Transportation Cyber-Physical Systems: Reliability Modeling and Analysis Framework

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***Abstract*- Recently, computation, communication and control are deeply and pervasively embedded into physical and engineered components of transportation Cyber-Physical Systems (CPS). In such systems, the growing complexity of system structure calls for increasing system reliability. Reliability modeling and analysis need not only capture the complex dynamic of transportation systems, but also must be easy to apply, understand and compute. The objective of this paper is to develop a suitable analysis framework to study the structural reliability and connectivity reliability of transportation CPS. We adopt frequency-domain analysis to model the transportation systems reliability. We also discussed how to use our frequency-domain analysis framework to obtain the approximations of complex transportation systems' reliabilities, which is a future research direction.**

I. Introduction and Related Work

Transportation systems, including today's automotive, aviation and rail systems, have played a crucial role for maintaining normal functions of our society. Automotive, aviation and rail systems are interconnected with each other, and form transportation network with structural components, such as serial, parallel, and circular structures. The transportation networks are vulnerable to accidents, such as traffic jam, earthquakes, hurricanes, flood and attacks. With the assistance of cyber world, the transportation systems have better ability to detect malfunctions than before. The integration of traditional transportation system together with the cyber systems gives the birth of Transportation Cyber-Physical Systems (TCPS). As the TCPS grows more and more complex, we need a reliability analysis framework for these systems, which will help to design and maintain the reliability of modern transportation systems.

Generally, reliability of transportation networks is defined as the ability to continuing to perform a specified transportation operation despite the effects of network malfunctions in a specified time and environment. In recent years, transportation network being built are increasingly complex and large; it becomes increasingly difficult to model and analyze using existing methods. Current reliability modeling and analysis have been based on network reliability theory in the time domain. Network reliability theory has been applied extensively in many real-world systems such as communication networks, microelectronics system, power transmission and distribution systems etc [1][2][3][4] in addition to transportation systems. But in many cases, time-domain analysis is not efficient to compute reliability functions (e.g. in terms of failure distributions) for complex systems. To solve this problem, we propose frequency-domain reliability analysis framework in this position paper to model the reliability of transportation cyber physical systems. The advantages of the proposed theoretical framework over the traditional time-domain approaches include the capability to capture higher order moments of system characteristics, the scalability to analyze reliability of complex systems, and the efficiency in calculation.

The remainder of this paper is organized as follows. Section 2 briefly presents our frequency-domain reliability model and analysis framework, and proposes the hierarchical reliability analysis of large-scale transportation systems. Section 3 summarizes the paper with conclusions.

II. Reliability Modeling and Analysis Framework

Transportation cyber-physical systems are critical infrastructure networks. Nowadays, TCPS must be dependable, secure, safe, and efficient and operate in the real-time physical world, so the reliability analysis of transportation CPS has become an important issue. Such systems are concerned with structural reliability and connectivity reliability.

Structural reliability is defined as the probability of the useful life of a given structure exceeding a certain time-period. It is a yardstick of the capability of a structure to operate without failure when put into service. It shows a good measure of the level of safety for structures that are made of various components.

Connectivity reliability is concerned with the probability that there exists at least one path without disruption or heavy delay between a given origin-destination (O-D) pair within a certain time-period.

Previous work on reliability modeling and analysis has been focused on the solutions in the time domain. There are several drawbacks to study the system reliability in the time domain. First, computing complexity in time domain analysis is usually high in many cases. Any change in a single network component may require designers to rebuild and re-solve the reliability model. Second, the reliability functions are time-variant according to various traffic conditions. The complexity in computing makes the reliability analysis infeasible under dynamic network traffic using time domain analysis. Therefore, our goal is to develop a new reliability analysis for complex transportation CPS with much less computational complexity. We also expect that our approach can help obtain the reliability curve which captures the detailed higher-order moments of the reliability. To this end, we propose frequency domain reliability analysis for TCPS.

A transportation network can be modeled as a graph $G=(V,E)$ with a set of nodes (V) and links (E). To perform frequency-domain reliability analysis, we first convert the reliability functions from the time domain to the frequency domain using Laplace Transform (i.e. $f(s)=\Im[f(t)]=E[e^{-sT}]=\int_0^{\infty} e^{-st} f(t)dt$, $\Im$ is the operator of Laplace Transform). After the transformation, $V=\{v_i(s)\}$ represents the set of reliability functions of network nodes, and $E=\{e_{ij}(s)\}$ denotes the reliability functions of links in the frequency domain.

Next, we give reliability functions for the basic structural components (e.g., serial, parallel, and circular structures) in frequency-domain analysis.

For a serial structure as shown in Figure 1, where rectangular blocks represent states of vertices and rhombic blocks represent states of the links. The *structural reliability* function of a serial structure is represented as $v_{1n}(s)$ and we have:

$$v_{1n}(s)=\overset{n}{\underset{i=1}{\otimes}} v_i(s)=v_1(s)\otimes v_2(s)\cdots\otimes v_n(s)\,, \qquad (1)$$

where $x_i(s)\otimes x_j(s)=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} x_i(t)x_j(s-t)dt$, and the link reliability is to set to 1, for simplification.

The *connectivity reliability* function $e_{1n}(s)$ is represented as:

$$e_{1n}(s)=\prod e_{ij}(s)=e_{12}(s)\cdot e_{23}(s)\cdots e_{n-1\;n}(s)\,, \qquad (2)$$

where the node reliability is to simplify into 1.

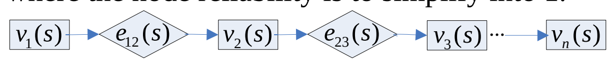

Figure 1. Serial structure

For a parallel structure, similarly, the *structural reliability* function of parallel structure $v_{1n}(s)$ is shown in Figure 2(a):

$$v_{1n}(s)=\overset{n}{\underset{k=1}{\oplus}} v_k(s)=v_1(s)\oplus v_2(s)\cdots\oplus v_n(s)=\sum_{i=1}^{n} v_k(s)-\sum_{1\le i<j\le n} v_i(s)\otimes v_j(s)$$
$$+\sum_{1\le i<j<k\le n} v_i(s)\otimes v_j(s)\otimes v_k(s)+\cdots+(-1)^{n-1}\overset{n}{\underset{k=1}{\otimes}} v_k(s)\,, \qquad (3)$$

where the link reliability is to simplify into 1.

Accordingly, *connectivity reliability* function $e_{ij}(s)$ is shown in Figure 3(b):

$$e_{ij}(s)=\sum_{k=1}^{n} p_k\cdot e_{ijk}(s)=p_1\cdot e_{ij1}(s)+p_2\cdot e_{ij2}(s)\cdots+p_n\cdot e_{ijn}(s)\,, \qquad (4)$$

where $p_1+p_2+\cdots+p_n=1$ and the node reliability is to simplify into 1.

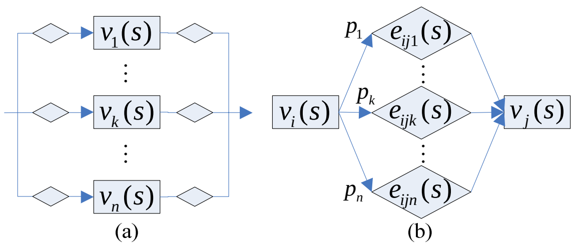

Figure 2. Parallel structure

For a circular structure, *structural reliability* function of circular structure $v_{ij}(s)$ is shown in Figure 3(a):

$$v_{ij}(s)=v_i(s)\oplus\max\{v_i(s)\otimes v_j(s)\otimes v_i(s),[v_i(s)]^3\otimes[v_j(s)]^2,\cdots,[v_i(s)]^{n+1}\otimes[v_j(s)]^n\}=v_i(s)\oplus(v_i(s)\otimes v_j(s)\otimes v_i(s)), \quad (5)$$

where the link reliability is to simplify into 1.

The *connectivity reliability* function $e_{ij}^*(s)$ is shown in Figure 3(b):

$$e_{ij}^*(s)=p_1\cdot e_{ij}(s)+p_2\cdot e_{ii}(s)\cdot p_1\cdot e_{ij}(s)+[p_2\cdot e_{ii}(s)]^2\cdot p_1\cdot e_{ij}(s)+\cdots =p_1\cdot e_{ij}(s)\cdot[1+\sum_{n=1}^{\infty}(p_2\cdot e_{ii}(s))^n]=\frac{p_1\cdot e_{ij}(s)}{1-p_2\cdot e_{ii}(s)}, \quad (6)$$

where $p_1+p_2=1$ and the node reliability is to simplify into 1.

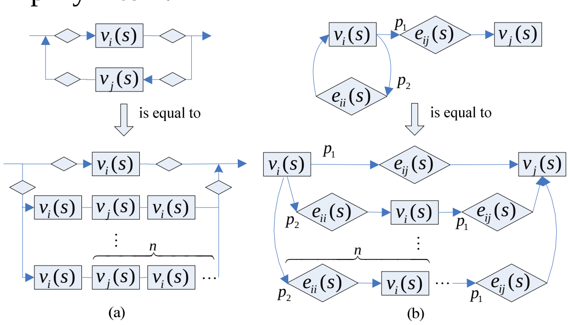

Figure 3. Circular structure

For more complex systems such as large-scale transportation CPS, though our frequency-domain analysis framework can help obtain the reliability functions of the underlying systems, more in-depth study is still needed to further reduce the computational complexity and makes the solution more practical for real-life complex systems. As a future work, we plan to investigate hierarchy structure to simplify the representation of these systems, and analyze the reliability of complex systems using the tactic of divide-and-conquer. That is, the entire original transportation network is divided into k sub-networks based on the size and the topology of the original network. The block model of each sub-network is built, and the minimal path set is searched by the corresponding frequency-domain adjacency matrix and the correlative nodes between the sub-networks. Then leveraging the above-presented frequency-domain reliability analyses techniques, we obtain approximations of the reliability density function in the frequency-domain for complex transportation CPS.

III. Conclusions

This position paper presents a novel frequency-domain modeling approach and analysis framework for the reliability analysis of transportation CPS. The contribution of this paper can be summarized as follows. First, we discuss the structural reliability and connectivity reliability, which are very important to the system design and analysis for future transportation CPS. Second, the frequency-domain reliability modeling and analysis framework in transportation CPS is studied. In this modeling and analysis framework, a complex reliability density function in the time domain can be translated into a simple frequency-domain function which can be easily expressed and calculated. We hope this initial work can help attract interests and bring some new ideas and novel tools for investigating the reliability of future transportation CPS.

References

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