AWE-Inspired

V. Raghavan and R.A. Rohrer
Carnegie Mellon University
Pittsburgh, PA 15213

L.T. Pillage
University of Texas
Austin, TX 78712

J.Y. Lee, J.E.Bracken, M.M. Alaybeyi
Performance Signal Integrity
Pittsburgh, PA 15213

Abstract

This tutorial paper attempts to bring together the literature on the Asymptotic Waveform Evaluation (AWE) technique. The efficient generation of moments and using moments to determine stable and accurate AWE approximations are discussed. Techniques to determine AWE macromodels and extend AWE to distributed elements are also discussed.

1 The Evolution of AWE

Although it is of a relatively recent vintage [20], Asymptotic Waveform Evaluation (AWE) has been the subject of a large number of papers [23][21][1][9][41][52][26][25][27][46][17][48][30][13][22][28][32][2][33][34][24][37][31][36][16][35][15]. With this invited tutorial paper we hope to set the record straight on what AWE is, what it is not, and give a feel for its evolution.

AWE drew its inspiration first from the seminal work of Penfield, Rubinstein, and Horowitz[38], which applied the efficient Elmore delay[8] estimate to the RC-trees that can arise in integrated circuit modeling. Since these estimates were not always accurate, an essential attempt of the early work was to bound the transient response waveform[50]. A number of subsequent works concentrated on tightening the bounds[53] [42]and/or extending the domain of applicability of these techniques beyond RC-tree structured circuits[6][49]. A second point of inspiration was the work of McCormick and Allen[43], which showed that interconnect circuit moments (the coefficients of expansion of a circuit driving point or transfer function in a Maclaurin series about s=0 in the frequency domain) could lead to lower order circuit models that provided reasonably accurate transient responses. From there it was but a short step to the formalization and generalization of what has come to be known as AWE.

Why did we name it Asymptotic Waveform Evaluation? AWE finds successively higher, but relatively low order approximations to much higher order systems. These low order approximations consist of a few dominant poles (and zeros). As the order of the approximation increases, the related approximate transient responses asymptotically approach the actual, inspiring the name. At first we called it Asymptotic Waveform Estimation but as our confidence grew we redubbed "Estimation" "Evaluation".

What is AWE? As originally presented [35], AWE comprised two essential steps: recursive solution of an equivalent dc circuit to find driving point or transfer function moments (coefficients of the Maclaurin Series expansions of those functions about the s-plane origin, s=0); and Padé Approximation [10] to match a low order system to the first few terms in that expansion. To be honest, we did not know what we originally did was Padé approximation. The result was derived in such a convoluted manner [35] that we didn't recognize Padé approximation to be part of it. And had we known that, we might not have pursued it, given the reputation of Padé approximations to yield unstable poles[4]. By the time we realized what we were doing was Padé approximation the results we had obtained were too good to cause us to stop.

This paper attempts to bring together the literature on AWE and put it in perspective. We will discuss the basic AWE algorithm in the next section. Section 3 will discuss moment generation techniques. Sections 4,5 and 6 will concentrate on techniques to obtain stable and accurate AWE approximations. Section 7 discusses techniques to include AWE Macromodels in other simulators. Section 8 will discuss the extensions of AWE to handle distributed elements.

2 The Basic Algorithm

The AWE algorithm is a technique to approximate time domain (transient) responses of large linear circuits in terms of a few dominant poles and their residues. This low order approximation forms a reduced order model for a given high order circuit.

The AWE algorithm consist of two main parts —moment computation and moment matching. Let us first define a time moment of a function.

The time domain moments of a signal $f(t)$ are related to the Taylor series coefficients about $s = 0$ (Maclaurin Series) of the signal's Laplace transform, $F(s)$:

$$F(s) = \int_0^\infty f(t)\, e^{-st} dt = \int_0^\infty f(t) \left[1 - st + \frac{1}{2}s^2t^2 - \ldots\right] dt \tag{1}$$

or

$$F(s) = \int_0^\infty f(t)\, dt - s\int_0^\infty t f(t)\, dt + \frac{1}{2}s^2 \int_0^\infty t^2 f(t)\, dt + \ldots = m_0 + m_1 s + m_2 s^2 + \ldots \tag{2}$$

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where

$$m_i = \frac{(-1)^i}{i!} \cdot \int_0^\infty t^i f(t)\, dt \qquad (3)$$

is the i^{th} coefficient of the Taylor series expansion of $F(s)$ about $s = 0$, which is closely related to the i^{th} time moment of $f(t)$ given by

$$i^{th} - moment = \int_0^\infty t^i f(t)\, dt \qquad (4)$$

A lumped, linear, time-invariant circuit can be described by a first-order matrix differential equation, such as

$$\boldsymbol{T}\boldsymbol{x}(t) + \boldsymbol{W}\frac{d\boldsymbol{x}}{dt} = \boldsymbol{b}(t) \qquad (5)$$

The above expression holds true independent of the way that the circuit equations are formulated (e.g., by modified nodal analysis[12], sparse tableau[11], etc.) The vector $\boldsymbol{x}(t)$ contains the fundamental circuit variables of interest; these could be currents, voltages, charges, etc., depending on the formulation method. The matrix $\boldsymbol{T}$ represents the contributions of memoryless elements (such as resistances), while the matrix $\boldsymbol{W}$ contains contributions from memory elements (capacitances and inductances). The right-hand side vector $\boldsymbol{b}(t)$ describes the influence of independent sources. Taking the Laplace transform of (5), and assuming zero initial conditions, yields

$$(\boldsymbol{T} + s\boldsymbol{W})\boldsymbol{X}(s) = \boldsymbol{B}(s) \qquad (6)$$

Equation (6) can be used to determine the moments of the circuit's impulse response $\boldsymbol{x}(t)$. If we assume that the Laplace transform of $\boldsymbol{x}(t)$ has a Taylor series expansion about $s = 0$, and that the input sources are impulses, then we have

$$\boldsymbol{X}(s) = \boldsymbol{X}_0 + s\boldsymbol{X}_1 + s^2\boldsymbol{X}_2 + \ldots \qquad (7)$$

and

$$\boldsymbol{B}(s) = \boldsymbol{B}_0 \quad \text{(independent of s).} \qquad (8)$$

Substituting (7) and (8) into (6) yields

$$(\boldsymbol{T} + s\boldsymbol{W}) \cdot (\boldsymbol{X}_0 + s\boldsymbol{X}_1 + s^2\boldsymbol{X}_2 + \ldots) = \boldsymbol{I} \qquad (9)$$

Equating like powers of s in the above expression, we find that

$$\boldsymbol{T}\boldsymbol{X}_0 = \boldsymbol{B}_0 \quad \text{and} \qquad (10)$$

$$\boldsymbol{T}\boldsymbol{X}_k = -\boldsymbol{W}\boldsymbol{X}_{k-1} \quad \text{for } k > 0. \qquad (11)$$

Equations (10) and (11) can be solved recursively to determine the moments of $\boldsymbol{x}(t)$. First (10) is solved for $\boldsymbol{X}_0$. Notice that this is equivalent to performing a dc steady-state solution of (5), with the input sources set to dc values $\boldsymbol{B}_0$. After $\boldsymbol{X}_0$ has been determined, it can be used in equation (11) to compute the right-hand side vector $-\boldsymbol{W}\boldsymbol{X}_0$; this allows us to solve for $\boldsymbol{X}_1$. The procedure continues recursively, until $2q$ moments have been obtained.

The moment computations described above can be performed very efficiently. To solve (10), the matrix $\boldsymbol{T}$ must be factored into lower and upper triangular (LU) form. Once this has been done, the LU factors of $\boldsymbol{T}$ may be re-used to solve (11) with a different right-hand side. Thus, only 1 LU-factorization is required. Also note that the circuit may be re-solved for a different impulse response requiring another factorization; only the value of $\boldsymbol{B}_0$ must be changed.

Suppose now that the $2q$ low-order moments for a particular circuit response $H(s)$ have been determined

$$H(s) = m_0 + sm_1 + s^2m_2 + \ldots \qquad (12)$$

The AWE algorithm matches these moments to a reduced-order model $\hat{H}(s)$ by using Padé approximation. The reduced-order model has q poles, and is described by a strictly proper rational function:

$$\hat{H}(s) = \frac{b_{q-1}s^{q-1} + \ldots + b_1 s + b_0}{a_q s^q + \ldots + a_1 s + 1} \qquad (13)$$

Setting (12) equal to (13) and cross-multiplying by the denominator yields a set of linear equations that can be solved for the coefficients $\{a_m\}$ and $\{b_m\}$ of the reduced-order model[52]. The poles $\{p_r\}$ of the approximate model can then be determined by finding the roots of the denominator polynomial. This allows the reduced-order model to be expressed as a partial fraction expansion:

$$\hat{H}(s) = \sum_{r=1}^{q} \frac{k_r}{s - p_r} \qquad (14)$$

where the $\{k_i\}$ are the residues corresponding to the poles $\{p_i\}$.

With the impulse response of the circuit described by the analytic approximation (14), it is possible to determine the response to any kind of input. For example, the step response can be found by multiplying (14) by $1/s$, and evaluating the inverse Laplace transform of the product analytically. Thus, it is not necessary to repeat the moment computations when the input signal changes.

3 Moment Generation

Though the recursive solution technique described in the previous section reduces the cost of finding moments to a single LU factorization and a number of Forward and Backward

Substitutions of the circuit matrix, LU factorization remains the dominant cost of determining an AWE approximation for most circuits of reasonable size. Thus a great deal of effort has gone into improving the efficiency of moment generation.

It is often erroneously claimed that the cost of finding the LU factors of a circuit matrix is $O(n^3)$, where n is the dimension of the circuit matrix. Using sparse matrices, it has been shown that the cost of finding the LU factors is $O(n^{1.4-1.7})$ for matrices that typically arise in circuit simulation. For tree structured circuits, commonly seen in integrated circuit interconnect it can be shown that the cost of LU factorization is actually $O(n)$. For structures which do not significantly differ from tree structures the order is close to $O(n)$.

An application specific implementation of the AWE algorithm, RICE, [30] which exploits the fact that interconnect circuits are largely tree structured with relatively few non-tree elements has been developed. This technique used path tracing, compaction and factorization to speed up the generation of circuit moments. For many interconnect circuits, RICE showed a significant speed up over a general implementation of AWE using the Sparse 1.3[19] Package from Berkeley for sparse matrix manipulation. Suitably tuned sparse matrix techniques can be competitive with path tracing while retaining generality for non-tree structured circuits.

4 Searching the Padé Table

It has been known historically [4] that Padé approximation can produce poles with positive real parts for stable systems. In such cases the time domain response of the approximation will be unbounded. Another problem associated with Padé approximations is that successively higher orders of approximation are not guaranteed to converge uniformly to the actual high order system function, though such convergence can often be observed. This raises two important questions about Padé approximation: How do we guarantee a stable approximation? How accurately does our approximation model the actual system function? We attempt to answer these questions in the remainder of this paper.

The standard form of AWE approximation assumes that a qth order approximation consists of q poles and $q-1$ zeros, as this is a common form of many system functions. However, AWE approximations can have any number of poles and zeros[51]. This gives rise to an entire table of Padé approximants shown in (15). As we move to the right on this table the number of zeros in the approximation is increased while moving down the table increases the number of poles in the approximation. An approximation P_{nm} has n poles and m zeros.

$$\begin{matrix} P_{00} & P_{01} & \cdots & P_{0n} & \cdots \\ P_{10} & P_{11} & \cdots & P_{1n} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} & \cdots \end{matrix} \qquad (15)$$

In general, when dealing with bandlimited signals we can restrict consideration to approximants that have more poles than zeros. Given the plethora of approximations available it is relatively easy to find a stable approximation, however finding an accurate approximation is another issue.

Searching the entire Padé table is possible but can be time consuming due to the quadratically increasing number of possible Padé approximants. Two popular techniques to search the Padé table described in mathematical literature [3] are the diagonal sequence and the horizontal sequence.

The diagonal sequence searches the Padé table along a subdiagonal, typically the subdiagonal which has one more pole than zero. The original form of AWE approximation [35] [20] is of this type. While approximations of this kind generally approach the actual system function as the order of the approximation is increased, no guarantees can be made.

The horizontal sequence fixes the number of poles in the approximation and then horizontally traverses the Padé table progressively increasing the number of zeros in the approximation. It has been shown that a horizontal sequence with n poles uniformly converges to the n lowest frequency poles as the number of zeros is increased and the appproximation itself converges uniformly to the system function within a circle of radius R (shown in Figure 1) equal to the magnitude of the largest pole in the approximation [14]. The

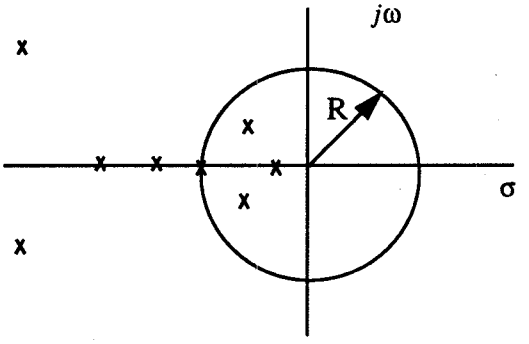

Figure 1: Radius of convergence for a four pole approximation

rate of convergence of each pole is dependent on the ratio of the magnitude of that pole to the magnitude of the smallest pole outside the circle of convergence.

The reason why the horizontal sequence tends to converge to the low order poles can easily be explained. Consider, an approximation $P_{nm}(s)$ with many more zeros than poles $m \gg n$

$$\begin{aligned} P_{nm}(s) &= \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0} \\ &= m_0 + m_1 s^1 + \ldots + m_{m+n}s^{m+n} + \ldots \end{aligned} \qquad (16)$$

When these equations are cross multiplied it follows that the moments used to calculate the poles are the moments m_{m-n-1}

through m_{m+n-1}. As can be simply inferred from the time domain definition of moments in (3), the higher moments tend to emphasize the long time response and thus the low frequency poles of the system.

Assuming that our system is band limited, the high frequency zeros that lie outside the region of convergence can be ignored and an accurate bandlimited low order response to the system function can be obtained. If it is desired to increase the bandwidth of the approximation the order of the approximation must be increased. A practical limit on the bandwidth may be reached due to the presence of a cluster of poles which reduces the convergence of additional poles. The convergence rate of the horizontal sequence can be slow when the ratio of the magnitude of the smallest pole outside the circle to that of the largest pole within the circle is almost unity. This is often the case when a large cluster of poles is encountered at a certain frequency. The details of the horizontal sequence approach are described in [51] [45]

A variation on the horizontal sequence approach is the "moment shifting" approach [29]. In this approach too the low frequency poles are emphasized by using progressively higher order moments to determine the poles. The use of progressively higher moments is achieved by matching step, ramp or even a quadratic input response instead of the impulse response. In this case the lowest order moments are used to match the particular solution of the input. For example, when the input is a ramp we have

$$\frac{A_r}{s^2}+\frac{A_s}{s}+\frac{b_{q-1}s^{q-1}+b_{q-2}s^{q-2}+\ldots+b_0}{s^q+a_{q-1}s^{q-1}+\ldots+a_0} = \frac{m_o}{s^2}+\frac{m_1}{s}+m_2+m_3 s+\ldots+m_{2q+2}s^{2q}+\ldots \quad (17)$$

The conventional q pole $q-1$ model is used and the lowest 2 moments are used to "match" the particular solution to the ramp input while higher moments are used to match the poles of the circuit. Results have shown that this technique can more or less guarantee stable approximations. However, accuracy must be ensured by other means.

5 Shifting and Multipoint AWE algorithms

Another way to improve stability and accuracy of AWE approximations, is to improve the quality and quantity of system function information. In the original AWE paper [20], moments, i.e. the coefficients of the Taylor series expansion about $s = 0$, were the primary source of modeling information. However, it was recognized that using expansions exclusively about $s = 0$ would produce large time domain errors near the initial time point ($t = 0$). To overcome this, the initial condition, or the first coefficient of the expansion about $s = \infty$, was used in addition to the moment information.

This idea was extended improve the accuracy of AWE approximations near the initial time (t = 0), through the use of both moment as well as derivatives[52] (the coefficients of the Taylor series expansion of $s = \infty$ in the frequency domain). The use of derivative information has proved useful in piecewise linear transient simulation [23] as it is important to match the initial conditions accurately. Such multi-point Padé approximations have been discussed in the literature as well [3]. Also in [51], the first attempts were made at expanding the system function at points other than $s = 0$ and $s = \infty$. In [40] arguments are made for using expansions about $s = \infty$ (and a single term about $s = 0$) "instead of AWE". However, we prefer to recognize AWE as rational approximations of functions using expansions about any combination of points in the s-plane.

The concept of expanding the system function in a Taylor series at some point other than $s = 0$ was originally called "frequency shifting" [51]. The expansion about a new frequency often improved both the stability and accuracy of an AWE approximation. If an expansion point $s = h$ along the positive real axis is chosen the procedure described in basic algorithm is modified only slightly. Instead of (6) we then have

$$[\boldsymbol{T}+h\boldsymbol{W}+(s-h)\boldsymbol{W}]\boldsymbol{X}(s) = \boldsymbol{B}(s) \quad (18)$$

The vector $\boldsymbol{X}(s)$ is then expanded as a series in $(s-h)$. A recursion relationship between the "shifted" moments can be developed, which is similar to (10)and (11). The only difference is that now the matrix $\boldsymbol{T}+h\boldsymbol{W}$ must be *LU*-factored, instead of $\boldsymbol{T}$. This may be advantageous in cases where the matrix $\boldsymbol{T}$ is singular, relaxing topological constraints. However, the sparsity of the matrix is often reduced by frequency shifting; $\boldsymbol{W}$ may have nonzero entries where the entries of $\boldsymbol{T}$ were zero. This may diminish the computational efficiency of the LU factorization. Complex frequency shifts can be entertained in a similar manner.

A *merged* AWE approximation can be formed from multiple shift frequencies. The approximation will contain information from each shift frequency. A general technique to include multiple shift frequencies in an AWE approximation is described in [22]. The process begins by forming an AWE approximation with the first shift frequency. A merged approximation is formed by repeatedly deflating each subsequent set of shifted moments by the existing approximation. The deflated moment set is used to form an AWE approximation—this set of poles and residues is added to the previous approximation, as a corrector. This process continues until the energy contained in the deflated data set is sufficiently small.

Another multipoint AWE algorithm has been described in [7]. In this technique, a number of shift frequencies along the $j\omega$ axis are combined to find accurate high order approximations for circuits which have a number of high-Q poles close to the imaginary axis. This technique performs expansions about $s = 0$ and $s = j\omega_{max}$. If there exist poles that appear in both of these expansions, then the search is considered complete. Otherwise, more frequencies are selected using a binary

search and expansion is carried out about until each intermediate frequency has at least one pole in common with the frequency above and below it. This search scheme is summarized in Figure 2.

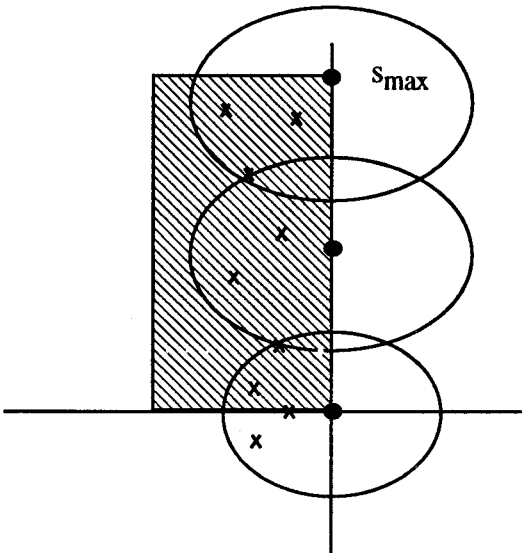

Figure 2: Circle of "confidence" for each Taylor series expansion

6 AWE and numerical integration algorithms

A generalization of the AWE technique which relates AWE to conventional numerical integration techniques is introduced through the concept of a mapping. A mapping defines a relationship between the eigenvalues, λ_A, of the circuit, and the eigenvalues, λ_B, of a related circuit. This provides a unifying theory—integration time points, moments, derivatives and shifted moments and shifted derivatives can be considered as mapped data points, which can be used to form AWE approximations.

The mappings for forward Euler, backward Euler and trapezoidal integration [47] are derived in [22]. The numerical strengths and weaknesses of each method are discussed. The relationship of these mappings to moments, derivatives, and shifted moments is established. This generalization has led to a significant improvement in the obtainable accuracy of the AWE approximations.

7. AWE Macromodels

In the previous sections, we discussed the process of finding AWE approximations for a single circuit function. It is straightforward to extend this technique to find AWE approximations for linear multiport circuit partitions. These AWE approximations may determine the H,Y,Z or S parameters of the multiport circuit. The cost of finding moments for an entire multiport approximation is shown to be a single LU factorization [21]. The moments of a number of multiport partitions can be combined to form larger multiport partitions in a hierarchical manner [21]. The form of the macromodels are slightly altered by the presence of arbitrary initial conditions and independent sources within the partitions [21].

If the circuit solution of the global circuit is super-linear in the number of circuit elements, solving it in a hierarchical manner can speed up the circuit solution. In addition, some of the resulting partitions may have structures enabling the use of special solution techniques to determine their admittance representations. Ladder networks and distributed elements are some practical circuit partitions that fall into this category. These special purpose solution techniques [21]can reduce the solution time of such partitions by orders of magnitude in some cases. The global matrix can be solved using a general matrix solution technique, and thus generality is not compromised.

This capability enables us to solve for macromodels of the partitions, and then abstract them away during the solution of the global circuit. This leads to significant memory savings which can be crucial in solution of extremely large interconnect circuits. A macromodel thus determined can also be catalogued and used again in the solution of any other related circuit.

All the matrix elements in AWE macromodels may be forced to have a common denominator [46] [18], that is the same set of poles can be forced on all the elements of the macromodel. In this case, an AWE approximation is determined only for one element of the macromodel. The same set of poles is then used for all the other elements of the macromodel, while the residues are determined from the moments of each corresponding element of the macromodel. The cost of macromodel determination may be marginally reduced if the circuit has a large number of ports, however the accuracy of the approximation may be sacrificed.

Alternatively, the macromodels may permit different sets of poles and residues for each element [48]. This allows different orders of approximation for the different elements of the macromodel which is desirable as different orders may be required to approximate them accurately.

Macromodels can be combined with nonlinear transient simulation in an accurate and efficient manner using a form of recursive convolution [48] [18]. Recursive convolution of rational functions was probably first described in [39].

8. Distributed Elements

The AWE technique has been extended to handle lossy coupled transmission lines without resorting to lumped equivalents [27] [9]. In the limit of increasing accuracy, distributed models are more efficient than the lumped model approach due to the large number of lumps required to accurately approximate the distributed nature of the lines. In the distributed model, moments for the various circuit variables are found from the constitutive differential equations. The circuit variables constitute a multiport admittance macromodel in the

moment domain. This model can then be converted into a pole-residue model by means of Padé approximation making it useful in time domain simulation.

A second approach to simulating distributed elements with the AWE technique recognizes that transmission line responses can contain pure delays[44]. These represent the time required for a wave to propagate from one end of the line to the other. It is difficult for the standard AWE algorithm to capture such a response because a rational function model has been assumed. In the time domain, the model response consists only of real or complex conjugate exponentials. To represent a pure delay, the AWE approximation must employ decaying sinusoids to artificially force the response to be close to zero for some initial time period. This can lead to spurious ringing effects in the AWE waveforms for lower orders of approximation.

To overcome this problem, the AWE technique has been extended to the generalized method of characteristics [5] for distributed elements. The delay factors are computed exactly and "extracted" from the propagation functions for the lines. The "remainder" responses can be viewed as containing the attenuation and dispersion behavior of the propagation responses. A conventional AWE analysis is used to accurately approximate the propagation response without delay. The resulting model is more accurate at lower orders of approximation, and can be simulated efficiently in the time domain, together with other linear and nonlinear elements. Other researchers have developed Padé approximations about $s = \infty$ for distributed elements in a similar way [40].

9. Conclusions

This paper has provided an overview of some of the continuing research on Asymptotic Waveform Evaluation (AWE) techniques. Techniques to improve the efficiency of moment generation have been discussed. A brief discussion of the various techniques to improve the efficiency, accuracy and stability of AWE approximations has been provided. An overview of techniques to generate AWE macromodels and extension of the AWE technique to handle distributed elements has also been presented.

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