Parametric Equivalent Circuit Extraction for VLSI Structures

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Abstract

Equivalent circuit modelling is a powerful technique widely used for time-domain simulation of complex electromagnetic VLSI structures. Surprisingly, parametric aspect of equivalent circuit modelling has not received much attention until recently (although the need for it has been previously advocated in several publications). Having a circuit with element values given as functions of the structure geometrical parameters eliminates the need to recalculate S-parameters and extract an equivalent circuit again whenever the geometry is modified.

The purpose of this paper is to discuss a concept of parametric equivalent circuit modelling for VLSI structures, to systematically describe a methodology of extracting such circuit from the given set of S-parameters, and to provide an overview of methods and problems arising at each step with referring to existing publications. For demonstration of the parametric equivalent circuit extraction, we use a classical example of a microstrip interconnect represented as an RLCG circuit[1].

1. Introduction

Various structures that exhibit electromagnetic (EM) behavior (inductors, connectors, interconnects, etc.) have always been an important part of microwave circuits. Now they play an important role in many modern VLSI systems-on-chips and seriously affect their performance, especially at multigigahertz frequencies. Typically, such structures are measured [1] or simulated in frequency domain using various EM simulators [2].

There exist a great variety of numerical electromagnetic field solvers that allow modelling of on- and off-chip structures. However, electromagnetic simulations are usually computationally intensive and are mostly used for verification rather than for design and synthesis, when one needs to vary parameters many times in the process of optimization.

There are two ways to use S-parameters (or other frequency domain parameters) obtained from EM simulations in SPICE-like time-domain circuit simulators. Most common approach is to extract an equivalent circuit whose S-parameters match those obtained from EM modelling [3, 4]. Many different techniques on implementing this approach exist in the literature [5]. An alternative approach is to perform an inverse Fourier transform on the S-parameters and then do a recursive convolution with the circuit time-domain response [6].

When structure's geometry changes (e.g. in the process of parasitic-aware layout optimization) new S-parameters must be obtained from electromagnetic simulation and new circuit values must be extracted. If the range in which EM structure parameters can vary is known, EM simulations can be carried beforehand to create a parametric table of values for equivalent circuit. This parametric tabulating capability is already present in several commercial EM software products (e.g., Ansoft's *Optimetrics* engine). One can expect that the next logical step is to use extracted parameter values to obtain circuit elements in a functional form for later use by a circuit designer.

Surprisingly, this subject has not received much attention in the CAD literature until recently. A good representative paper on the subject has been published by Sercu and Demuynck [7], who emphasized an integration of circuit simulation, EM simulation, and optimization tool. There exist several other more narrow-focused publications that address, e.g. parametric modelling of microstrip discontinuities [8].

As it is known, analytical models that relate, e.g. capacitance of a microstrip interconnect to its width and dielectric thickness, exist only for simple geometries [9, 10]. Especially when the parasitics effects become significant, no systematic methods exist to extract models that incorporate parasitics for general

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structures. Having an equivalent circuit whose element values are functions of the structure parameters does not only allow one to perform a faster circuit simulation and optimization but also provides a designer with a physical insight into the structure's behavior.

In this paper, we give a systematic description of the parametric equivalent circuit extraction process and discuss all related advantages and difficulties. We illustrate this process with a classical example of a microstrip interconnect with a variable strip width.

2. Parametric Equivalent Circuit Extraction Methodology

The process of parametric equivalent circuit extraction is illustrated in Figure . Geometrical parameters of the structure of interest are specified in the range of interest determined by the layout design rules (parameter step must be small enough not to miss important frequency response features, such as resonances). The structure is then modelled with an appropriate EM simulation tool in the frequency band of interest for each parameter value. An equivalent circuit is then extracted from these data. Parametric aspect of this process that we propose to explore is the last step, when circuit element values are approximated as functions of the structure parameters. The last three stages of the process shown in Figure are described below with more details.

2.2. EM modelling and S-parameters

As mentioned before, a variety of numerical electromagnetic field solving tools have been developed in the past, all of which have different limitations, capabilities, input and output formats, and computational costs. Choosing the best tool for a particular task and successfully employing and integrating it into a VLSI CAD design flow are challenging tasks. Most EM tools are based on three major methods and their flavors – method of moments (e.g., *Sonnet* by Sonnet Technologies), finite element method (e.g., *HFSS* by Ansoft Corporation), and finite-difference time domain method (e.g., *XFDTD* by Remcom, Inc.).

A number of equivalent parameters can be used to describe an arbitrary N-port device, such as S, Z, Y, $ABCD$, etc. Frickey [11] provides an excellent overview of various parameters and relationships between them: impedance matrix Z, admittance matrix Y, hybrid matrix h, chain matrix $ABCD$, scattering matrix S, and chain transfer matrix T.

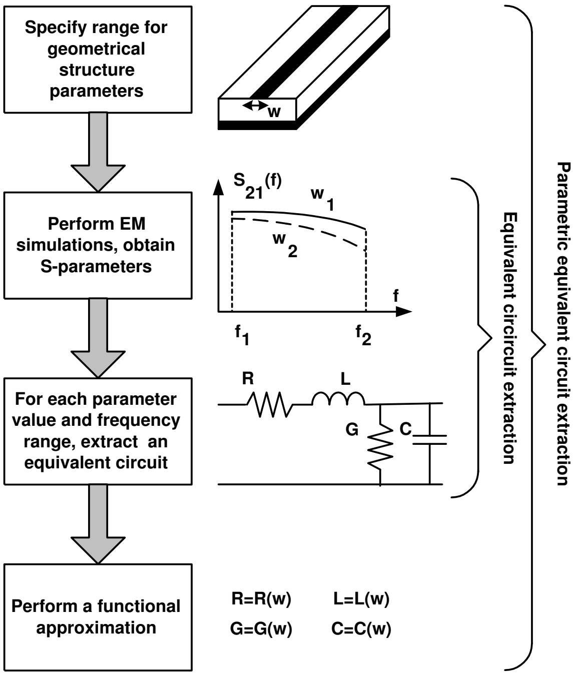

Figure 1: Parametric equivalent circuit extraction methodology.

S-parameters are the most popular way of modelling a device in frequency domain. They can be obtained directly from EM simulations and are typically computed for a device terminated with 50 Ohm loads.

2.2. Circuit structure

A crucial assumption for any equivalent circuit extraction approach is the knowledge of the circuit structure whose element values are to be extracted. This knowledge usually comes from a physical insight [8] or from the shape of the S-parameter frequency response. A large number of equivalent circuits are known and used for common structures like spiral inductors or bent interconnects. For example, a two port structure device can generally be modelled with a π-circuit [12].

Another approach to finding an equivalent circuit structure is genetic algorithm-based search [13], but its speed and convergence to correct circuit structures are currently the limiting factors of its applicability.

An assumed circuit structure is usually valid only for a certain frequency range. For example, the number of sections in an equivalent ladder circuit for an interconnect depends on the interconnect length with respect to the minimum wavelength of interest. The validity of one-stage lumped-circuit approximation breaks when the interconnect length becomes comparable to the quarter of a wavelength [14]. As the frequency increases, more stages need to be added.

Many existing EM tools have a built-in equivalent circuit extraction capability that is applied separately to each frequency point. As a result, circuit element values are frequency-dependent and change from one frequency point to another.

2.3. Objective function minimization

A general objective of equivalent circuit extraction is to find a set of circuit element values that results in a good match between the S-parameters of the circuit and the S-parameters of the given structure [13, 15].

For a two-port structure, the simplest objective function whose minimization delivers this match is [13]:

$$F = \sum_{i,j=1}^{2} \sum_{n=1}^{N} \left| S_{ij}^{M}(\omega_n) - S_{ij}(\omega_n) \right|^2 , \qquad (1)$$

where S_{ij}^{M} are the S-parameters obtained by EM modelling, S_{ij} are the S-parameters of an equivalent circuit, and $\omega_1 ... \omega_N$ are the discrete frequency points. Since the structure of the circuit is assumed to be known, its S-parameters, and, hence, the objective function can also be found in analytical form either by hand or using symbolical methods.

To find the set of circuit parameters that minimizes the objective function, various optimization methods can be used [16]. Most of the methods are gradient-based. All methods require initial values for circuit parameters to be specified.

One of the most popular gradient-based methods is steepest descent method [5]. This method is based on moving in the direction opposite to the gradient of the objective function. The process is repeated at the new point and the algorithm continues until a minimum is found.

One commonly used parameter updating algorithm for steepest descent method is the linear algorithm:

$$\vec{\alpha}_{n+1} = \vec{\alpha}_n - \eta \, \vec{g} \, , \qquad (2)$$

where $\vec{\alpha}$ is the multi-dimensional vector of circuit parameters, $\vec{g}$ is the gradient vector and η is commonly referred to as the learning rate and determines the convergence of the process.

The gradient itself can be computed in two ways: symbolically and numerically. Numerical approach to gradient computation is the most popular one as it does not require symbolic derivatives computation for the given circuit. If a symbolical expression is available for the objective function, the gradient vector $\vec{g}$ can also be found in an analytical form.

One should keep on mind that a general problem of optimization in a multi-parameter space is the presence of multiple minima in the objective function. There may be several possible circuit parameter combinations that result in a very small value of objective function and lead to solution ambiguity. This problem is well known and has been discussed in literature [15].

For parametric circuit extraction the uniqueness of solution is especially important: the same conformal objective function minimum must be used for each S-parameter set in order to ensure a proper functional behavior of the circuit parameters vs. structure parameters.

2.4. Parametrics

As mentioned before, tabular parametric ability is present in many commercial simulators. For parametric analysis, a range of structure parameters is specified beforehand. For each parameter set, S-parameters are obtained from EM simulations and equivalent circuit extraction process (objective function minimization) starts. Once the minimum of objective function is found, the obtained equivalent circuit element values are tabulated and the process is repeated for all parameter sets in the given range.

The next logical step that can be performed is a functional approximation – to have circuit element values approximated as analytical functions of the structure parameters. This would give a designer an insight into a structure's physical behavior and eliminate the need to recalculate S-parameters and extract an equivalent circuit again whenever the geometry is modified.

To perform a functional approximation, the type of basis analytical functions, such as polynomials, has to be specified. While the behavior of some structures may be complicated and involve logarithms and exponents, polynomial basis is useful and often sufficient to fairly approximate the first few terms of the Taylor expansion of unknown functions. Other basis functions, such as exponentials, can also be used.

3. Example

For demonstration of parametric equivalent circuit concept, we consider a simple microstrip interconnect line example shown in Figure . The interconnect is 160 mil long (1 mil=0.0254 mm) and consists of

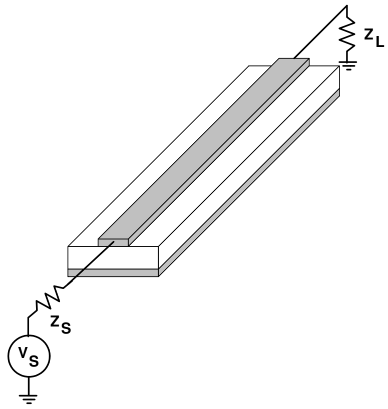

Figure 2: Physical structure of microstrip interconnect.

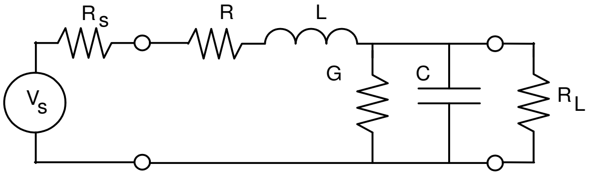

Figure 3: RLCG equivalent circuit representation of microstrip interconnect.

aluminum trace of height $t = 1$ mil and width w on top of alumina substrate of thickness $h = 25$ mil and relative dielectric permittivity $\varepsilon_r = 9.9$. The line is connected to a voltage source V_s, and the source and load impedances are $R_s = R_L = 50$ Ohm.

Such interconnect can typically be modelled as an RLCG ladder network shown in Figure . If the length of an interconnect is small compared to a wavelength, it can be represented as one lumped RLCG section. The transfer function (S_{21}) of the RLCG circuit shown in Figure is given by:

$$S_{21}(\omega) = \frac{1}{1 + (R + j\omega L)\left(R_L^{-1} + G + j\omega C\right)} . \quad (3)$$

Let us use *Sonnet* as a designer's EM tool of choice. *Sonnet* uses a 1/20 λ criteria: if the structure is larger than this size, it has to be modelled by parts which are then cascaded To satisfy this criteria for our interconnect length of 160 mil, we will limit the frequency range of consideration to 1 GHz.

Let us choose the width w as the geometrical parameter to be varied and use S_{21} for equivalent circuit objective function minimization. For demonstration purposes, we will select the following range of interconnect widths: 20 - 50 mil (with a step of 10 mil). Using *Sonnet*, we can perform EM simulation and obtain S_{21} responses, which are shown in Figure .

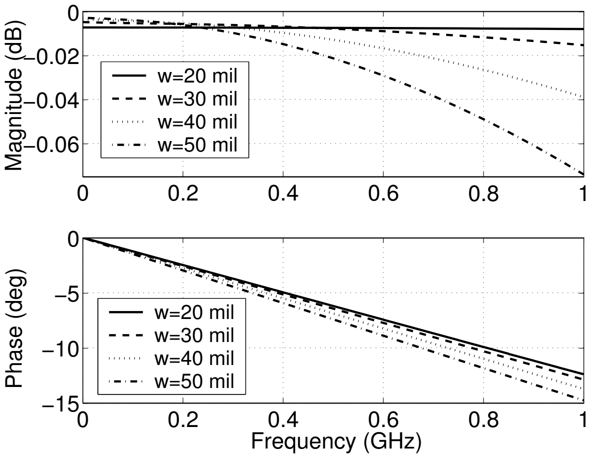

Figure 4: Magnitude and phase of S_{21} obtained with *Sonnet* for different interconnect widths.

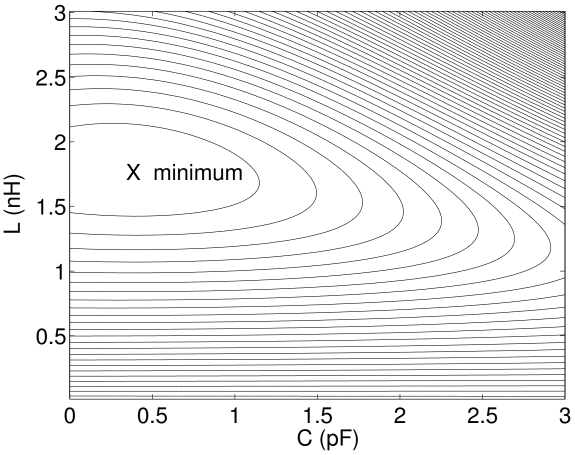

Figure 5: Contour plot of the objective function vs. L and C.

For illustration of the methodology, we created a simple optimization tool based on the steepest descent method, where the gradient of the objective function is computed numerically. Assume that two circuit parameters (L and C) are unknown and need to be extracted. The objective function is given by (1). Figure shows the contour plot of the objective function vs. L and C for $w = 30$ mil. One can see that there is a minimum. The exact at values of L and C at minimum, found from running an optimization tool, are L =1.76 nH and C =0.34 pF.

As mentioned before, initial values are needed for optimization process. Usually, a designer has an approximate idea of the order of magnitude of initial values. We use as initial estimate the following intuitive values: $L = 1$ nH, $C = 1$ pF. We also assume that conductance G and resistance R are small

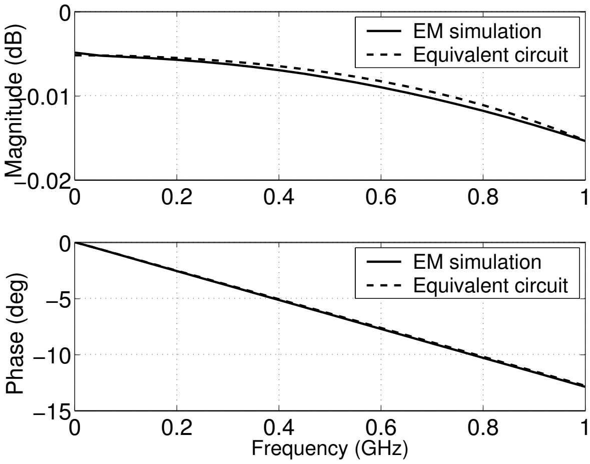

Figure 6: Magnitude and phase of S_{21} obtained with *Sonnet* ($w = 30$ mil) and approximated with an equivalent circuit .

but approximately known (for 30 mil wide trace they are: $R = 0.03$ and $G = 1e-7$). In this specific case, our optimization tool, written in *Matlab*, takes about 1.5 s to run on a 2.5 GHz PC and stops after 180 iterations, when the objective function becomes less than $4 \cdot 10^{-5}$. For each of the four frequency responses, the steepest descent algorithm converged to the same conformal minimum of the objective function.

A typical comparison between the original frequency response, obtained with *Sonnet*, and the frequency response of the equivalent circuit is given in Figure (the width of the interconnect is 30 mil). One can see that the agreement between frequency responses is very good.

Extracted L and C were approximated as functions of width w in the vicinity of $w_o = 30$ mil using the *polyfit* function in MATLAB for first-order polynomials, which gave the following dependence of L and C (per unit length) on the interconnect width w:

$$L \approx L_o + a\,(w_o - w),\; C \approx C_o + b\,(w - w_o), \quad (4)$$

where L_o and C_o are inductance and capacitance for the width w_o and a and b are constant coefficients.

In our specific case, the geometry was simple and well known. Analytical expressions given by (4) and the related coefficients could be obtained directly by writing the first two terms in the Taylor expansion of established analytical formulas for microstrip impedance and capacitance (see e.g. [17]). However, as mentioned before, for more complex geometries analytical formulas do not exist and the advantage of extracting a parametric equivalent circuit is obvious.

4. Discussion

The main advantage of the presented methodology is that functional approximation gives a designer access to equivalent circuit parameters as functions of geometrical and material parameters of the structure, even for those structures for which analytical model is not available. Another advantage is that it provides values for a continuum of geometrical parameters of the structure, not only for a discrete set available from the table or library.

The main drawback of the proposed methodology is the necessity of carrying multiple EM simulations beforehand, which means that a designer must specify or adaptively change the range of structure parameters that he is interested in and the number of points to be used, which determines the runtime. The accuracy of parametric equivalent circuit extraction strongly depends on the accuracy of EM simulator.

One issue to be aware of is that for wideband structures, the equivalent circuit is frequency-dependent. Having one circuit structure that is valid throughout the whole multi-gigahertz band would be ideal but may not be possible due to different frequency-dependent physical effects that take place in a structure (e.g., skin effect, proximity effect, etc.). Many of existing EM simulators are conservative in that regard. As mentioned before, *Sonnet* uses a 1/20 λ criteria to determine the maximum size of structure that can be approximated with one lumped circuit section. This can be viewed as an inter-dependence of frequency and geometrical parameters of VLSI structure.

The parametric methodology can be applied to all variables associated with a VLSI structure (geometrical parameters, electrical parameters, and frequency) to create, e.g., a library of equivalent circuits whose electrical parameters and frequency range of validity are given as functions of geometrical parameters and vice versa.

If a VLSI structure contains multiple ports, the minimization of the objective function has to be performed over a multi-port S-parameter matrix. The circuit structure in this case is usually more involved and equivalent circuit extraction process is more challenging compared to two-port devices.

The described functional parametric methodology can easily be integrated into most existing commercial EM simulators that have a built-in equivalent circuit export option. It can also be linked to a parametric and optimization engine already present in a simulator (and typically used for optimizing power, efficiency, reflection coefficient, or other system parameters).

5. Conclusion

In this paper, we described a methodology of extracting a functional parametric equivalent circuit from the set of S-parameters obtained via EM simulation for a VLSI structure with variable geometric parameters. We presented an overview of this methodology, discussed associated advantages and problems, and referred to existing publications in this area.

We demonstrated the methodology with the classical example of a straight microstrip interconnect, for which analytical capacitance and inductance models are available. The interconnect was modelled in *Sonnet* and represented as an RLCG network for equivalent circuit extraction using steepest descent optimization. The equivalent circuit element values were obtained as functions of the microstrip width.

The presented parametric approach gives a designer an insight into a physical behavior of the structure and can easily be integrated into existing EM simulators which have an equivalent circuit export option. It eliminates the need to recalculate the S-parameters whenever the layout is modified and can be very valuable for time-domain simulation and optimization of VLSI systems that include both circuit and EM structures coupled together.

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