Difference-frequency time scales for multitime PDEs

Jaijeet Roychowdhury

University of Minnesota

Introduction

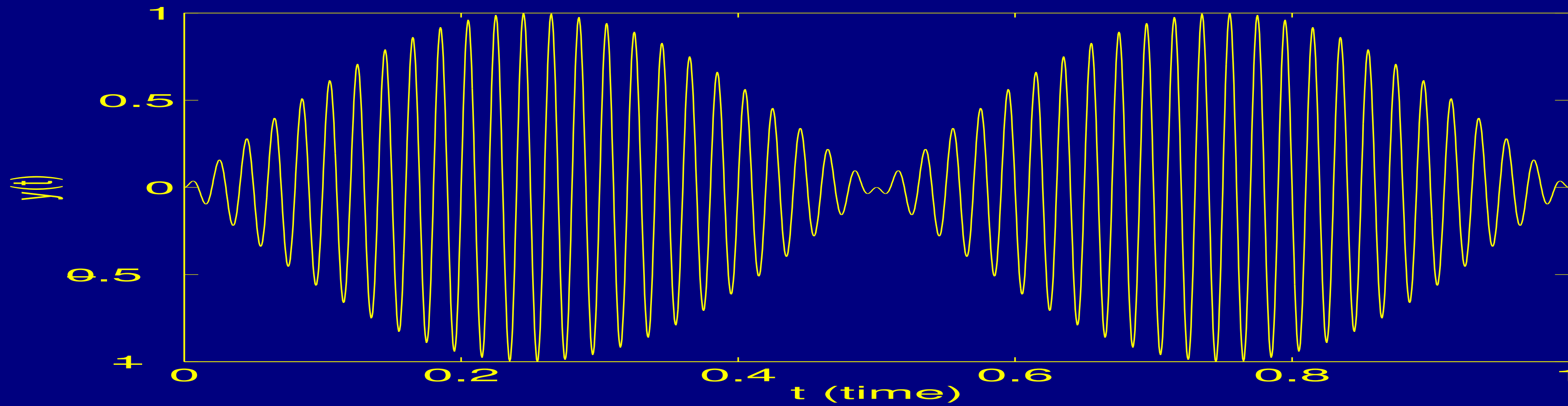
- The circuit DAE:

$$g(\dot{x}, x, t) = 0, \quad \text{or,} \qquad \dot{q}(x) + f(x) + b(t) = 0$$

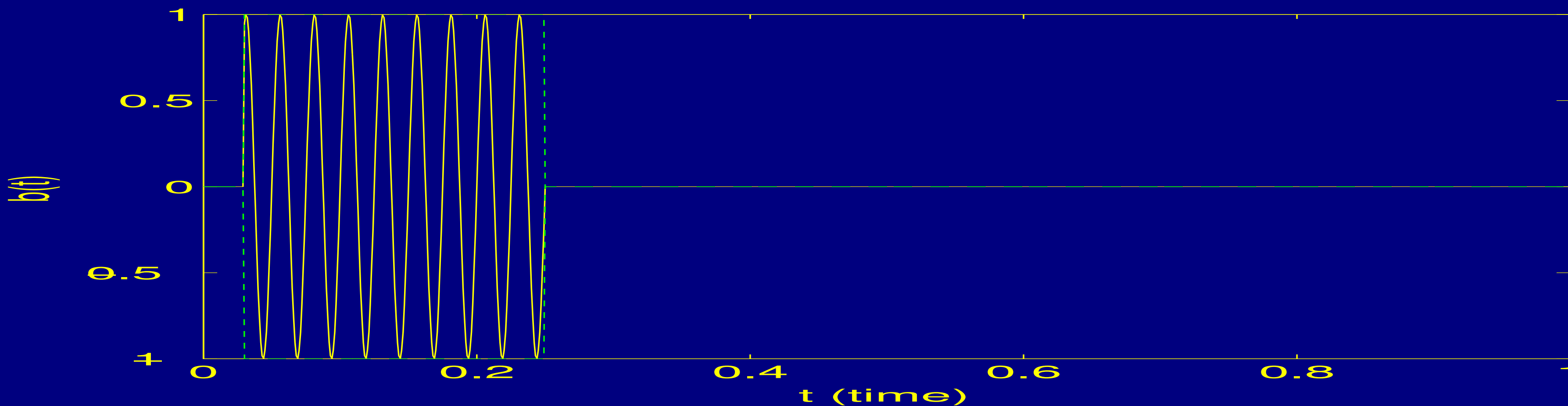
- The problem:
  - $b(t)$ is multi-tone, time scales widely separated
  - $f(\cdot)$, $q(\cdot)$ are strongly nonlinear
- ODE/DAE initial value solvers can take very long

Representing "multi-rate" signals

$$b(t) = \sin(2\pi 1 t)\sin(2\pi 10^9 t)$$ TD: 10^{10} samples; FD: 2 harmonics

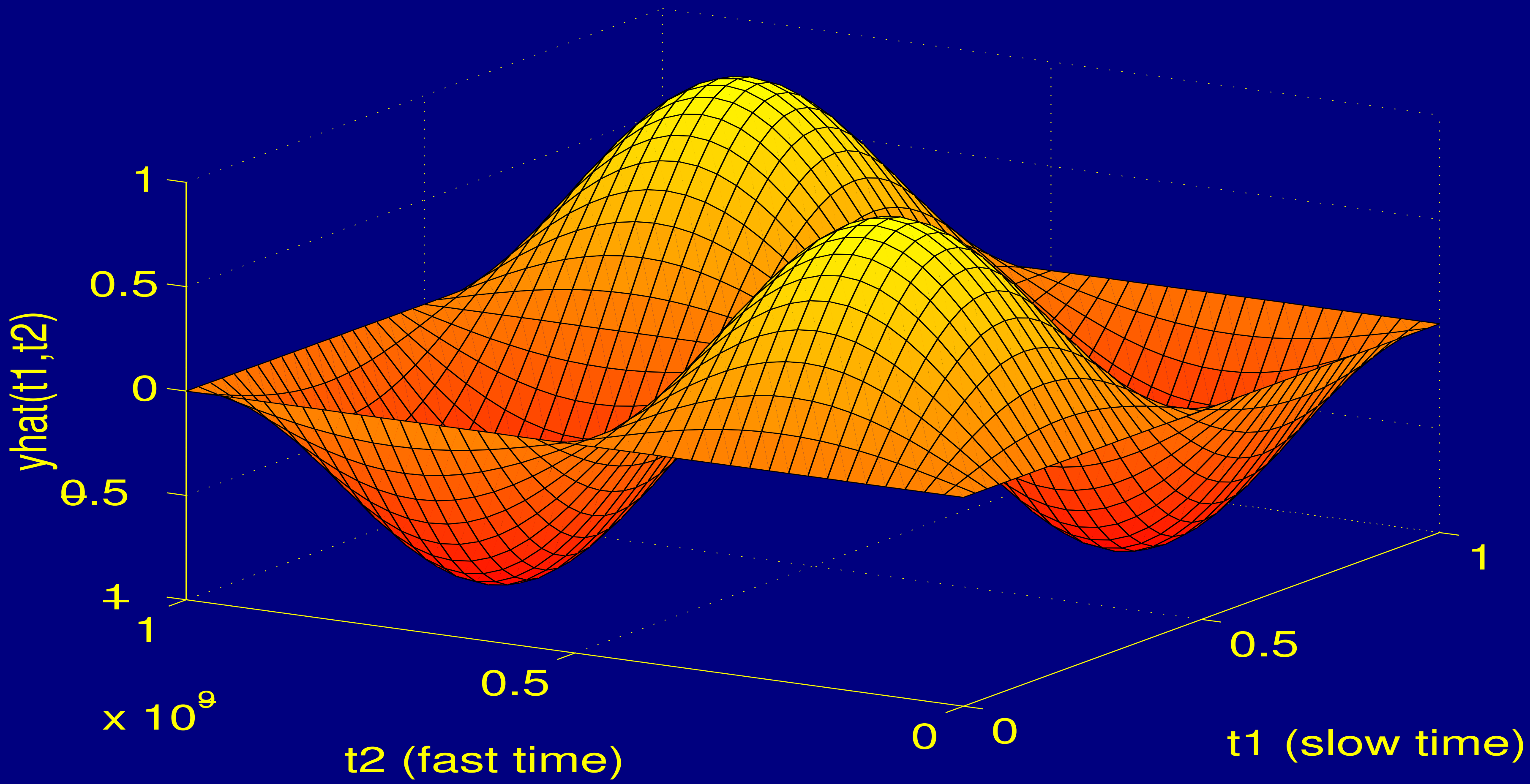


$$b(t) = \text{pulse}(1t)\sin(2\pi 10^9 t)$$ TD: 10^{10} samples; FD: 10^n harmonics



Two Artificial Time Scales

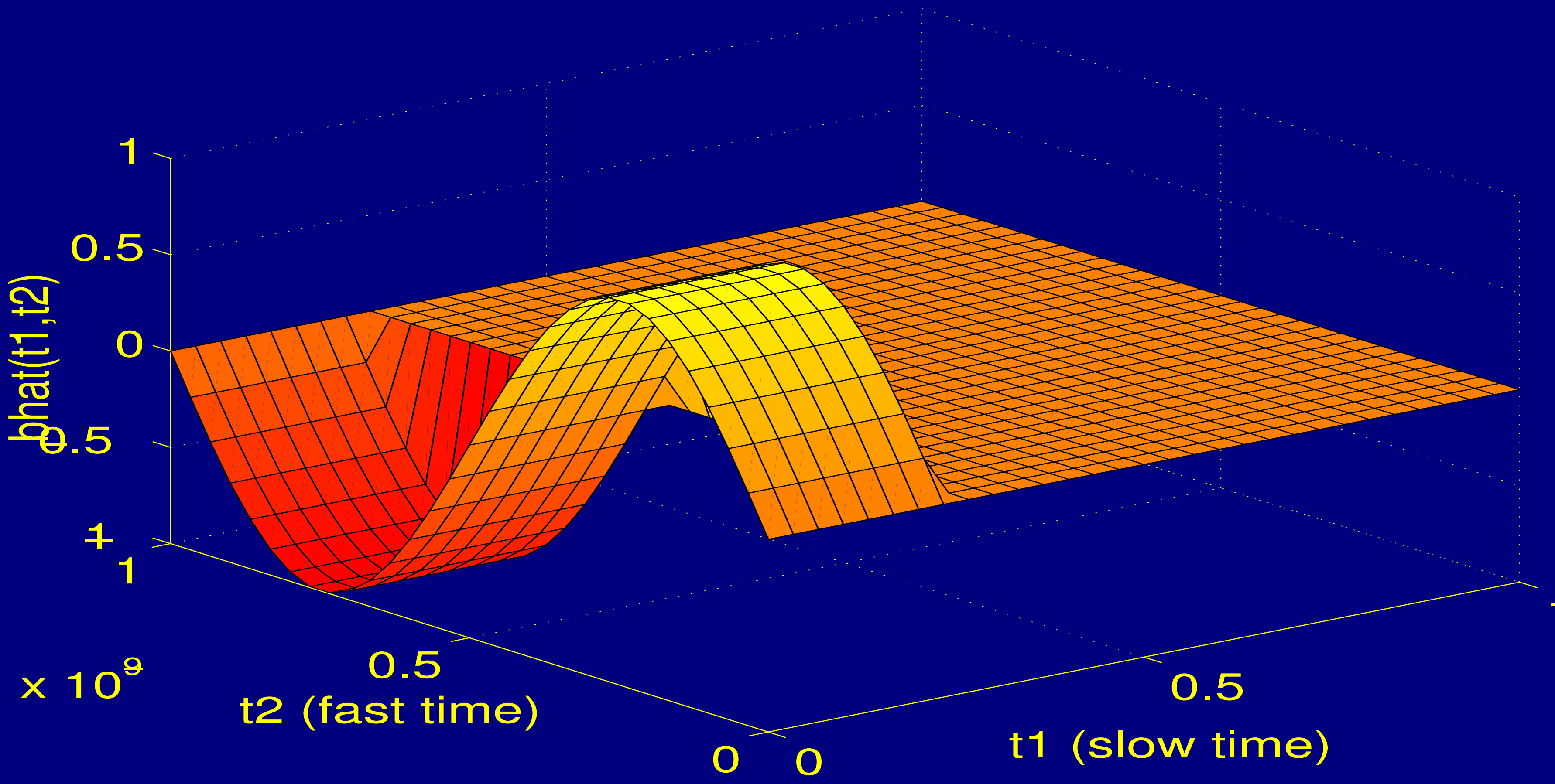
- Two time variables separate time scales



$$\hat{x}(t_1, t_2) = \sin(2\pi 1 t_1)\sin(2\pi 10^9 t_2) \qquad x(t) = \hat{x}(t, t)$$

Two Artificial Time Scales

- "Strongly nonlinear" waveforms



$$\hat{b}(t_1,t_2) = \mathrm{pulse}(1t_1)\sin(2\pi 10^9 t_2) \qquad b(t) = \hat{b}(t,t)$$

The Multitime Partial Differential Equation (MPDE)

- Re-write DAE using multi-time variables

DAE

$$\dot{q}(x) + f(x) = b(t)$$

$\rightsquigarrow$

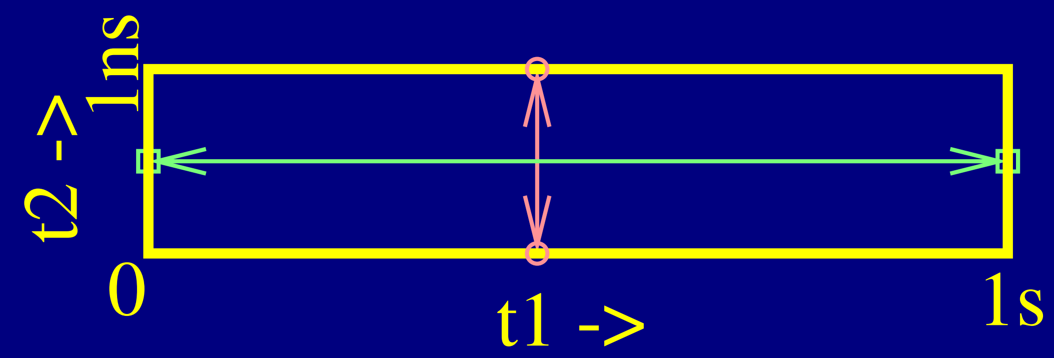
MPDE

$$\left(\frac{\partial}{\partial t_1} + \frac{\partial}{\partial t_2}\right) q(\hat{x}) + f(\hat{x}) = \hat{b}(t_1, t_2)$$

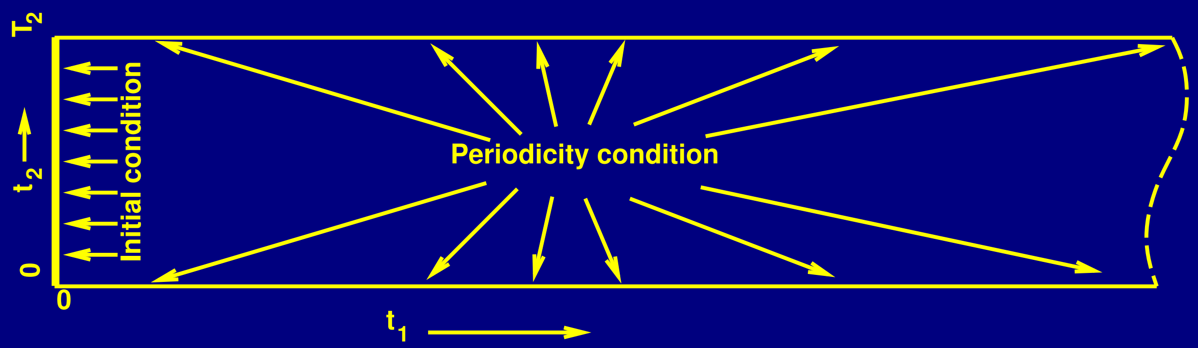
- $\hat{x}(t_1, t_2)$: vector of *multivariate* unknowns
- $\hat{b}(t_1, t_2)$: multivariate form of inputs
- MPDE solution $\Rightarrow$ DAE solution
- $b(t) = \hat{b}(t, t) \Rightarrow x(t) = \hat{x}(t, t)$
- Key: solve for multivariate forms *directly*

Quasiperiodic/Envelope Solutions

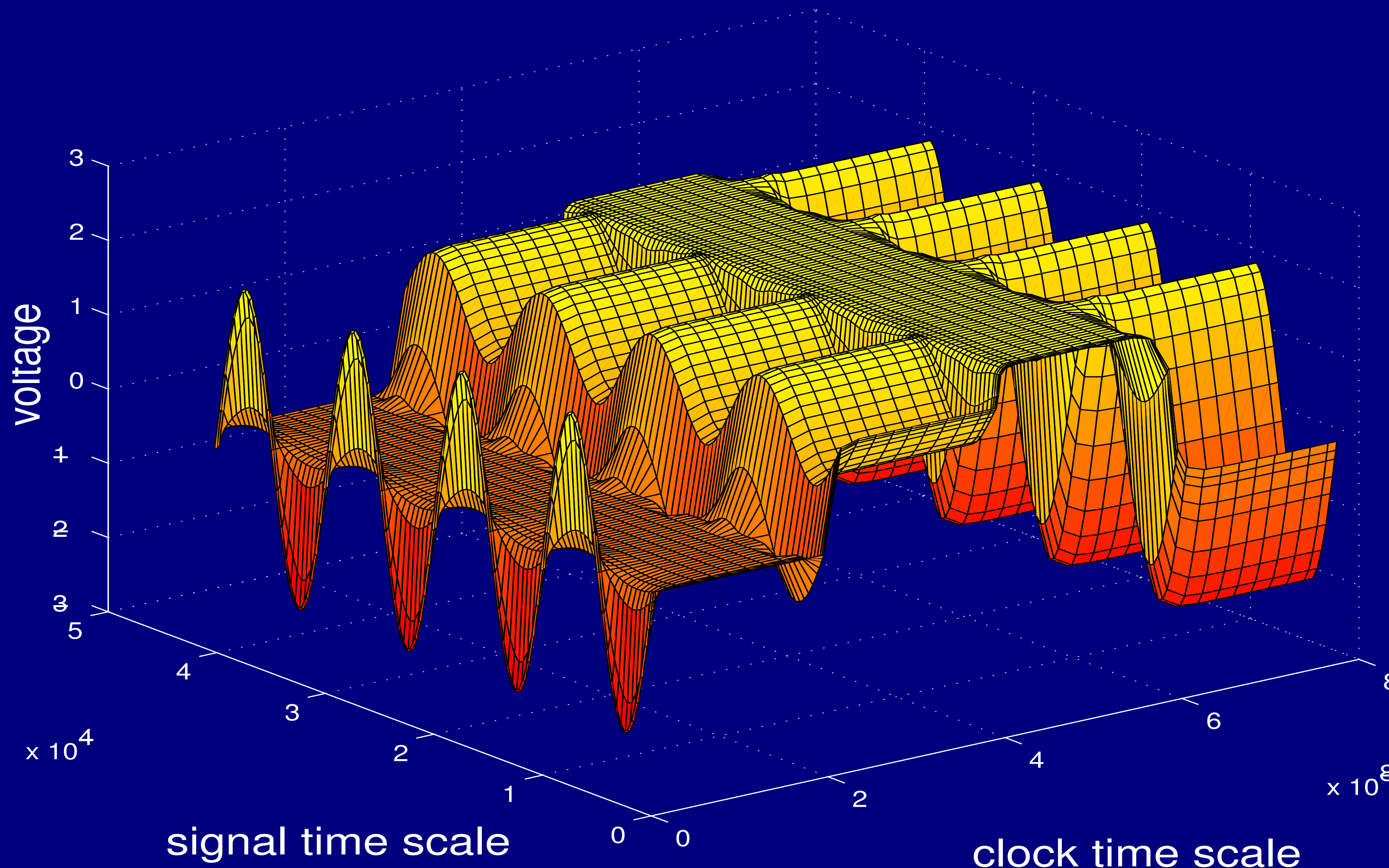
- Quasiperiodic: periodic boundary conditions
  - $\sin(2\pi 1 t_1)\text{pulse}(2\pi 10^9 t_2)$



- Envelope: mixed initial/periodic BCs
  - $a(t_1)\text{pulse}(2\pi 10^9 t_2)$

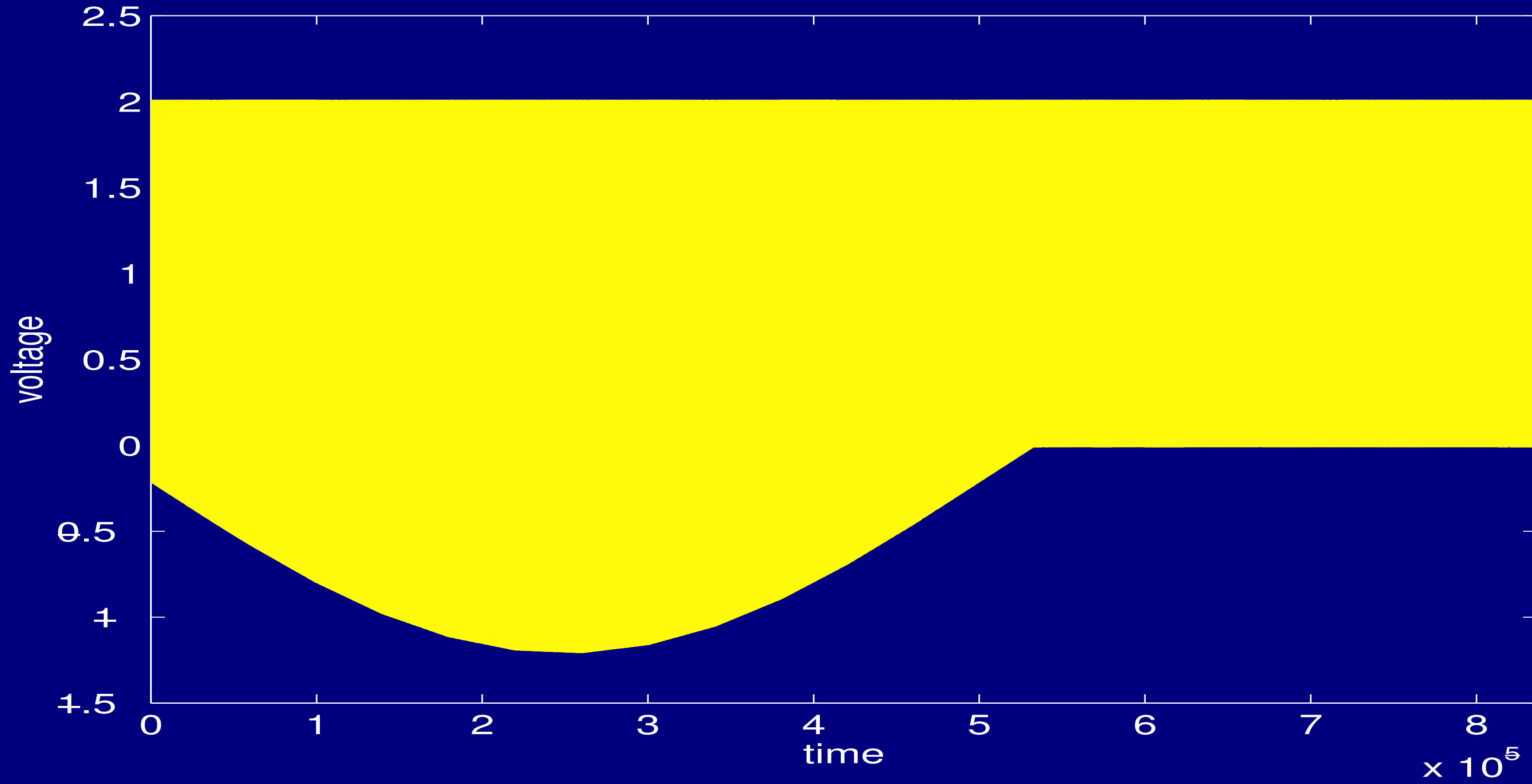


SC Integrator: Multi-Time Simulation

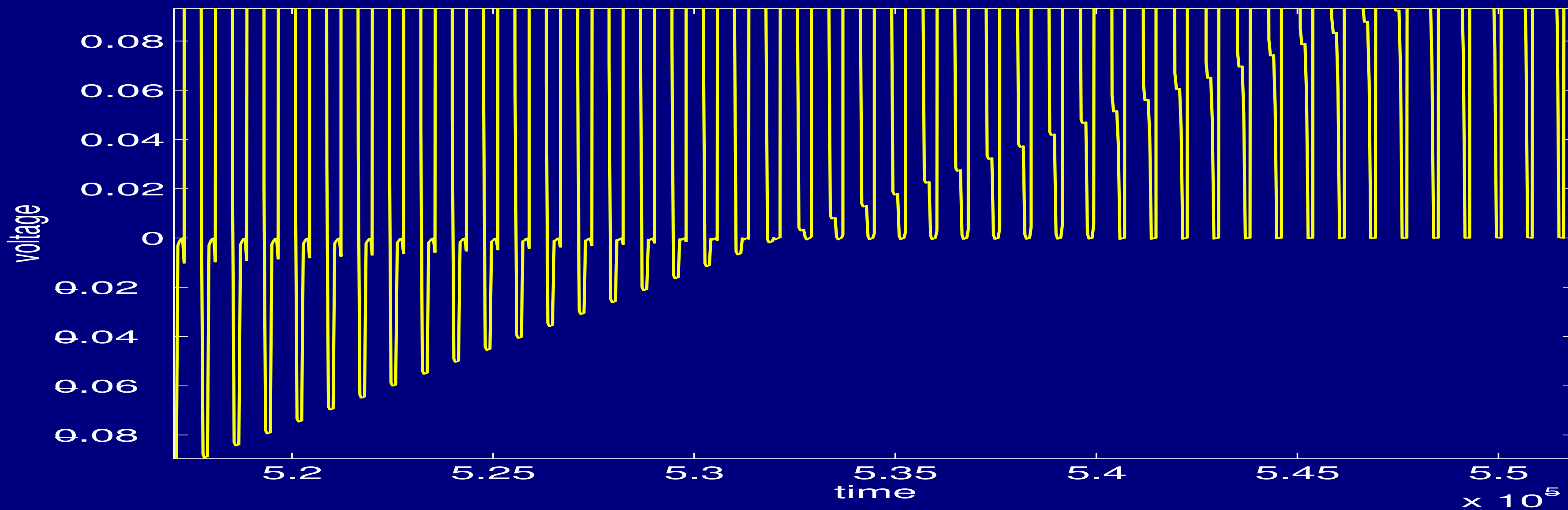


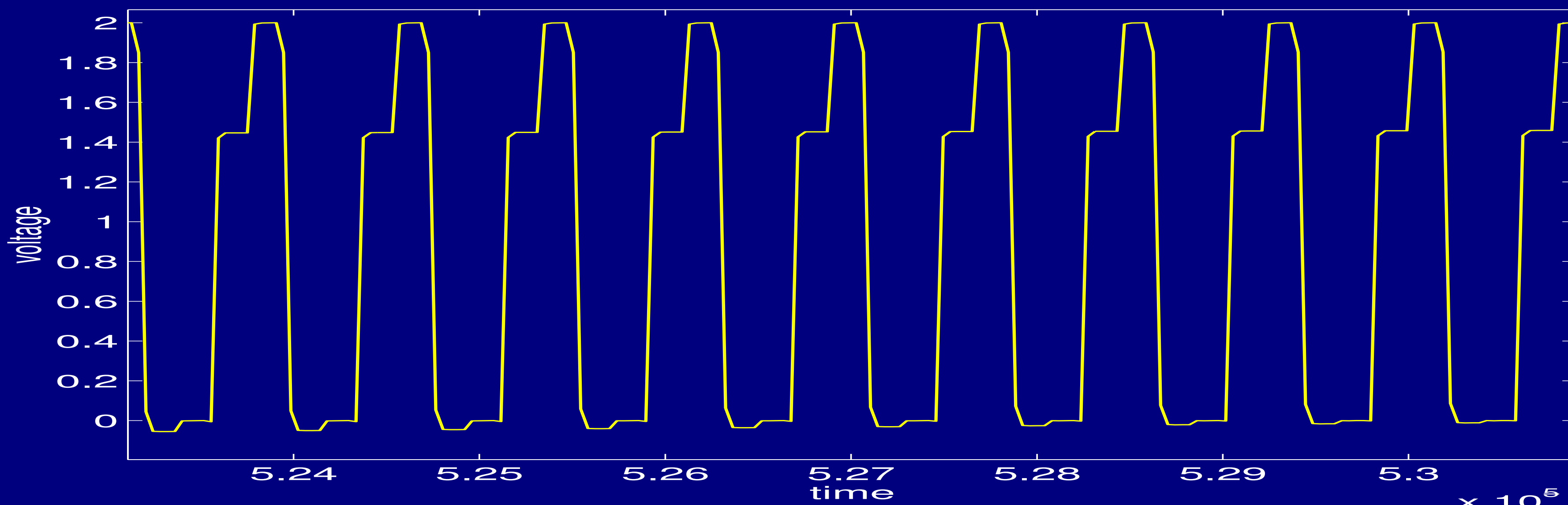
Switched-Capacitor Integrator

- Lossy balanced design; 350 MOSFETs
- clock: 12.8 MHz; test signal: 10 kHz



SC Integrator: transient detail

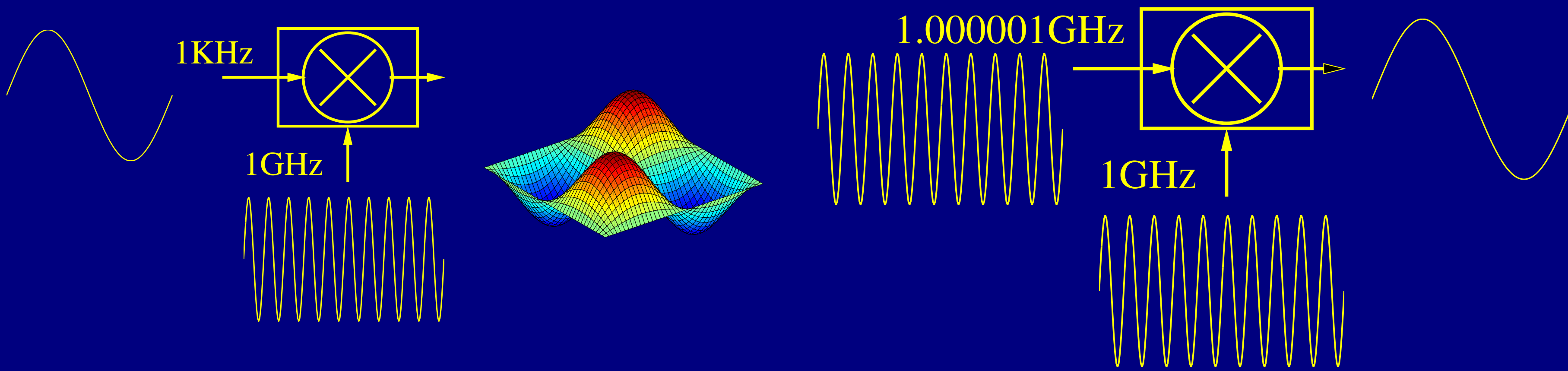




Development of multi-time PDE concepts

- Separate time scales for SHO perturbations: asymptotic expansions
- First mention of MPDE: Ngoya/Larchevèque
- Multitone HB derived from MPDE: Brachtendorf
- Time-domain/mixed/envelope methods for strongly nonlinear MPDE: R.
- MPDE methods for oscillators: Brachtendorf (startup); Narayan & R. (WaMPDE)
- MPDE for LTV/nonlinear macromodelling: R.
- Difference-frequency time scales

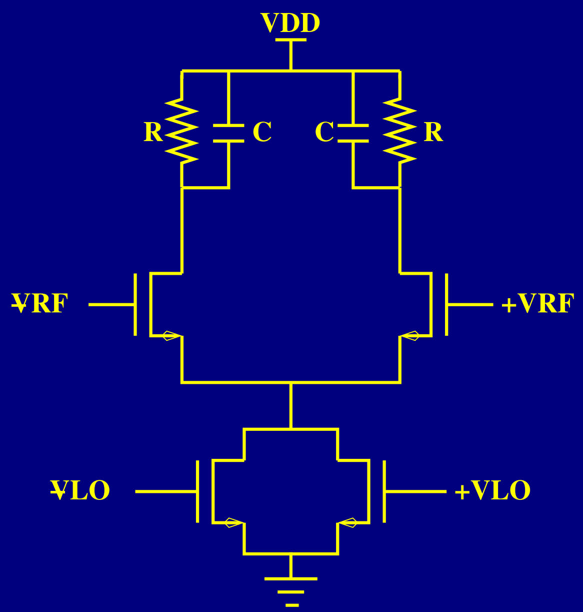
The Down-conversion Problem

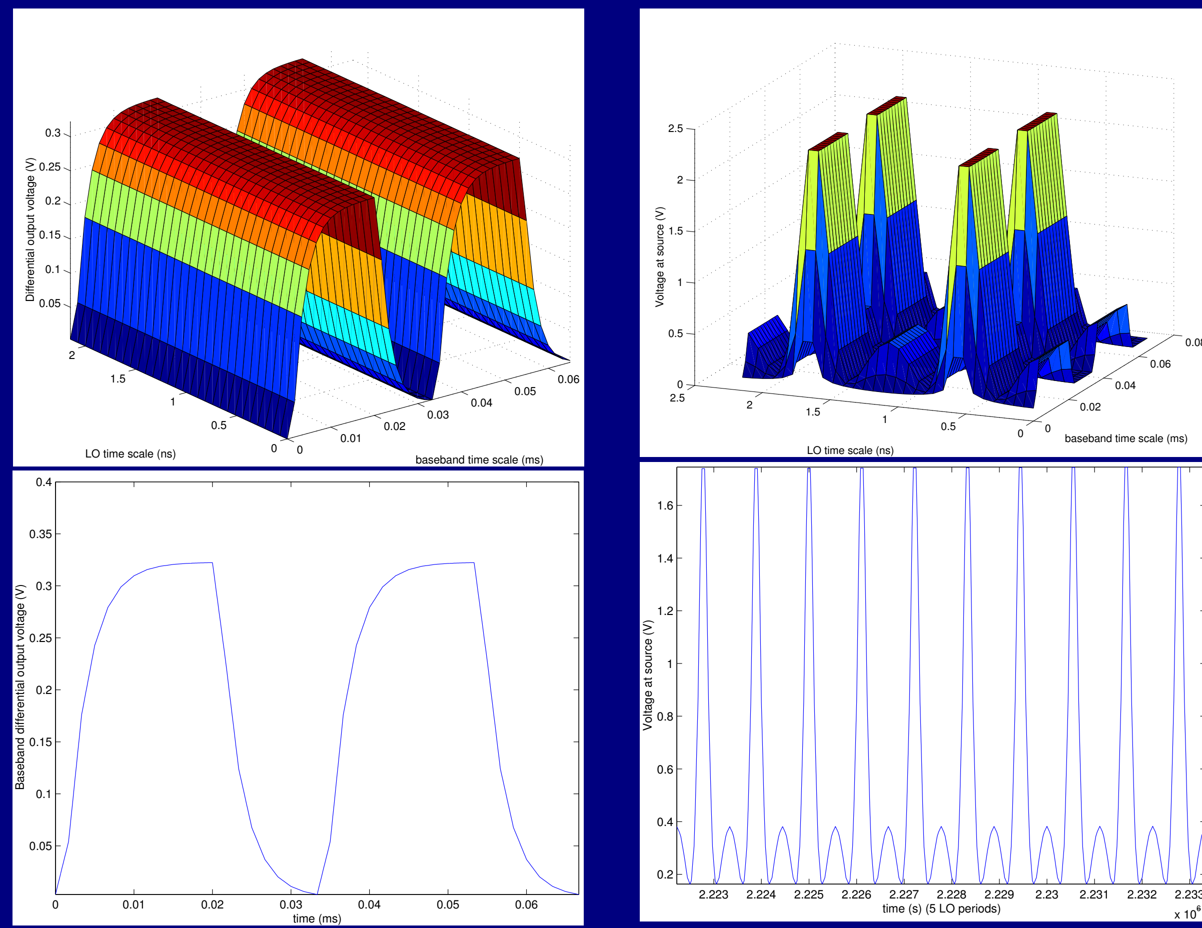


- Up-conversion
- input specification: 1 slow, 1 fast time scale
- $\hat{x}(t_1, t_2) = \sin(10^3\, t_1)\sin(10^9\, t_2)$
- output envelope: along t_1

- Down-conversion
- input specification: two fast time scales
- $\hat{x}(t_1, t_2) = \sin((10^9 + 10^3)t_1)\sin(10^9\, t_2)$
- output envelope: along neither t_1 nor t_2

LO-doubling downconversion mixer





LO=450MHz; 15kHz bitstream; 40x30 grid; prob-1 homotopy; speedup vs shooting: $\sim$ 100

Conclusion and Directions

- Widely-separated timescales: DAEs $\Rightarrow$ artificial-time PDEs
  - Unifying: HB, mixed F/T, envelope, ...
- Difference-frequency time scales
  - Slow envelope, including Fourier envelope (HB)
- New, little explored area
  - Import existing PDE techniques (*e.g.*, adaptive gridding)
  - Pre-conditioners
  - Surprises/opportunities remain in circuit simulation