + 9.56\ P_3 + 0.00694\ P_3^2$ $/h
- What *combination* of units 1, 2 and 3 will produce 550 MW at minimum cost?
- How much should each unit in that combination generate?

Cost of the Various Combinations

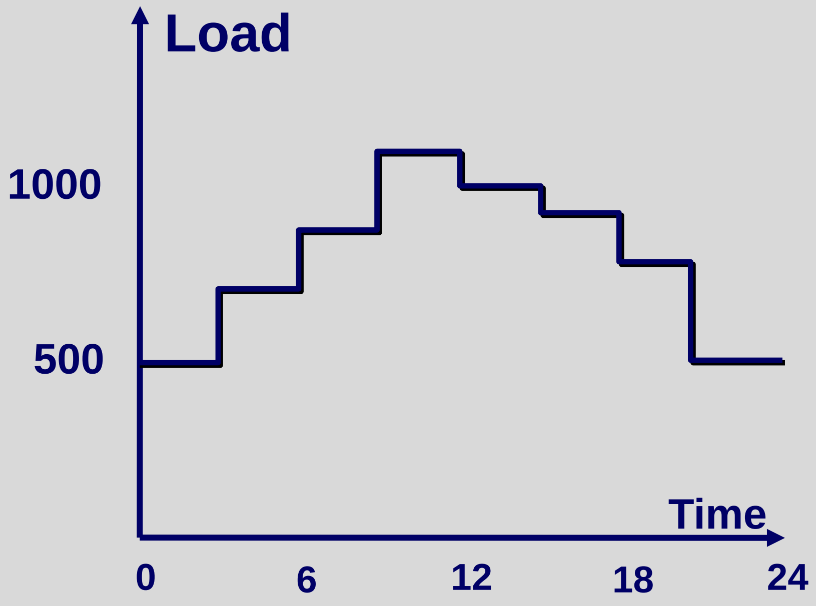
G1	G2	G3	P_{min}	P_{max}	P_1	P_2	P_3	C_{total}
Off	*Off*	*Off*	0	0	Infeasible			
Off	*Off*	*On*	150	500	Infeasible			
Off	*On*	*Off*	200	400	Infeasible			
Off	*On*	*On*	350	900	0	400	150	5418
On	*Off*	*Off*	250	600	550	0	0	3753
On	*Off*	*On*	400	1100	400	0	150	5613
On	*On*	*Off*	450	1000	295	255	0	5471
On	*On*	*On*	600	1500	Infeasible			5617

Observations on the example:

- Far too few units committed:
  → Can't meet the demand
- Not enough units committed:
  → Some units operate above optimum
- Too many units committed:
  → Some units below optimum
- Far too many units committed:
  → Minimum generation exceeds demand

- No-load cost affects choice of optimal combination

Another Example



Optimal generation schedule for a load profile

Decompose the profile into a set of period

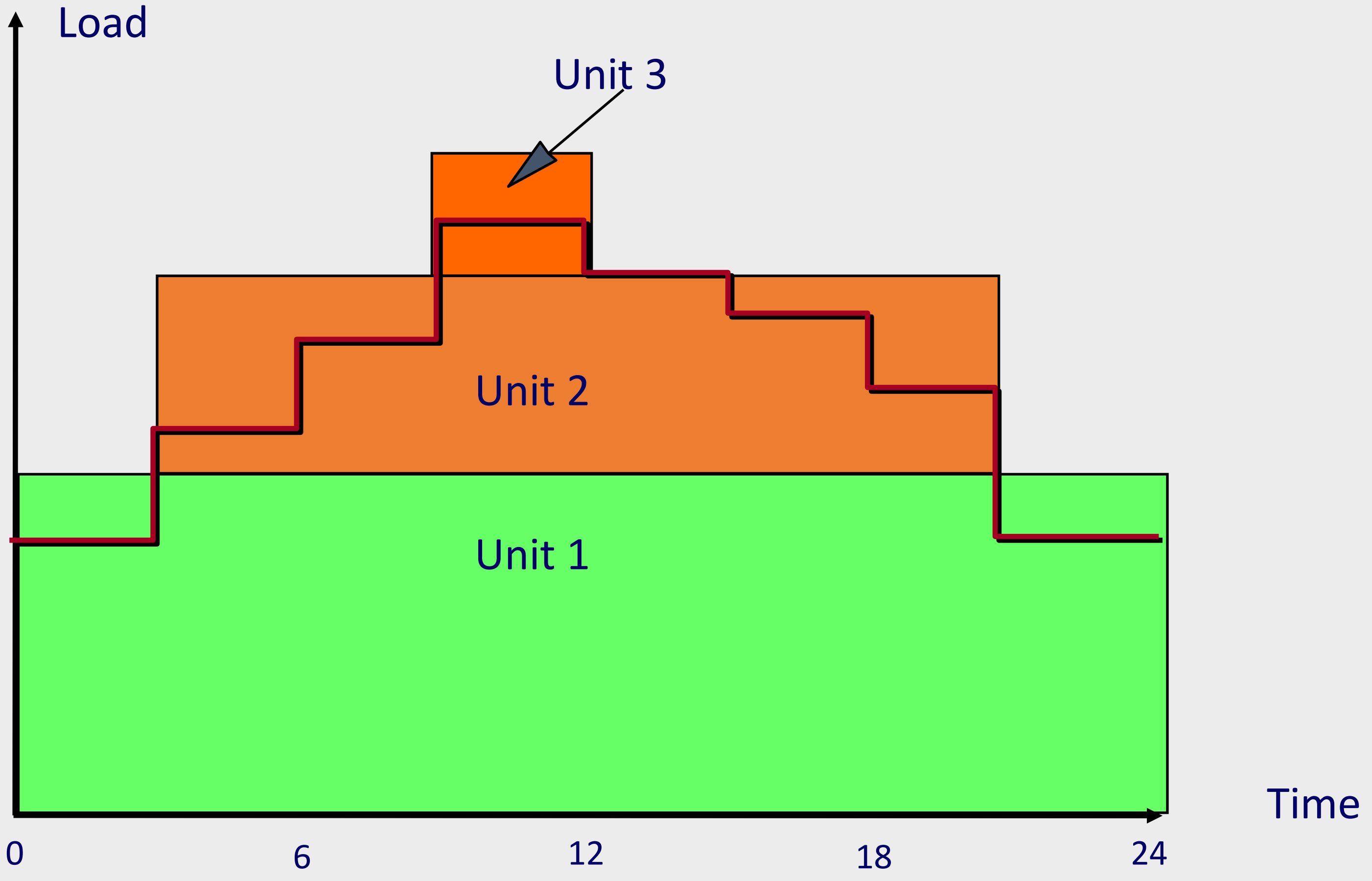
Assume load is constant over each period

For each time period, which units should be committed to generate at minimum cost during that period?

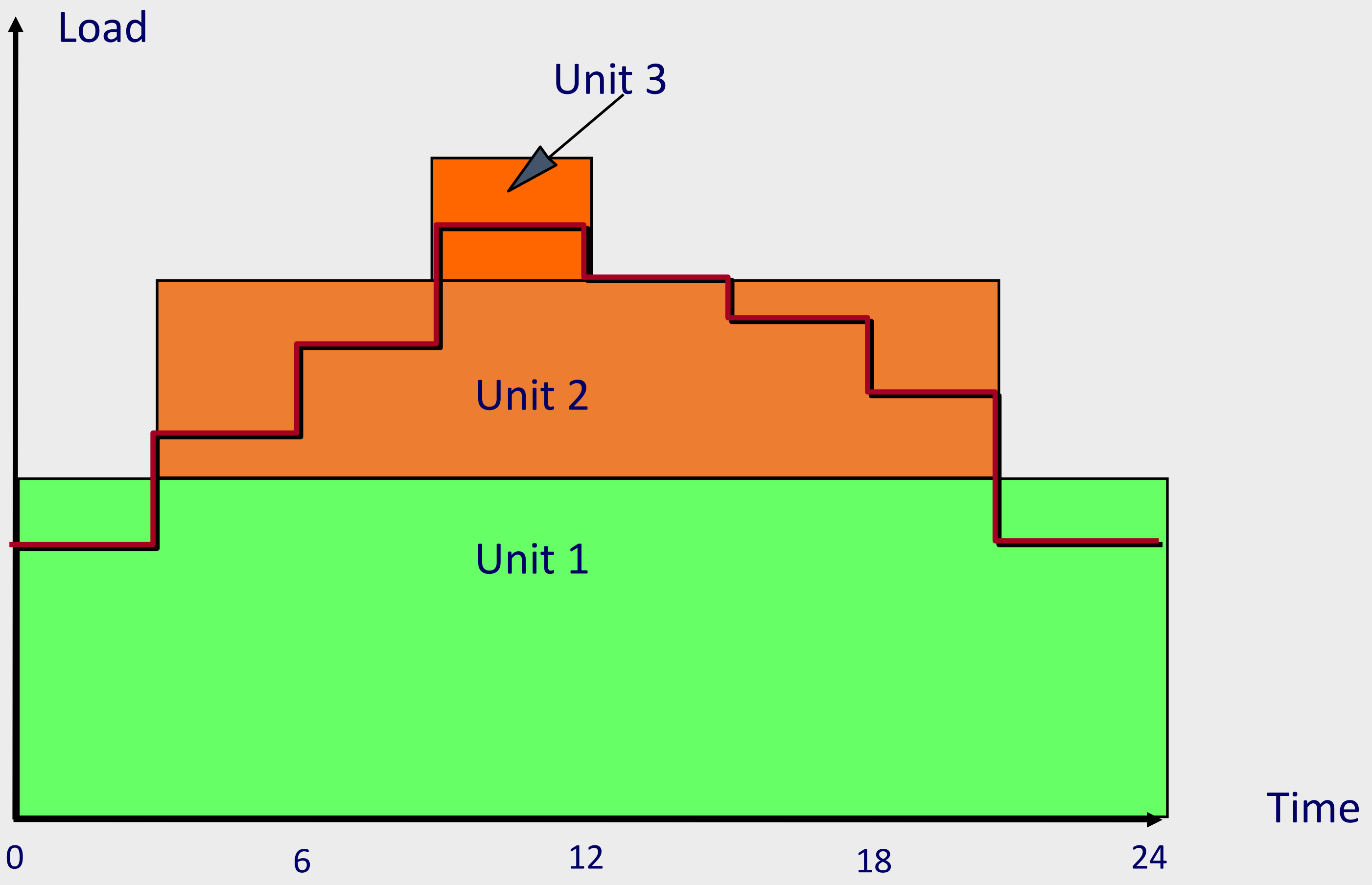
Optimal Combination for Each Hour

Load	Unit 1	Unit 2	Unit 3
1100	On	On	On
1000	On	On	Off
900	On	On	Off
800	On	On	Off
700	On	On	Off
600	On	Off	Off
500	On	Off	Off

Matching the Combinations to the Load



Matching the Combinations to the Load



Flexible Plants

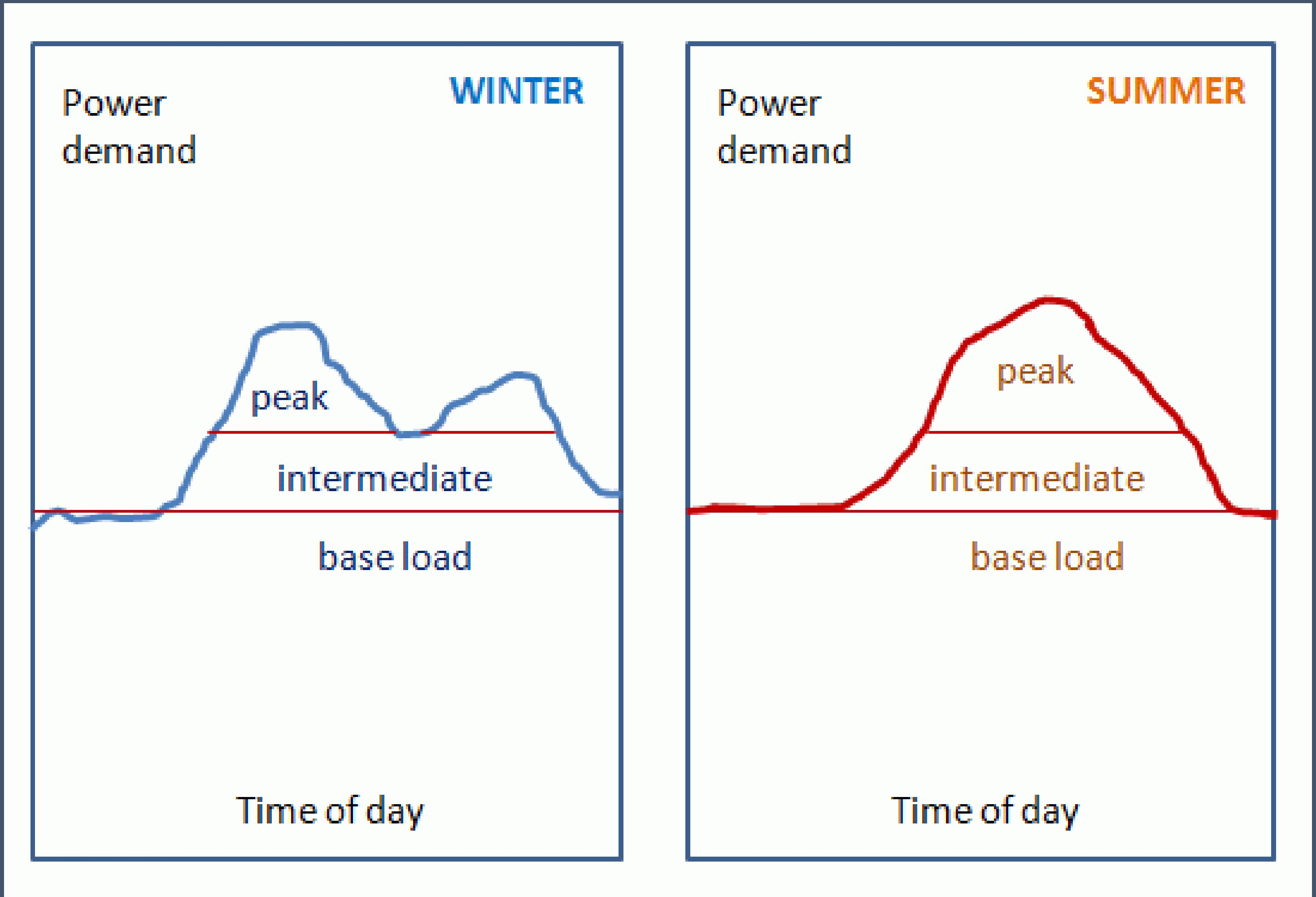
- Power output can be adjusted (within limits)
- Examples:
  - Coal-fired
  - Oil-fired
  - Open cycle gas turbines
  - Combined cycle gas turbines
  - Hydro plants with storage
- Status and power output can be optimized

Thermal units

Inflexible Plants

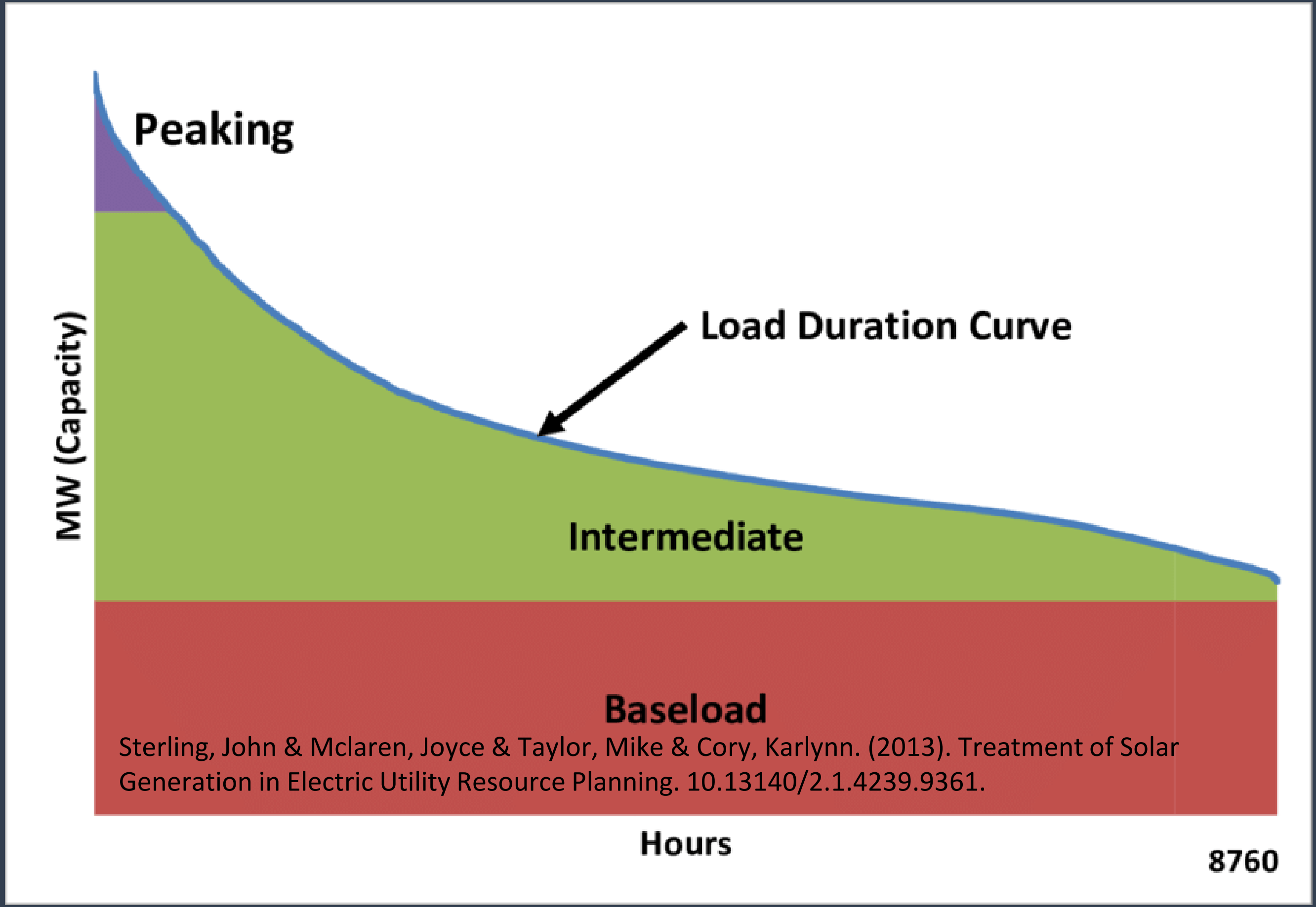
- Power output cannot be adjusted for technical or commercial reasons
- Examples:
  - Nuclear
  - Run-of-the-river hydro
  - Renewables (wind, solar,…)
  - Combined heat and power (CHP, cogeneration)
- Output treated as given when optimizing

Typical daily variation in power demand



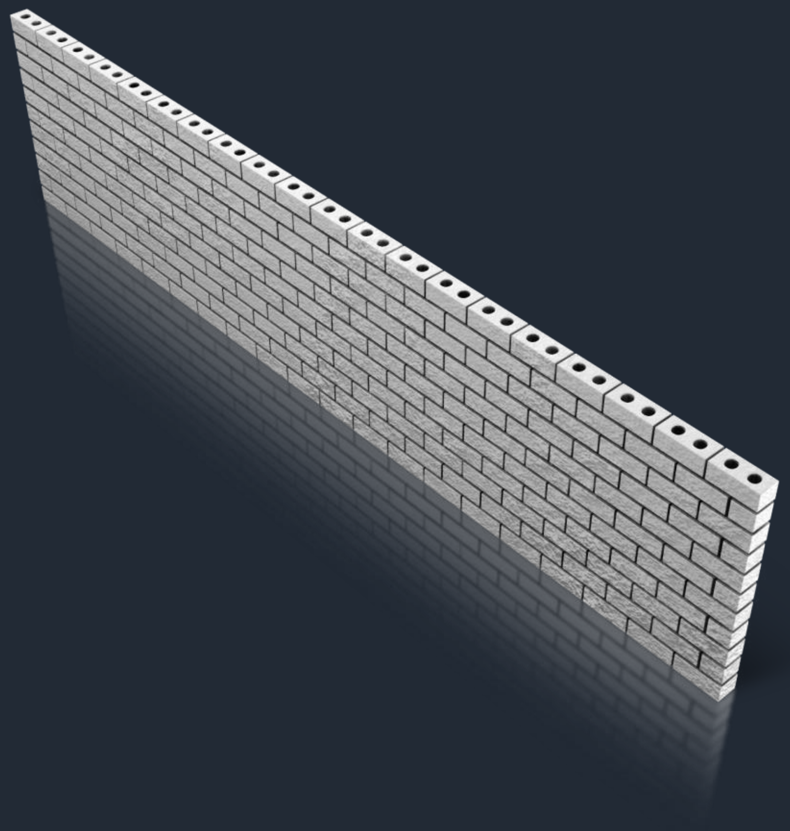
https://www.e-education.psu.edu/eme807/node/667

Load Duration Curve



The load duration curve is defined as the curve between the load and time in which the ordinates representing the load, plotted in the order of decreasing magnitude, i.e., with the greatest load at the left, lesser loads towards the rights and the lowest loads at the time extreme right. (Source: circuitglobe.com)

Issues



Must consider all constraints

- Unit constraints
- System constraints

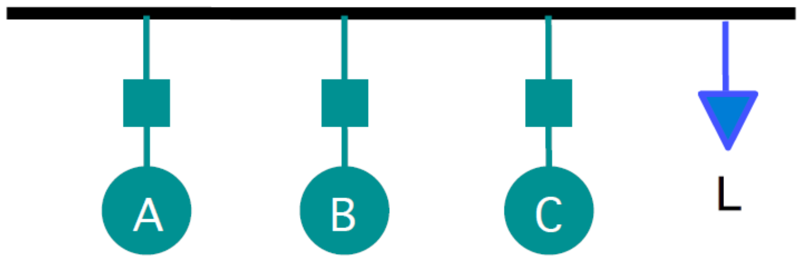
Some constraints create a link between the periods

Starting up a generating unit costs money in addition to the running cost considered in economic dispatch

Curse of dimensionality

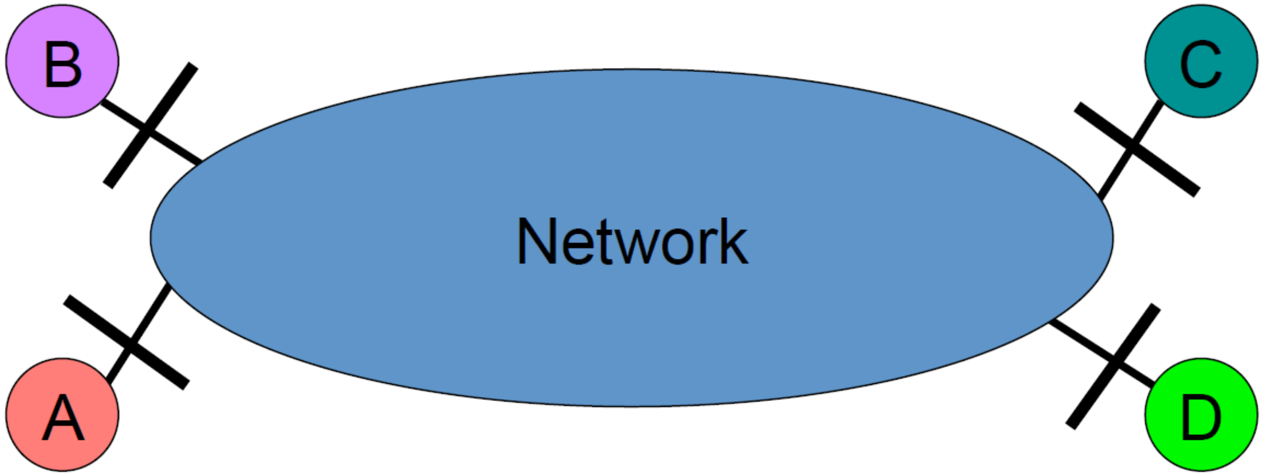
The Impact of Network Limitation

Economic Dispatch



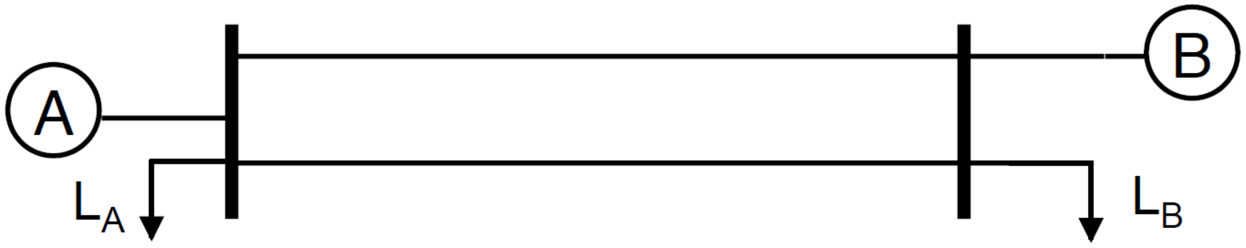
- Objective: minimize the cost of generation
- Constraints
  - Equality constraint: load generation balance
  - Inequality constraints: upper and lower limits on generating units output

Limitation of Economic Dispatch

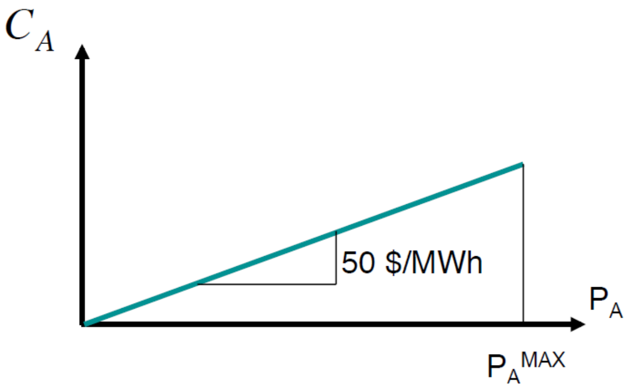


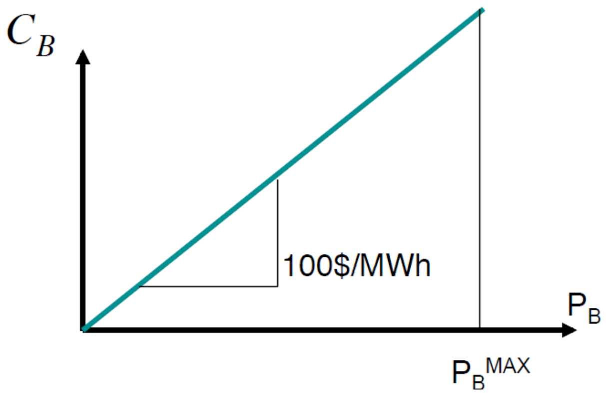
- Generating units and loads are not all connected to the same bus
- The economic dispatch may result in unacceptable flows or voltages in the network

Example of network limitation

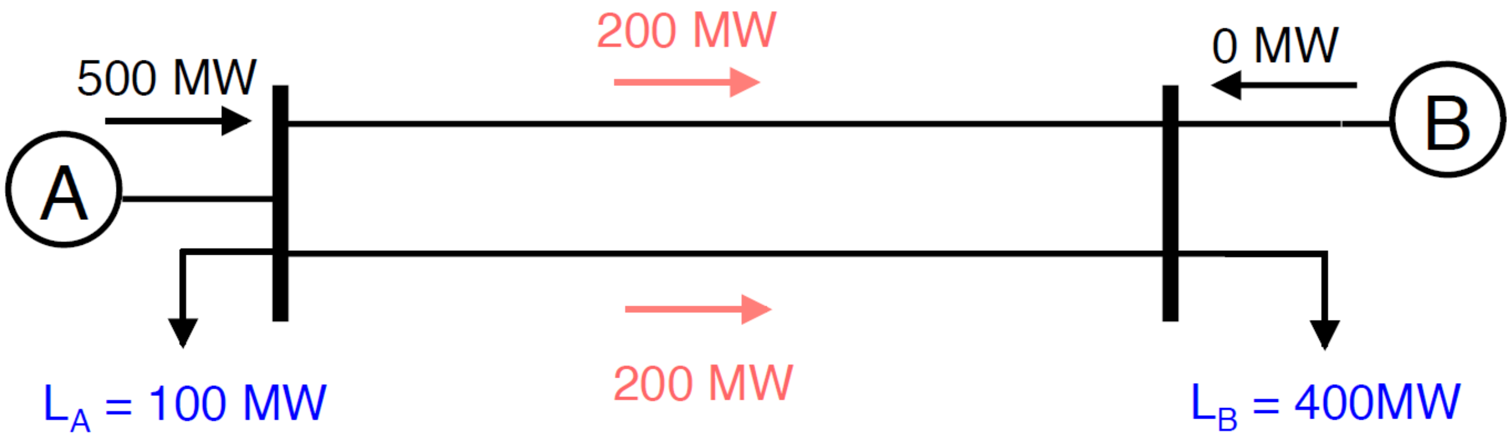


Maximum flow on each line: 100MW





Unacceptable ED Solution



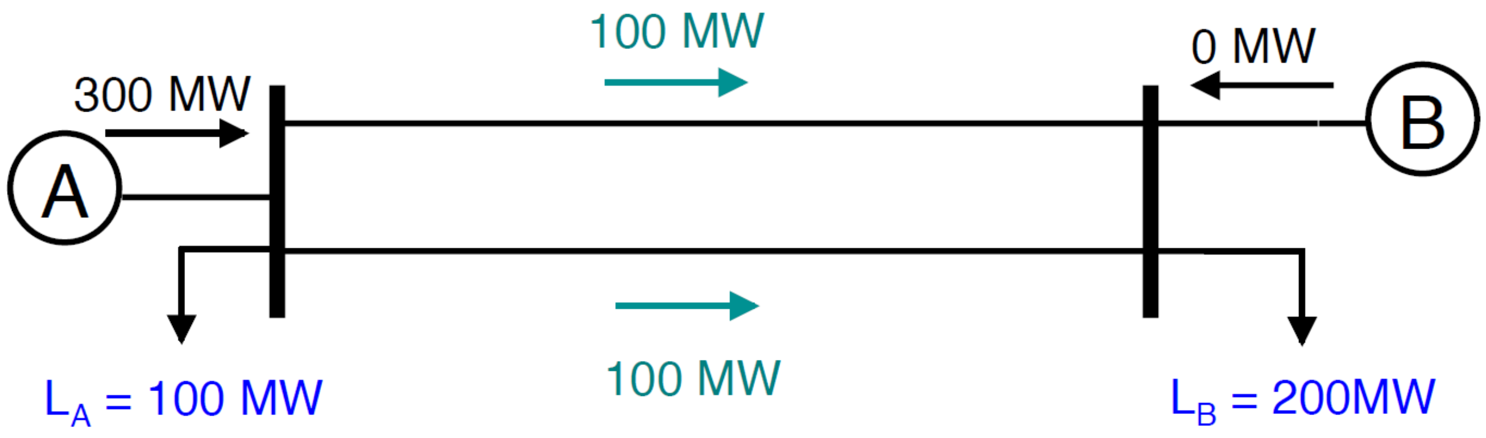
The solution of the economic dispatch is:

$$P_A = 500 \text{ MW}$$

$$P_B = 0 \text{ MW}$$

The resulting flows exceed their limit
The economic dispatch solution is not acceptable

Acceptable ED Solution



The solution of this (trivial) economic dispatch is:

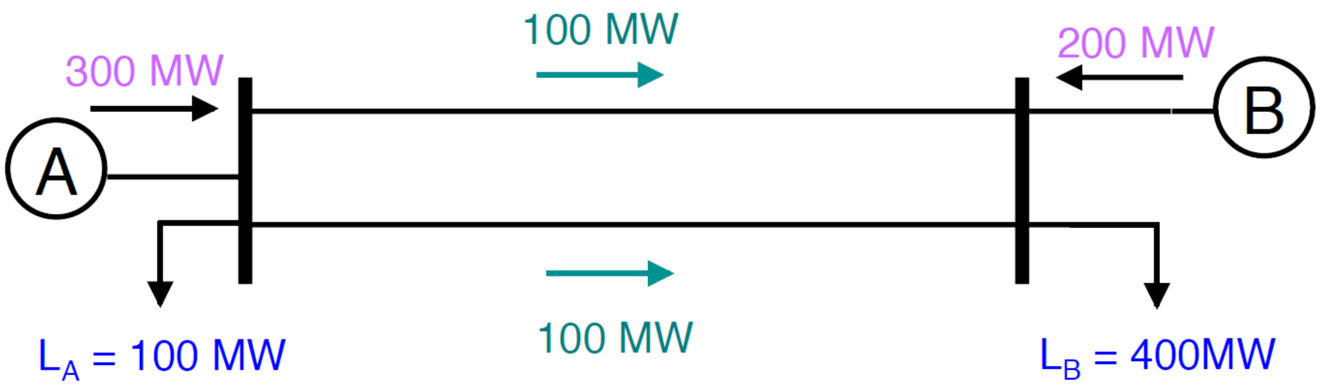
$$P_A = 300 \text{ MW}$$

$$P_B = 0 \text{ MW}$$

The flows on the lines are below the limit

The economic dispatch solution is acceptable

Modified ED Solution



In this simple case, the solution of the economic dispatch can be modified easily to produce acceptable flows.

This could be done mathematically by adding the following inequality constraint: $P_A - L_A \leq 200\text{MW}$

However, adding inequality constraints for each problem is not practical in more complex situations
We need a more general approach

Optimal Power Flow Problem

Optimal Power Flow- Overview

- Optimization problem
- Full ac model
- Classical objective function
  - Minimize the cost of generation
  - Many other possibilities
- Equality constraints
  - Power balance at each node - power flow equations
- Inequality constraints
  - Network operating limits (line flows, voltages)
  - Limits on control variables

Mathematical Formulation of the OPF (1)

- Decision variables (control variables)
  - Active power output of the generating units
  - Voltage at the generating units
  - Position of the transformer taps
  - Position of the phase shifter (quad booster) taps
  - Status of the switched capacitors and reactors
  - Control of power electronics (HVDC, FACTS)
  - Amount of load disconnected
- Vector of control variables: $\boldsymbol{u}$

Mathematical Formulation of the OPF (2)

- State variables
  - Describe the response of the system to changes in the control variables
  - Magnitude of voltage at each bus
    - Except generator busses, which are control variables
  - Angle of voltage at each bus
    - Except slack bus
  - Vector of state variables: $\boldsymbol{x}$

Mathematical formulation of the OPF (3)

- Parameters
  - Known characteristics of the system
  - Assumed constant
    - Network topology
    - Network parameters (R, X, B, flow and voltage limits)
    - Generator cost functions
    - Generator limits
    - …
  - Vector of parameters: $\boldsymbol{y}$

Mathematical Formulation of the OPF (4)

- Classical objective function:
  - Minimize total generating cost: $\min_{\boldsymbol{u}} \sum_{i=1}^{N_G} \boldsymbol{C}_i(\boldsymbol{P}_i)$
- Many other objective functions are possible:
  - Minimize changes in controls: $\min_{\boldsymbol{u}} \sum_{i=1}^{N_U} \boldsymbol{c}_i \mid \boldsymbol{u}_i - \boldsymbol{u}_i^0 \mid$
  - Minimize system losses
  - …

Mathematical Formulation of the OPF (5)

- Equality constraints:
  - Power balance at each node - power flow equations

$$P_k^G - P_k^L = \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)]$$

$$Q_k^G - Q_k^L = \sum_{i=1}^{N} V_k V_i [G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)]$$

$$k = 1, \ldots N$$

  - Compact expression:

$$G(x, u, y) = 0$$

Mathematical Formulation of the OPF (6)

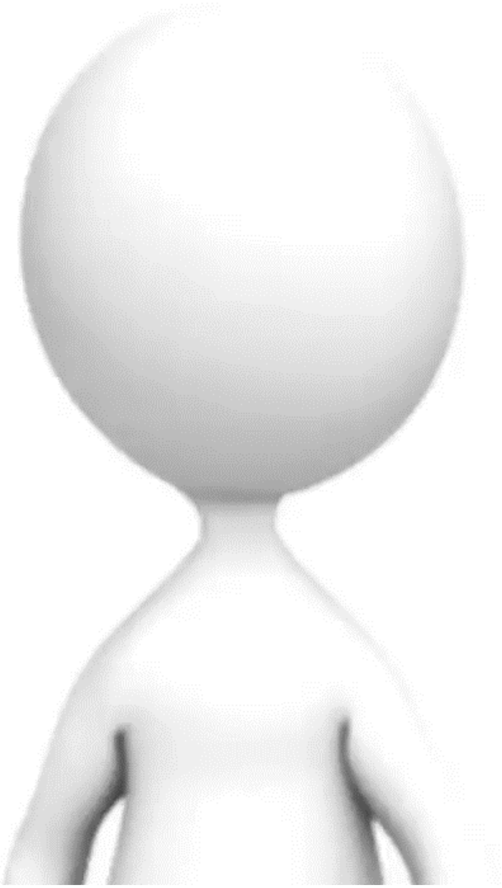
- Inequality constraints:
  - Limits on the control variables: $\underline{\boldsymbol{u}} \leq \boldsymbol{u} \leq \overline{\boldsymbol{u}}$
  - Operating limits on flows: $\left|\boldsymbol{F}_{ij}\right| \leq \overline{\boldsymbol{F}_{ij}}$
  - Operating limits on voltages $\underline{\boldsymbol{V}_{j}} \leq \boldsymbol{V}_{j} \leq \overline{\boldsymbol{V}_{j}}$
- Compact expression: $\boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) \geq 0$

Compact Form of the OPF problem

$$\min_{\boldsymbol{u}} \boldsymbol{f}(\boldsymbol{u})$$

Subject to: $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) = 0$

$$\boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{y}) \geq 0$$



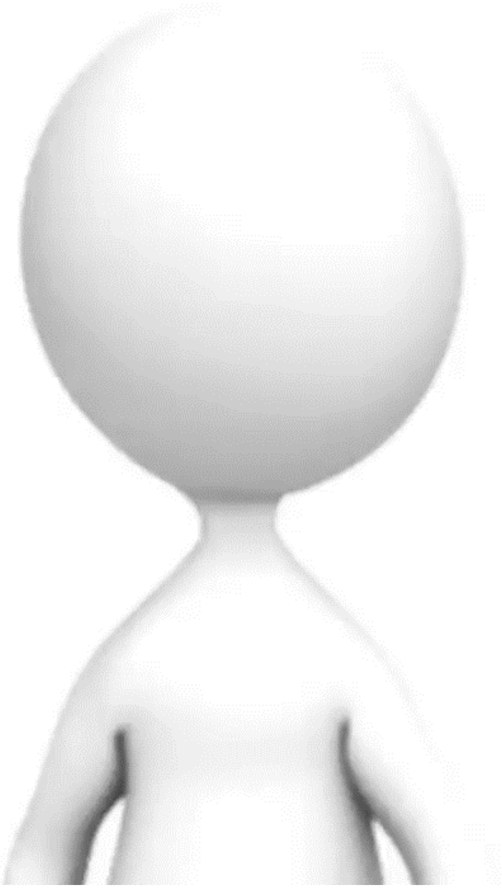
OPF Applications

- Planning
  - Lots of models & features
- Operation
  - Speed
  - Robust
  - Modeling accuracy
  - Should produce implementable solutions
- Markets
  - Even more robust
  - Support changes in market rules

OPF Challenges

- Size of the problem
  - 1000's of lines, hundreds of controls
  - Which inequality constraints are binding?
- Problem is non-linear
- Problem is non-convex
- Some of the variables are discrete
  - Position of transformer and phase shifter taps
  - Status of switched capacitors or reactors

DC Power Flow Problem



Advantages of DC Power Flow

- Linearization is more accurate for active power than reactive power
- Active power only OPF
  - Minimization of active power generation
  - Active power balance constraints
  - Line flow inequality constraints
- Not suitable for system operation
- Suitable for electricity markets
  - Clearing market
  - Calculating locational marginal prices

Active/Reactive Decoupling

- Active power controls
  - Generator P injections
  - Phase shifters
  - HVDC links
  - Demand response
  - Load shedding
- Reactive power controls
  - Generator V set-points
  - Transformer taps
  - Capacitor banks
  - Reactor banks

Expensive

Cheap

Power Flow Equations

$$P_k^I - \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

$$Q_k^I - \sum_{i=1}^{N} V_k V_i [G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)] = 0$$

Set of non-linear simultaneous equations

Occasionally need a simple linear relation for fast and intuitive analysis

dc power flow provides such a relation but requires a number of approximations

Neglect Resistance of the Branches

$$P_k^I - \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

$$\Downarrow$$

$$P_k^I - \sum_{i=1}^{N} V_k V_i B_{ki} \sin(\theta_k - \theta_i) = 0$$

Assume All Voltage Magnitudes = 1.0 p.u.

$$P_k^I - \sum_{i=1}^{N} V_k V_i B_{ki} \sin(\theta_k - \theta_i) = 0$$

$$\Downarrow$$

$$P_k^I - \sum_{i=1}^{N} B_{ki} \sin(\theta_k - \theta_i) = 0$$

Assume All Angles are Small

$$P_k^I - \sum_{i=1}^{N} B_{ki} \sin(\theta_k - \theta_i) = 0$$

If α is small: $\sin\alpha \approx \alpha$ (α in radians)

$$\Downarrow$$

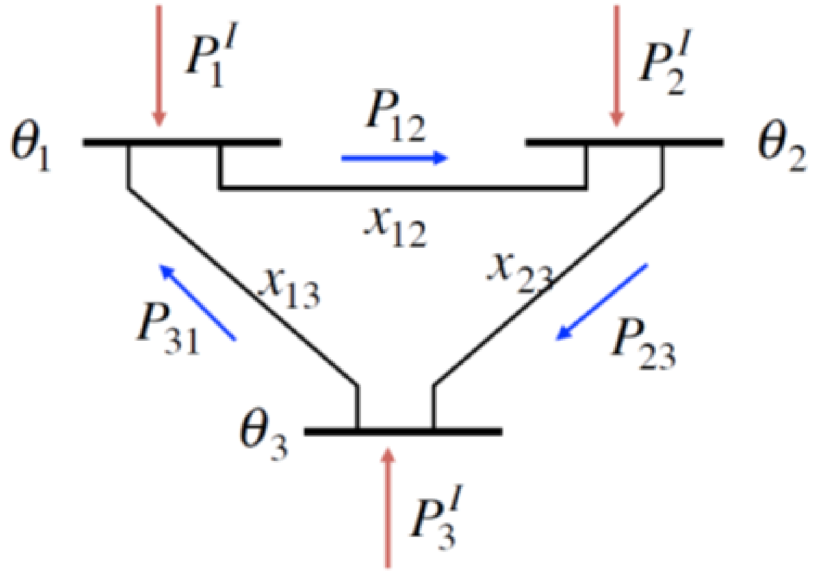
$$P_k^I - \sum_{i=1}^{N} B_{ki} (\theta_k - \theta_i) = 0 \quad \text{or} \quad P_k^I - \sum_{i=1}^{N} \frac{(\theta_k - \theta_i)}{x_{ki}} = 0$$

Interpretation

$$P_k^I - \sum_{i=1}^{N} \frac{(\theta_k - \theta_i)}{x_{ki}} = 0$$

$$P_k^I - \sum_{i=1}^{N} P_{ki} = 0$$

$$P_{ki} = \frac{(\theta_k - \theta_i)}{x_{ki}}$$



$$P_{12} = \frac{(\theta_1 - \theta_2)}{x_{12}}; \quad P_{23} = \frac{(\theta_2 - \theta_3)}{x_{23}}; \quad P_{31} = \frac{(\theta_3 - \theta_1)}{x_{13}}$$

Why is it called DC power flow?

- Reactance plays the role of resistance in dc circuit
- Voltage angle plays the role of dc voltage
- Power plays the role of dc current

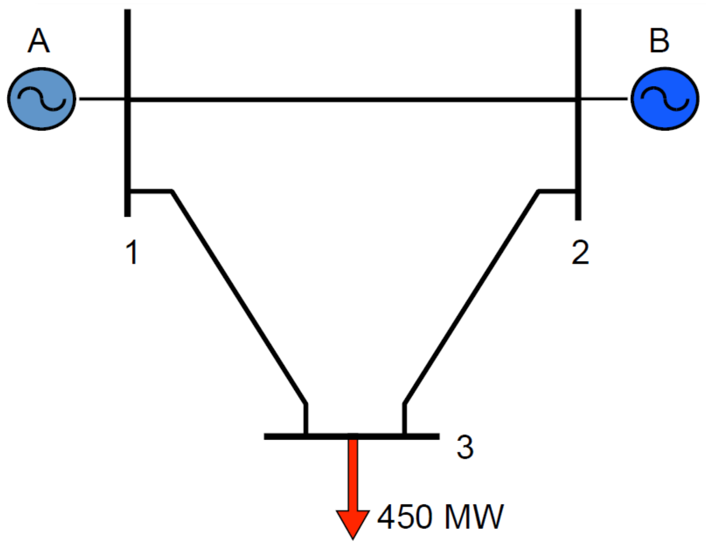
$$P_{ki} = \frac{(\theta_k - \theta_i)}{x_{ki}} \quad \Longleftrightarrow \quad I_{ki} = \frac{(V_k - V_i)}{R_{ki}}$$

An Illustrative Example

Example of Linear Programming OPF (LPOPF)

- Solving the full non-linear OPF problem by hand is too difficult, even for small systems
- We will solve linearized 3-bus examples by hand

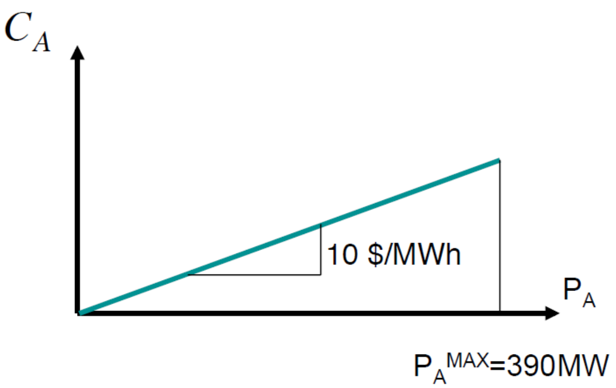
Example

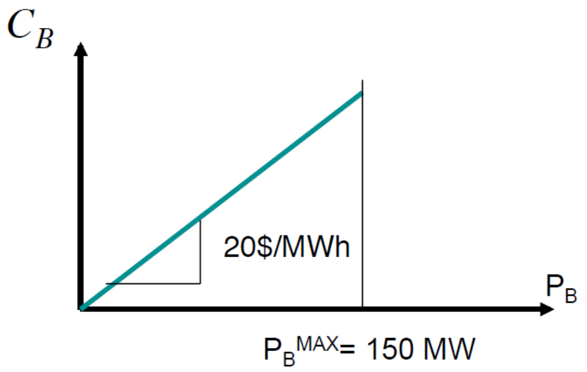


Economic dispatch:

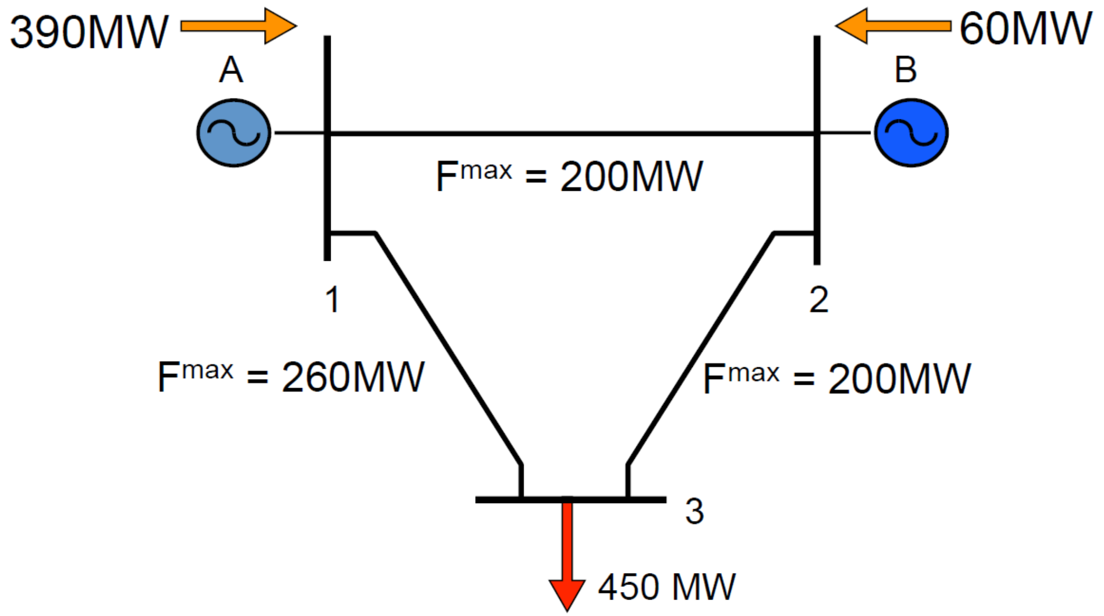
$$P_A = P_A^{\max} = 390 \text{ MW}$$

$$P_B = 60 \text{ MW}$$





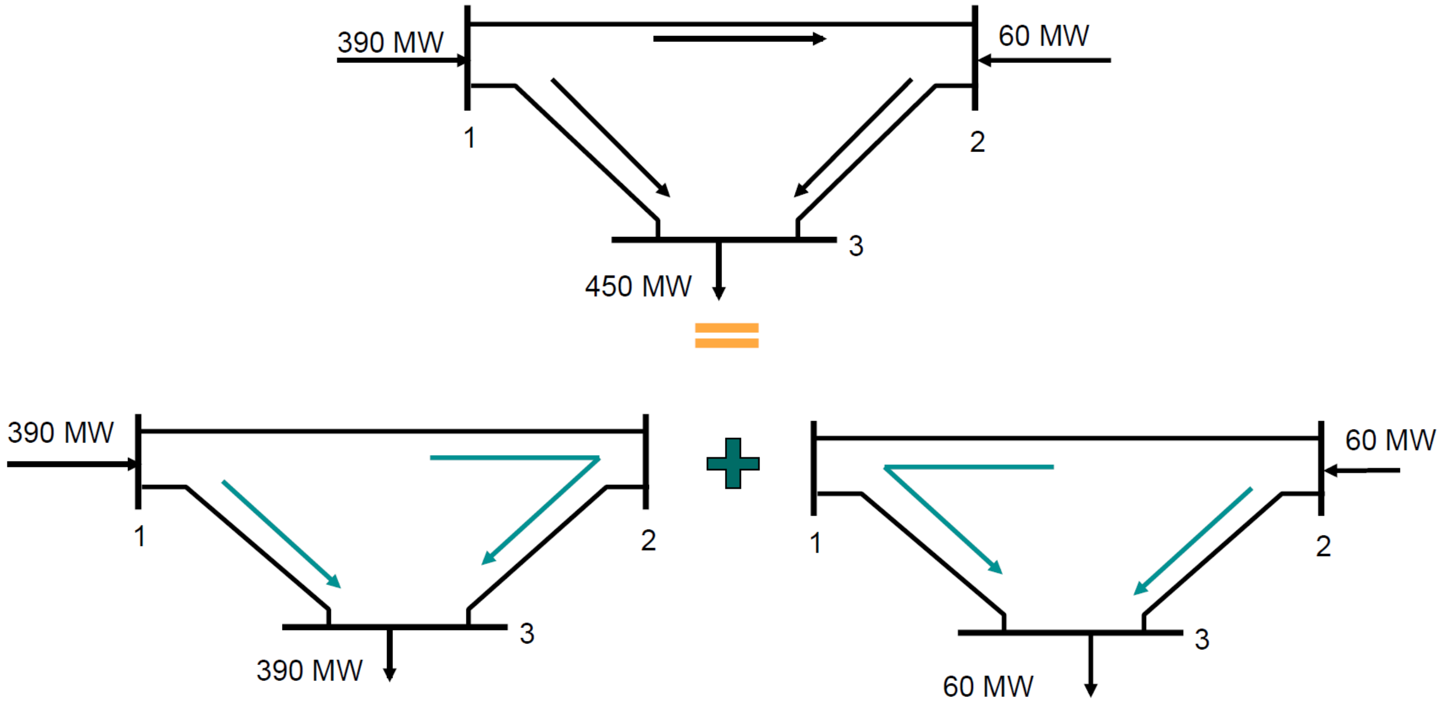
Flows Resulting from the Economic Dispatch



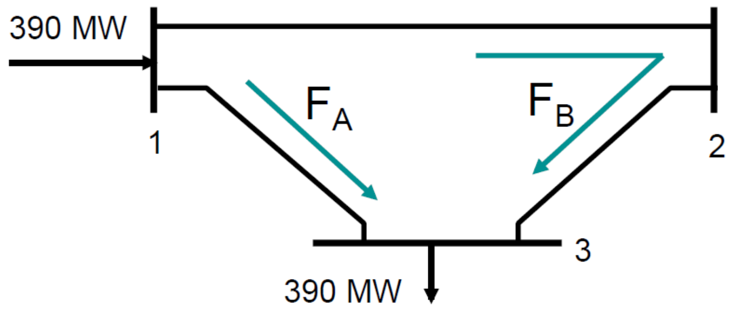
Assume that all the lines have the same reactance
Do these injection result in acceptable flows?

Calculating the Flows using Superposition

Because we assume a linear model, superposition is applicable



Calculating the Flows Using Superposition (1)

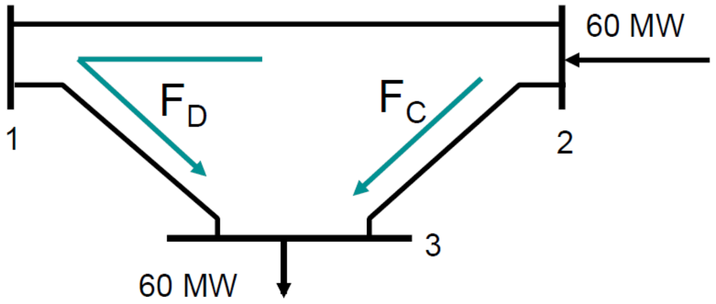


$F_A = 2 \times F_B$ because the path 1-2-3 has twice the reactance of the path 1-3

$F_A + F_B = 390$ MW

$F_A = 260$ MW
$F_B = 130$ MW

Calculating the Flows Using Superposition (2)

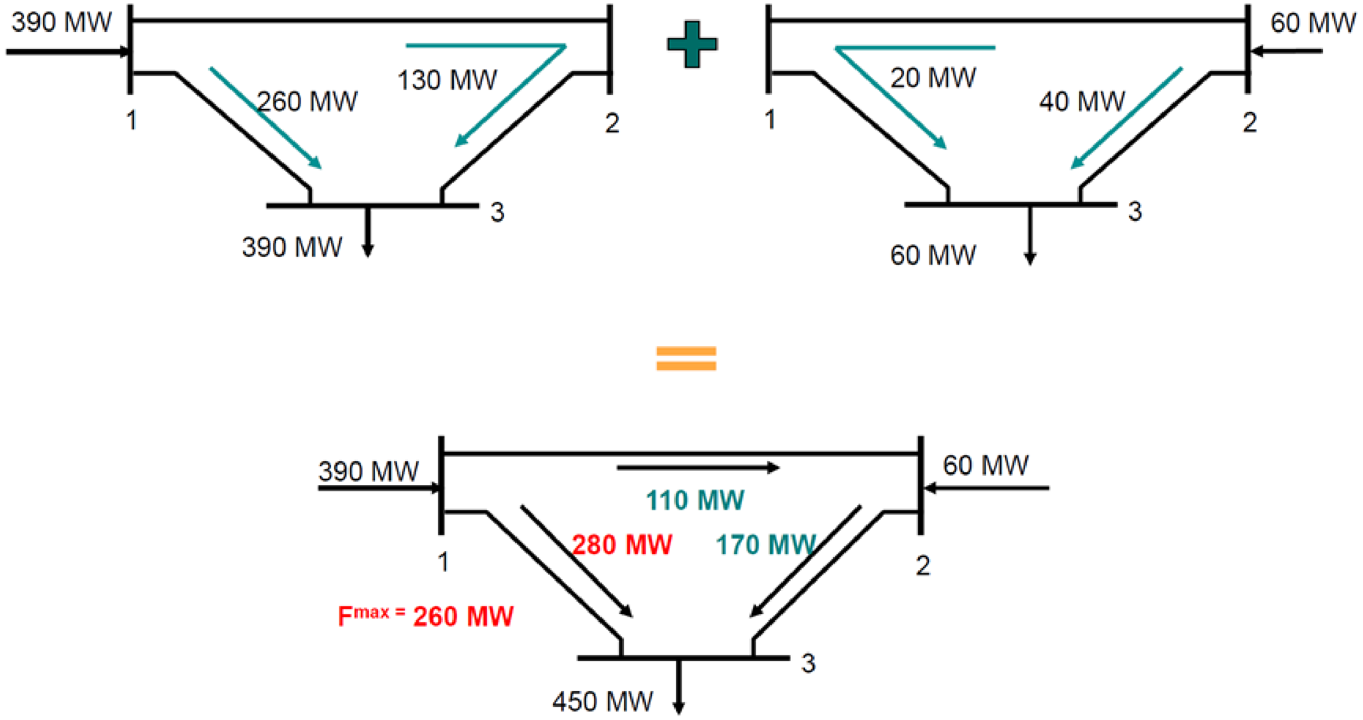


$F_C = 2 \times F_D$ because the path 2-1-3 has twice the reactance of the path 2-3

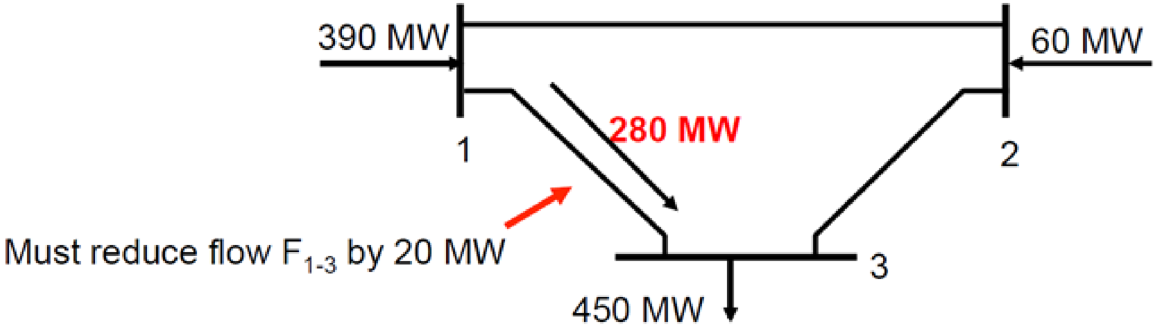
$F_C + F_D = 60$ MW

$F_C = 40$ MW
$F_D = 20$ MW

Calculating the Flows Using Superposition (3)

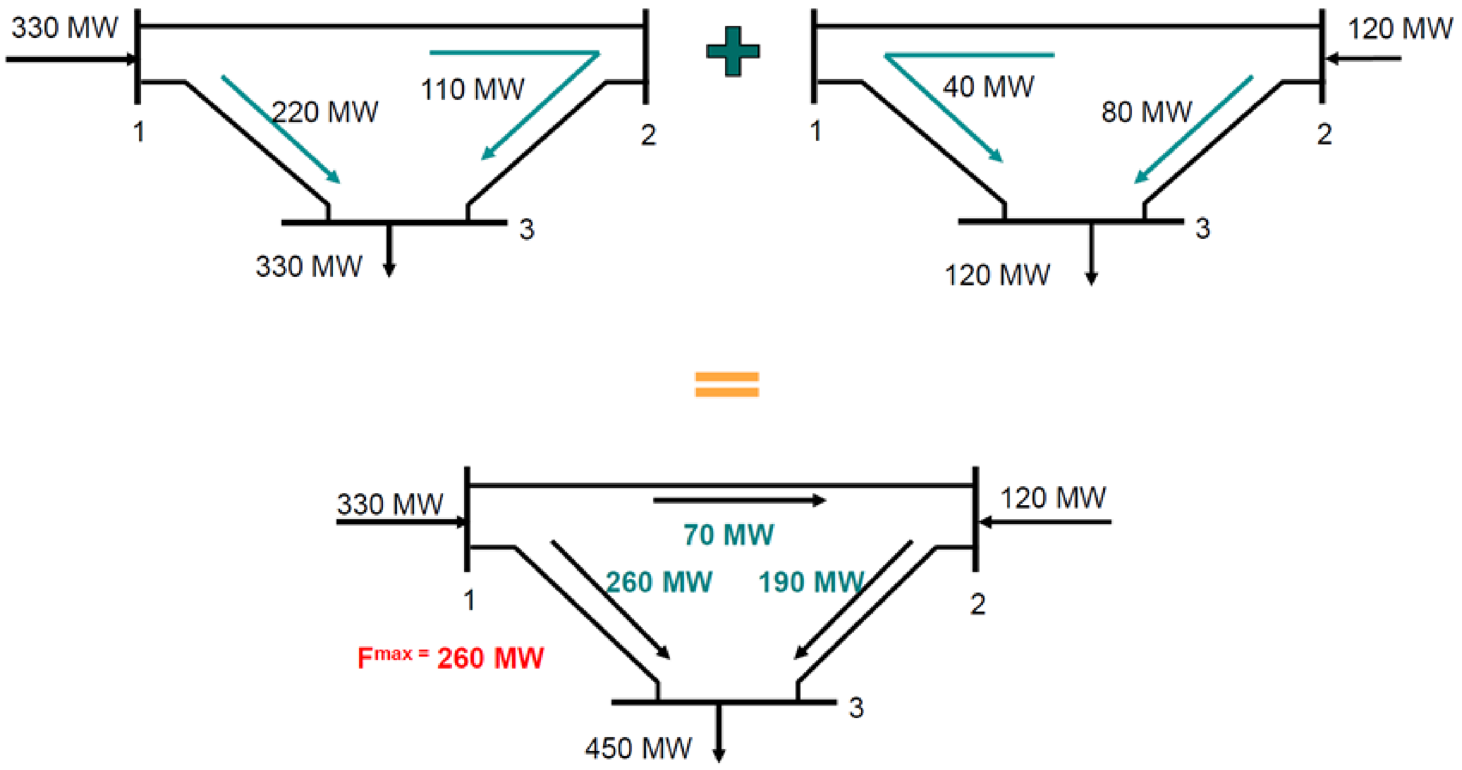


Correcting Unacceptable Flows



- Must use a combination of reducing the injection at bus 1 and increasing the injection at bus 2 to keep the load/generation balance
- Decreasing the injection at 1 by 3 MW reduces F_{1-3} by 2 MW
- Increasing the injection at 2 by 3 MW increases F_{1-3} by 1 MW
- A combination of a 3 MW decrease at 1 and 3 MW increase at 2 decreases F_{1-3} by 1 MW
- To achieve a 20 MW reduction in F_{1-3} we need to shift 60 MW of injection from bus 1 to bus 2

Check the Solution Using Superposition



Comments (1)

- The OPF solution is more expensive than the ED solution
  - $C_{ED} = 10 \times 390 + 20 \times 60 = \$5{,}100$
  - $C_{OPF} = 10 \times 330 + 20 \times 120 = \$5{,}700$
- The difference is the cost of security
  - $C_{security} = C_{OPF} - C_{ED} = \$600$
- The constraint on the line flow is satisfied exactly
  - Reducing the flow below the limit would cost more

Comments (2)

- We have used an “ad hoc” method to solve this problem
- In practice, there are systematic techniques for calculating the sensitivities of line flows to injections
- These techniques are used to generate constraint equations that are added to the optimization problem