A Simple Diode Model with Reverse Recovery

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Abstract—**The basic diode charge-control model used in SPICE is extended to include reverse recovery. The diode charge transport equations are simplified using the lumped charge concept of Linvill, and the model is demonstrated on the Saber simulator for simple inductive and resistive load circuits. The two model parameters, diode lifetime τ and diffusion transit time T_M can be easily determined from a switching waveform.**

I. Introduction

The diode models widely used in circuit simulators such as SPICE are based on the original charge control model [1] which includes the effects of charge storage during reverse recovery operation but does not provide for reverse recovery of the diode. Thus, these diode models always exhibit an instantaneous or snappy recovery during commutation to the non-conducting state.

Some of the appearance of a more gradual or softer recovery can be obtained by adding extra parallel capacitance to the diode. However, this practice gives the incorrect relation between diode reverse current and voltage. Capacitive current is non-dissipative, associated with energy storage in the capacitor. Actual diode reverse recovery current is dissipative and is the most important source of power dissipation for diodes used in fast switching converters.

This paper extends the basic charge-control diode model to include reverse recovery. The model is derived from the semiconductor charge transport equations and its performance is demonstrated using the Saber simulator.

II. Model Derivation and Description

A high voltage p-i-n structure operating in high level injection is assumed as typical for most power diodes. The original charge control diode model employs one charge storage node. When the charge stored in that node becomes exhausted during reverse conduction, the diode instantly switches to the reverse blocking mode. In actual diodes, reverse recovery is caused by diffusion of charge from the center of the i region; thus, one or more additional charge storage nodes must be added to provide for this diffusion current.

Fig. 1 shows the charge distribution in a p-i-n diode under forward conduction with the charge assigned to four charge storage nodes following the lumped model approach originally developed by Linvill [2]. Under high-level injection conditions the hole and electron distributions are equal, $p(x) = n(x)$. The excess stored charges q_1, q_2, q_3, q_4 are assumed to be located at a node in the center of each charge storage region. For simplicity, equal hole and electron mobilities are assumed which makes the charge distributions symmetric and allows the analysis to be simplified to the charges q_1 and q_2 on one side. Here, $q_1 = qA\delta(p_1 - p_{i0})$ and $q_2 = qAd(p_2 - p_{i0})$ where q is the unit electron charge, A the junction area, δ and d the widths of the two charge storage regions, p_1 and p_2 the average hole concentrations in the regions corresponding to q_1 and q_2, and p_{i0} the equilibrium hole concentration.

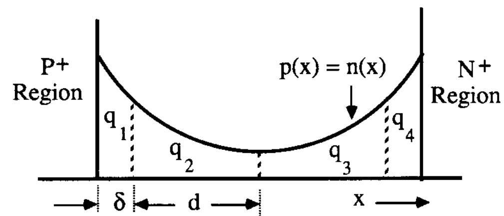

Fig. 1. Charge storage locations in a p-i-n diode.

High level or ambipolar diffusion is the mechanism for charge redistribution from q_1 to q_2, while holes are injected directly into q_1 from the p^+-i junction on the left. Current $i(t)$ is by diffusion between the two regions:

$$i(t) = -qA2D_a \frac{dp}{dx} = \frac{qA2D_a(p_1 - p_2)}{\left(\frac{\delta}{2} + \frac{d}{2}\right)}. \quad (1)$$

Here, D_a is the high level or ambipolar diffusion constant. To prevent an arbitrarily large current from flowing between q_1 and the external leads, the width δ must be made small compared to the width d. In the model we take the limit $\delta \to 0$, and (1) becomes

$$i(t) = \frac{(q_0 - q_2)}{T_{12}}. \quad (2)$$

Here, $q_0 = qAd(p_1 - p_{i0})$ represents the variable remaining in (2) after $\delta \to 0$, and the constant $T_{12} = d^2/4D_a$ represents the approximate diffusion time across the region q_2. The variable q_0 has the units of charge, but does not represent charge storage. The charge control continuity equation for q_2 is

$$0 = \frac{dq_2}{dt} + \frac{q_2}{\tau} - \frac{(q_0 - q_2)}{2T_{12}}. \quad (3)$$

The first term is charge storage, the second is recombination with lifetime τ, and the third represents the half of the diffusion current from the p^+-i junction which is injected into q_2. Finally, the relationship between the variable q_0 and the junction voltage can be determined from the p^+-i junction equation:

$$(p_1 - p_{i0}) = p_{n0}\left[\exp\left(\frac{v}{2V_T}\right) - 1\right]. \quad (4)$$

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Here $v/2$ represents the half of the diode voltage which drops across the p^+-i junction. Finally, multiply both sides of (4) by qAd to obtain q_0:

$$qAd(p_1 - p_{i0}) = q_0 = \frac{I_s\tau}{2}\left[\exp\left(\frac{v}{2V_T}\right) - 1\right] \quad (5)$$

Here $I_s = qA2dp_{i0}/\tau$ is similar to the diode saturation current I_s used in the SPICE mode. The diode model is completely described by (2), (3), and (5). However, a few substitutions $q_M = 2q_2$, $T_M = 2T_{12}$, $q_E = 2q_0$, simplify these equations slightly:

$$i(t) = \frac{(q_E - q_M)}{T_M} \quad (6)$$

$$0 = \frac{dq_M}{dt} + \frac{q_M}{\tau} - \frac{(q_E - q_M)}{T_M} \quad (7)$$

$$q_E = I_s\tau\left[\exp\left(\frac{v}{nV_T}\right) - 1\right]. \quad (8)$$

In (8) the emission coefficient n replaces the factor 2 in (5) to generalize the equation and make it similar to the SPICE equation. Typically, $n \to 2$ at high level injection, but by setting $n \to 1$, the model can also function as a low-voltage p-n junction diode.

The steady-state dc forward-bias i-v characteristics can be derived from (6), (7), and (8):

$$i = \frac{I_s}{\left(1 + \frac{T_M}{\tau}\right)}\left[\exp\left(\frac{v}{nV_T}\right) - 1\right] \quad (9)$$

For $T_M \to 0$, the familiar expression of the original charge control model is obtained. Also, the total forward bias injected charge is $q_M = i\tau$ as in the original model. For a complete diode model, the equations for the junction capacitance, parasitic series resistance and lead wire inductance also need to be added.

III. Determination of Model Parameters

The parameters τ and T_M must be determined from a diode turn-OFF current waveform such as the typical inductive load waveform shown in Fig. 2. For $t \le T_1$, the diode presents a low impedance and the circuit determines the current. The total stored charge $q_M(t)$ can be found by eliminating q_E from (6) and (7) and solving for $q_M(t)$ under the boundary condition $i(t) = I_F - at$ where the constant $a = -(di/dt)$. Following the approach given in Tien and Hu [3],

$$q_M(t) = a\tau\left[T_0 + \tau - t - \tau\exp\left(-\frac{t}{\tau}\right)\right] \quad \text{for } t \le T_1. \quad (10)$$

At time $t = T_1$ the charge q_M becomes sufficiently depleted that the diffusion term in (6) limits the transfer of charge from q_M, and the recovery phase begins:

$$q_E(T_1) = 0, \quad v(T_1) = 0, \quad \text{and}$$

$$i(T_1) = -I_{RM} = -\frac{q_M(T_1)}{T_M} \quad \text{for } t = T_1. \quad (11)$$

Here I_{RM} is the maximum reverse current shown in Fig. 2.

After $t \ge T_1$, the diode becomes a high impedance and the diffusion current during recovery becomes independent of the

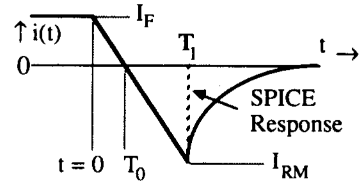

Fig. 2. Reverse recovery $i(t)$ waveform.

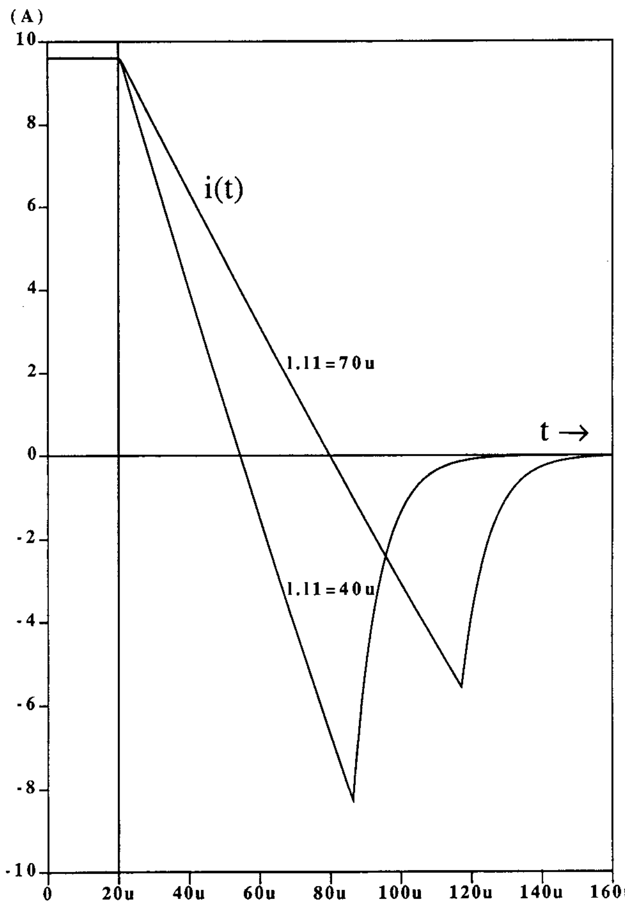

Fig. 3. Inductive load $i(t)$ turn-OFF switching waveform for two different inductor values.

diode reverse voltage. From (6), (7) and (11) for $q_E = 0$:

$$i(t) = -\frac{q_M(t)}{T_M} = -I_{RM}\exp\left[-\frac{(t - T_1)}{\tau_{rr}}\right] \quad \text{for } t \ge T_1. \quad (12)$$

Here, $1/\tau_{rr} = (1/\tau) + (1/T_M)$ where τ_{rr} is the reverse recovery time constant which can be measured from the current waveform (Fig. 2). The other time parameters can be obtained from a measurement of T_1 and I_{RM} as follows. Set $t = T_1$ in (10), eliminate $q_M(T_1)$ using (11), eliminate T_0 using $T_0 = T_1 - (I_{RM}/a)$ and solve for I_{RM} to obtain an expression with only one unknown τ:

$$I_{RM} = a(\tau - \tau_{rr})\left[1 - \exp\left(-\frac{T_1}{\tau}\right)\right]. \quad (13)$$

All parameters can now be determined: τ_{rr} is measured directly, τ determined from (13), T_M calculated from τ and τ_{rr}, and n and I_s found from the steady-state dc forward-bias i-v characteristic (9).

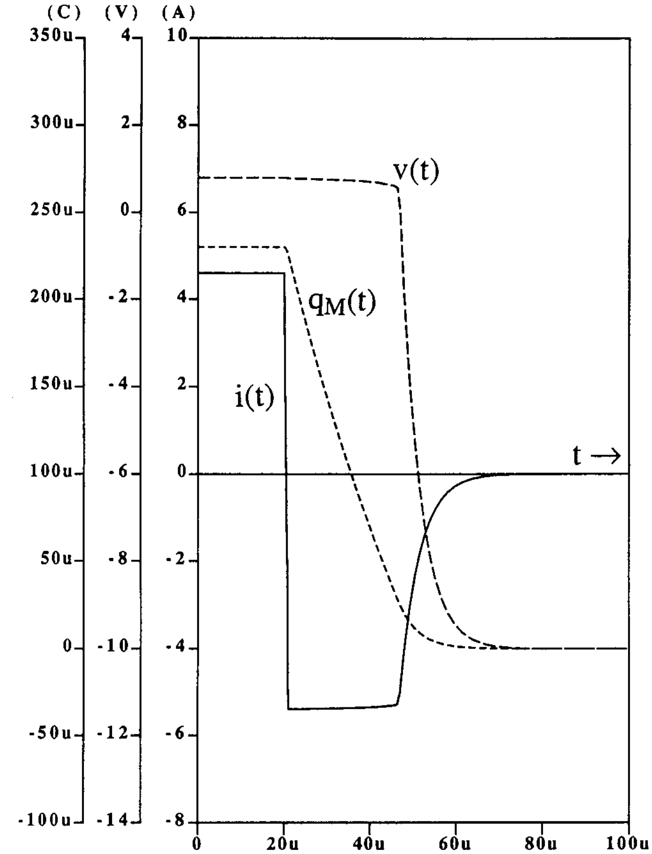

Fig. 4. Resistive load turn-OFF switching waveforms showing $i(t)$, $v(t)$, and $q_M(t)$.

IV. Simulation Examples

Equations (6)–(8) were written in the MAST modeling language available with the Saber [4] simulator. The complete MAST template is given in the Appendix. Simulations of the model were performed for simple inductive and resistive load circuits. Inductive load $i(t)$ switching waveforms are shown in Fig. 3 and a resistive load waveform in Fig. 4. The resistive load waveform also includes plots for $v(t)$ and $q_M(t)$ which show the reversal of diode voltage at the end of the storage time and the exponential decrease of $q_M(t)$ and $i(t)$ during reverse recovery. Both simulations were performed with the junction capacitance set to zero to demonstrate the robustness of the model. Simulations performed with the original diode model will usually fail to run if no parallel capacitance exists to ease its abrupt turn-OFF transition.

V. Discussion

Only a few high-voltage diode models have been published. Danielsson [5] and Xu and Schröder [6] have published complex diode models which are more likely to be used by device designers than by application engineers. Liang and Gosbell [7] have recently published a SPICE subcircuit model for both reverse and forward recovery.

A mathematically based model like the one presented here has the advantage of simplicity and simulation speed over a subcircuit model but requires a simulator which provides for the insertion of mathematical equations. A major feature of the model is that the same equations are valid over all regions of operation. No conditional statements are needed to define regions over which specific equations are valid. This model should work equally well on analog simulators other than Saber since state variables and derivatives of state variables are continuous.

VI. Conclusion

This new lumped charge diode model represents a significant improvement to the basic diode model used in SPICE simulators since 1972. Reverse recovery phenomena need no longer be ignored in circuit simulations employing p-n and p-i-n diodes. The lumped charge approach can also be extended to the modeling of other high-voltage conductivity modulated devices such as SCR's, GTO's, and IGBT's.

Appendix
MAST Template for Diode Model

```
element template plma2_paper  p n = model       # Model name

electrical p,n                          # Physical connection

struc {                                 # Default values
number Is=1e-14,                        # Diffusion leakage current
       no=2,                            # Emission coefficient
       tau=1e-5,                        # Carrier lifetime
       TM=5e-6,                         # Transit time
       cjo=1n                           # Junction capacitance
       } model = ()
{

number k=1.381e-23,                     # Boltzman's constant
       q=1.602e-19,                     # Electron charge
       temp=27                          # Device temperature

val v vd                                # Diode voltage
val q qE                                # Junction charge variable
var q qM                                # Charge in the base region

number vt
struc {number point, inc;}\             # Sample points of vd
svd[*]=[(-100k,100),(-100,.1),(0,10m),(.3,.1m),(1.5,.1),(10,0)],
nvd[*]=[(0,.1),(2,0)]                   # Sample points of Newton steps

parameters {
            vt=k*(temp+273)/q
            }

values {
            vd=v(p)-v(n)
            qE=model->tau*model->Is*(limexp(vd/(vt*model->no))-1)
                                        # Equation (8) in the paper
      }

control_section {
            pl_set(qE,vd)               # Nonlinear Relationship
            sample_points(vd,svd)
            newton_step(vd,nvd)
      }

equations {
           i(p->n)+=qE/model->TM-qM/model->TM
                                        # Equation (6) in the paper
      qM: 0=d_by_dt(qM)+qM/model->tau-qE/model->TM+qM/model->TM
                                        # Equation (7) in the paper
      }
}
```

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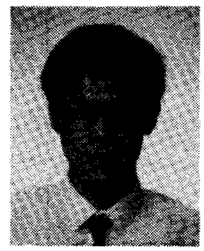

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