Techniques for the global stability analysis of microwave circuits. Application to novel circuit design.

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OUTLINE

1 - Introduction

2 - Nonlinear dynamics of analog circuits

- Geometrical approach: the phase space
- Local stability
- Global stability. Bifurcations

3 - Simulation techniques

- Comparison
- Stability analysis
- Bifurcation analysis. Use of auxiliary generators

4 - Novel circuit design

5 - Conclusions

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1 - INTRODUCTION

Nonlinear circuits: nonlinear differential equations

Steady-state solution:

- It can include autonomous and subharmonic frequencies
- DC, periodic, quasi-peridic, chaotic

Coexistence of solutions: Same or different type

Stable (or physical) and unstable

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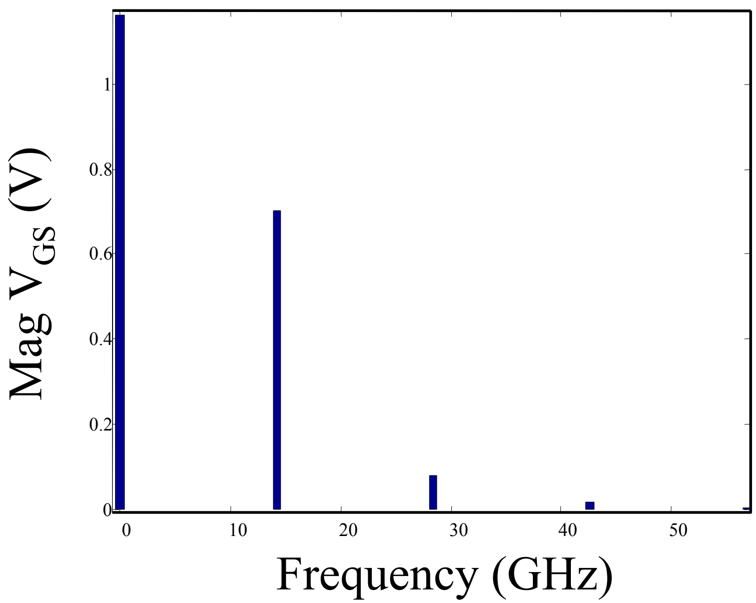
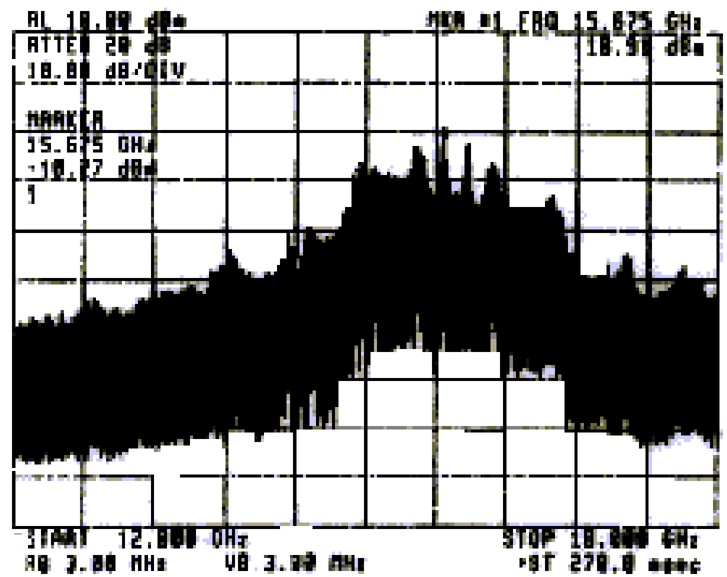
When using frequency-domain analysis (intrinsecally forced):

Measured solution may be very different from the simulated one





Objective:

- Techniques for the global stability analysis of nonlinear circuits

Stability analysis under the variation of the circuit parameters

Applications:

- Accurate determination of stable-operation borders
- Investigation of complex behavior
- Advanced and novel circuit design. Practical examples

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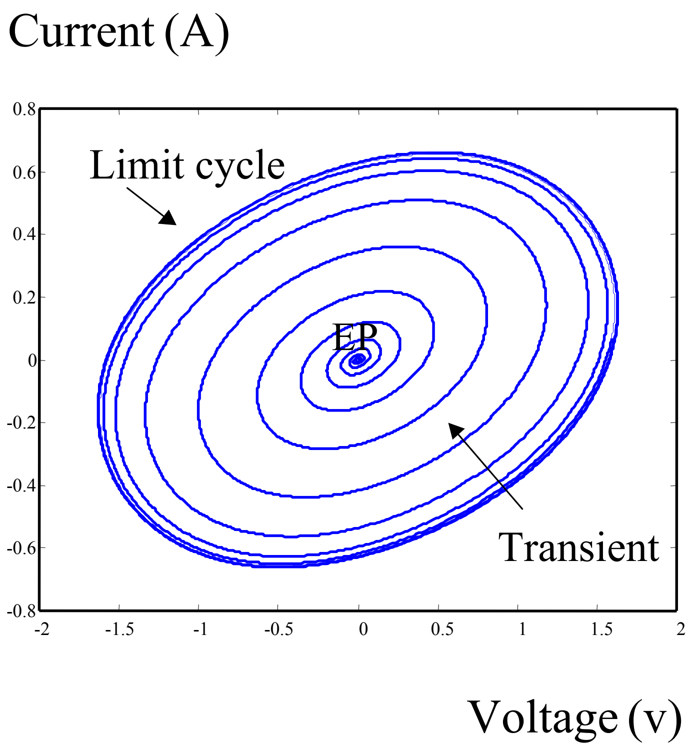
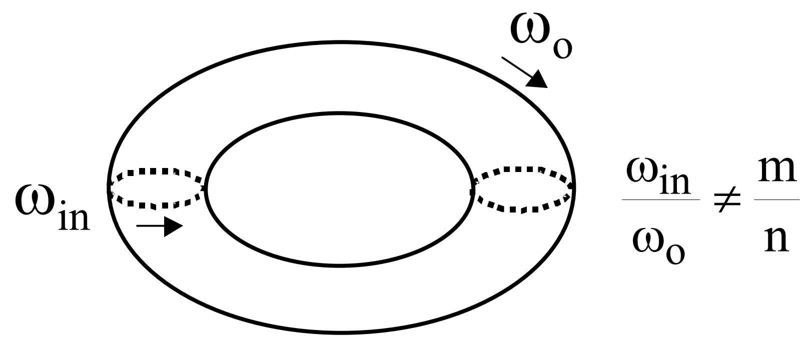
2 - NONLINEAR DYNAMICS OF ANALOG CIRCUITS

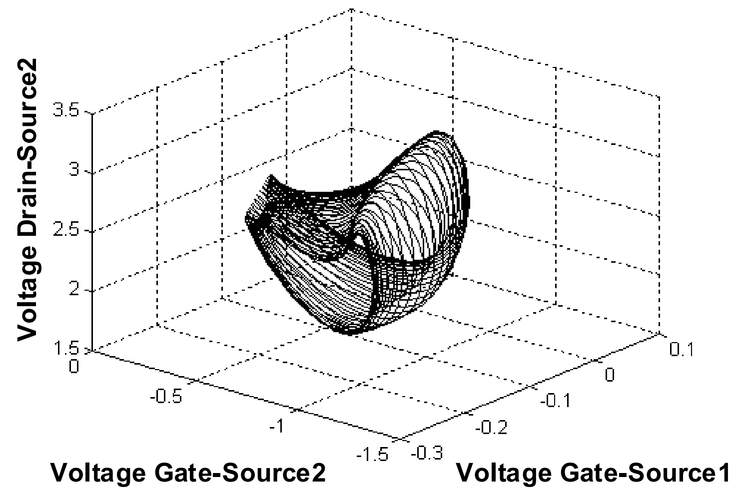
2.1- GEOMETRICAL APPROACH: THE PHASE SPACE



DC solution   EP

Periodic solution   LC

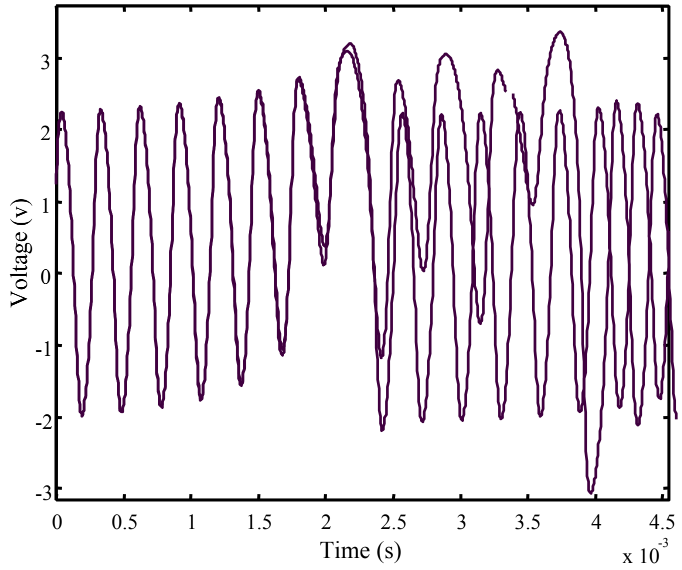


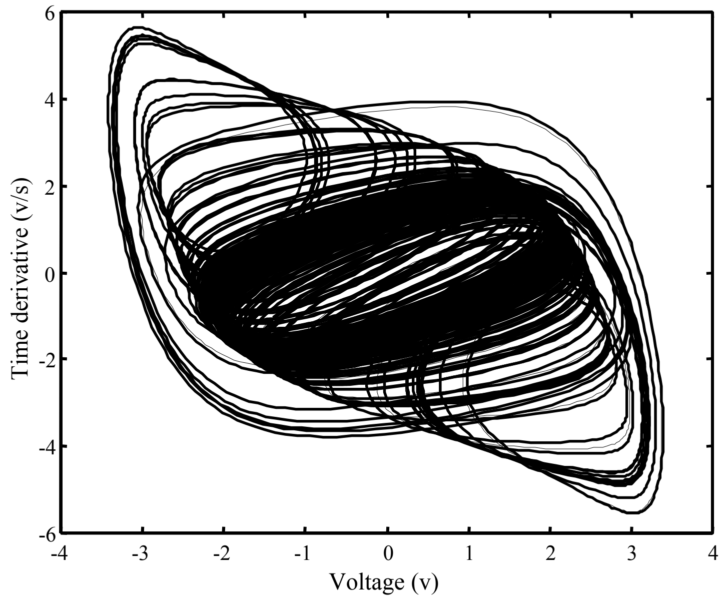


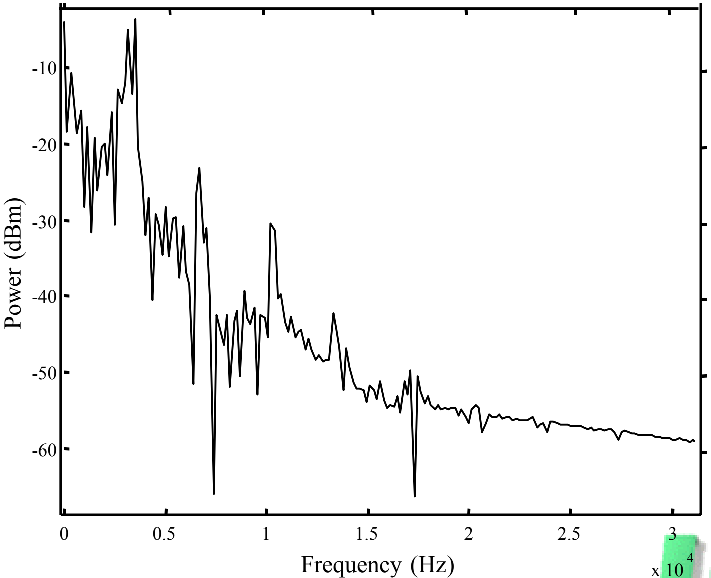
Quasi-periodic solution: Limit torus

Chaotic solution

- Sensitivity to initial conditions
- Continuous spectrum
- Fractal dimension

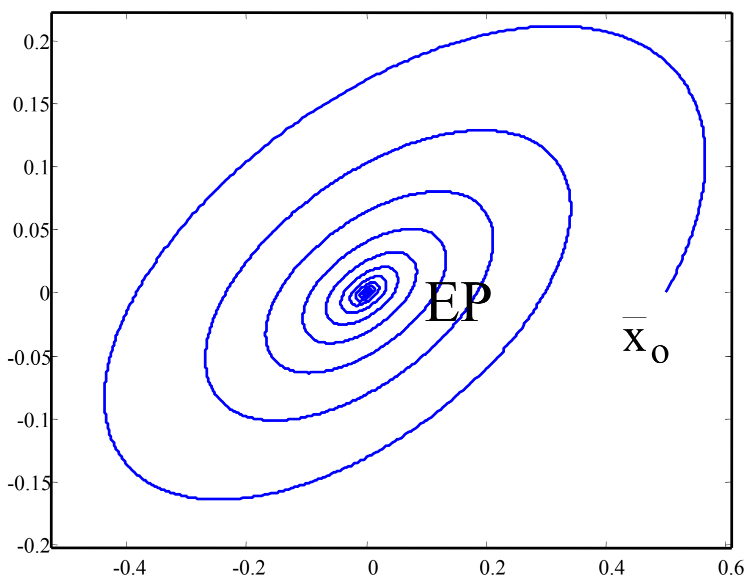




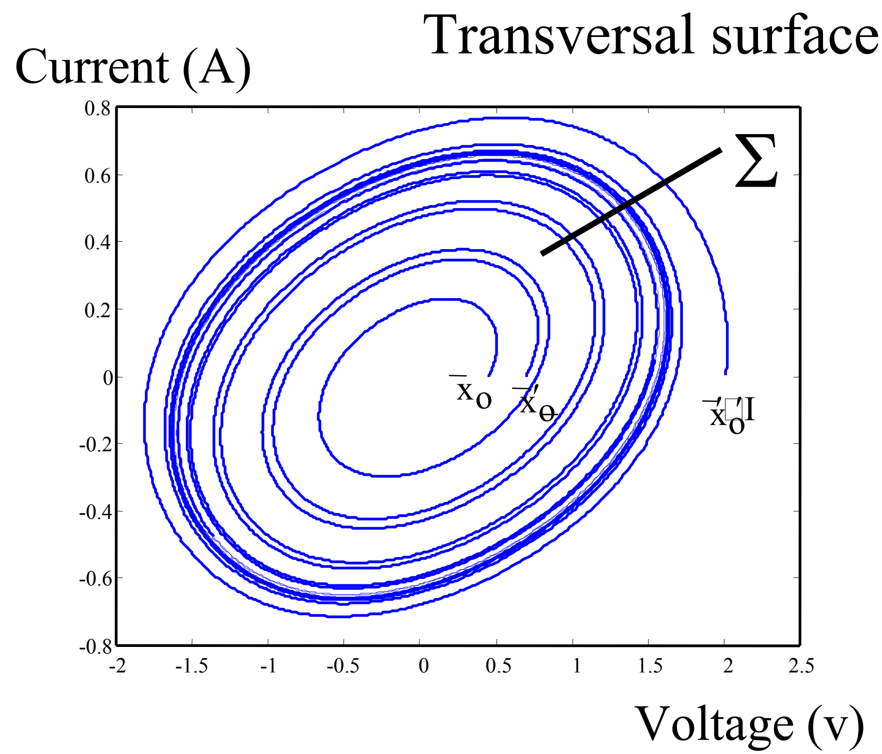


2.2 - LOCAL STABILITY OF STEADY-STATE SOLUTIONS

Through small amplitude perturbations: $\bar{\varepsilon}(t)$



Stable EP



Stable LC

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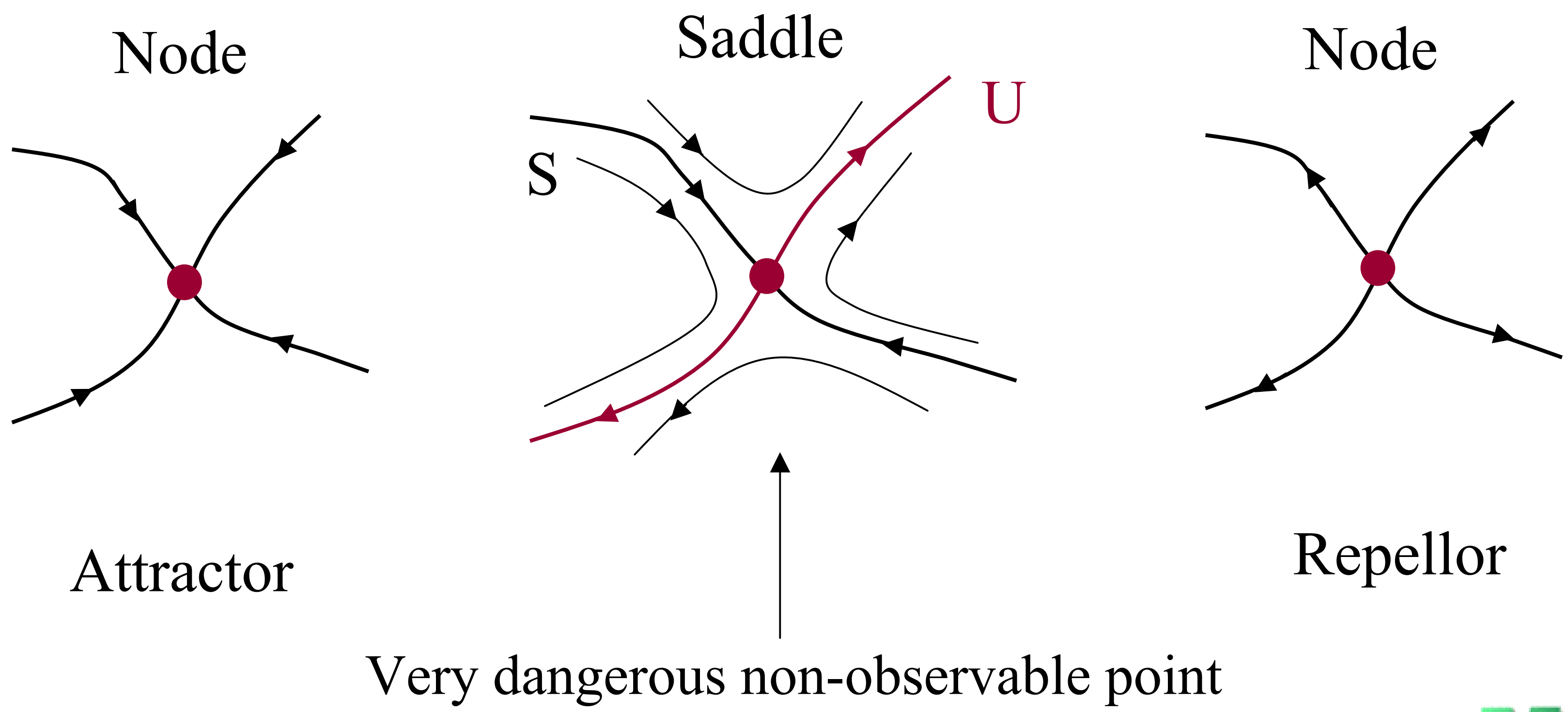
Stability types:

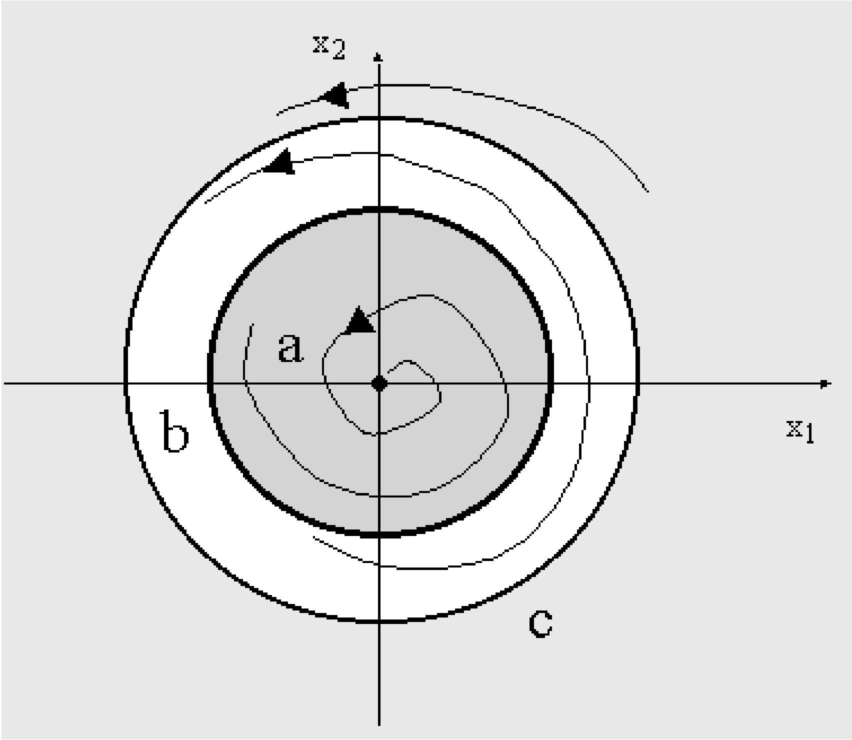
Equilibrium point or

intersection point with a transversal surface Σ

(limit cycle in phase space)



Coexistence of stable and unstable steady-state solutions :



Stable and unstable solutions alternate

Different basins of attraction

2.3 - GLOBAL STABILITY

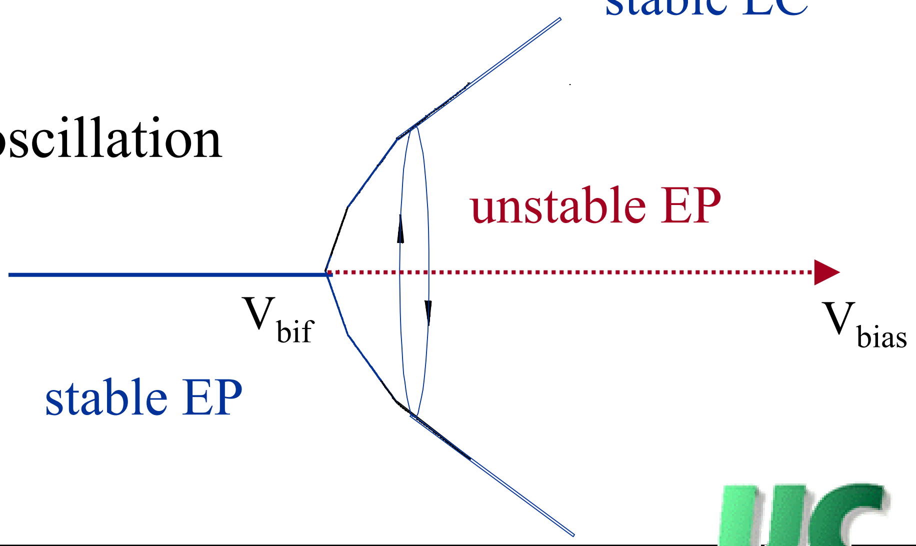
Variations in one or more circuit parameters η : Input generator / linear element value

Bifurcation

- *Qualitative* variation of the solution stability under *continuous* modification of η
- *Creation / destruction* of solutions

Example Onset of a free-running oscillation

EP ⟶ Limit cycle



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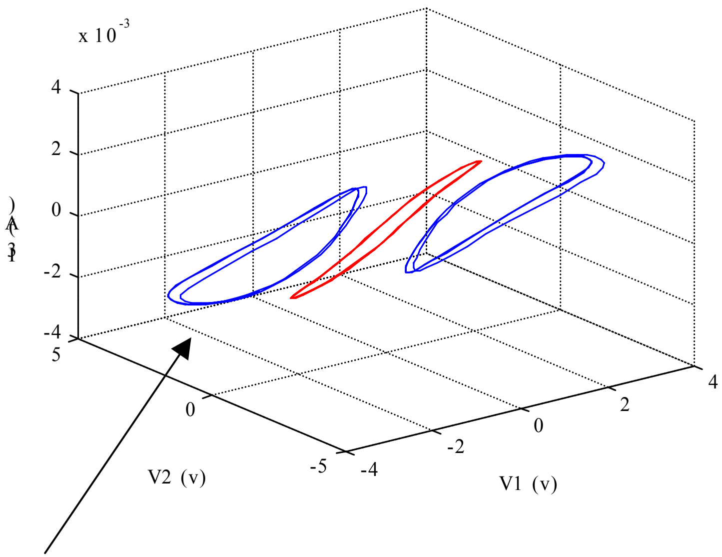
3 - SIMULATION TECHNIQUES

3.1 - COMPARISON

- Time-domain integration

Natural circuit evolution (Physical)

All types of steady-state solutions



Small basin of attraction

- Harmonic balance

Transient is avoided
Continuation techniques

Drawbacks:

Difficulty in autonomous and subharmonic regimes

Convergence to unstable solutions (stability analysis required)

Limited to periodic and quasi-periodic solutions

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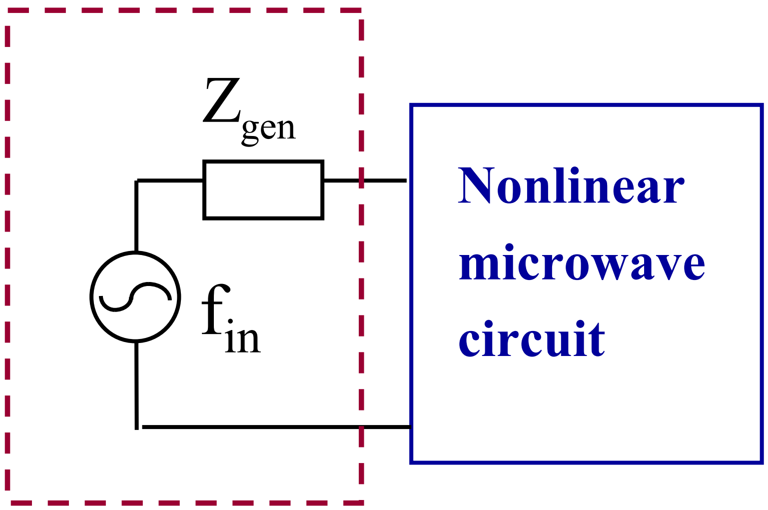
3.2 - STABILITY ANALYSIS

External Generator



$$\overline{H}(\overline{X}) = 0$$

$$x_p(t) = \sum_m X_m e^{j(m\omega_{in})t}$$

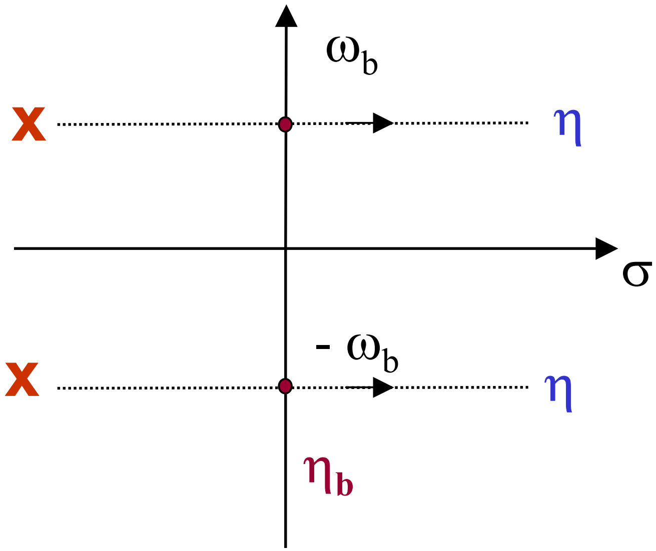
- **Perturbation:** $e^{(\sigma + j\omega t)}$
- **Linearization:** $\left[JH\left(K\omega_{in} + \omega - j\sigma\right) \right] \overline{\Delta X} = 0$
- **Determinant:** $\det\left[JH\left(K\omega_{in} + \omega - j\sigma\right) \right] = 0$

3.3 - BIFURCATION ANALYSIS

Local bifurcation conditions

$$\det\left[JH\,(jk\omega_{in} + j\omega_b) \right] = 0$$

$$\left.\frac{d\sigma}{d\eta}\right|_{\eta=\eta_b} \neq 0$$



$\omega_b \neq \frac{m}{n}\omega_{in}$ Hopf → Det complex, η_b, ω_b

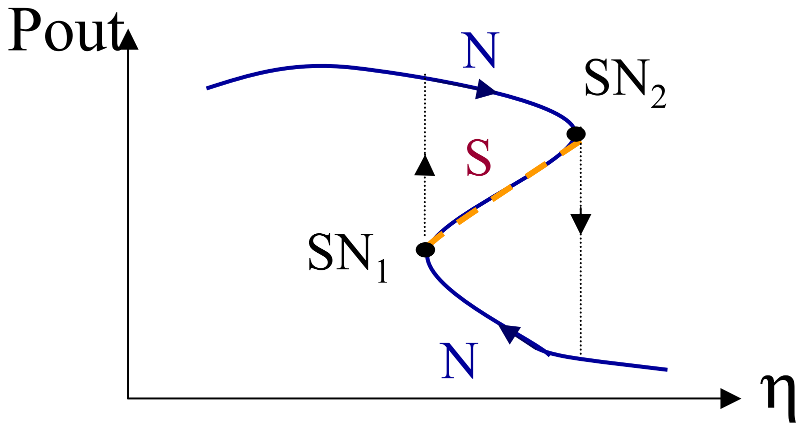
$\omega_b = \frac{\omega_{in}}{2}$ I or Flip → Det real, η_b

$\omega_b = 0$ Turning point / Pitchfork → Det real, η_b

Bifurcation diagrams

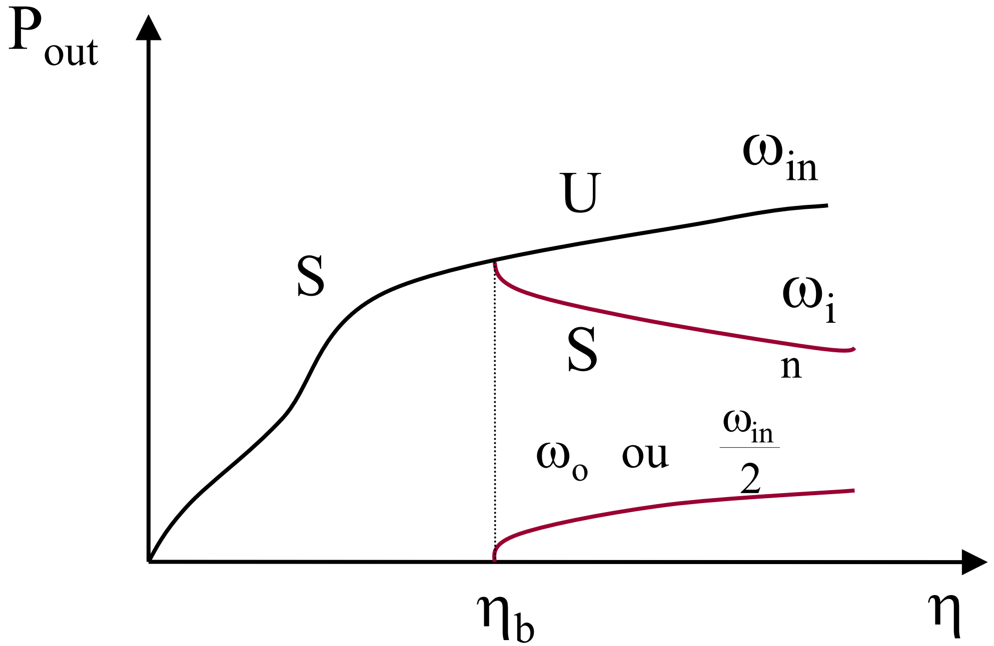
- **Turning points** $\omega_b = 0$

Jump and hysteresis



- **Branching - type bifurcations**

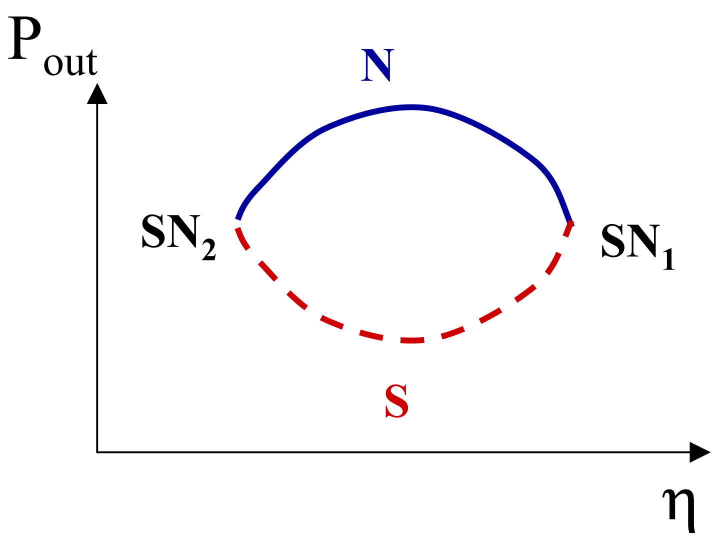
Hopf
Flip



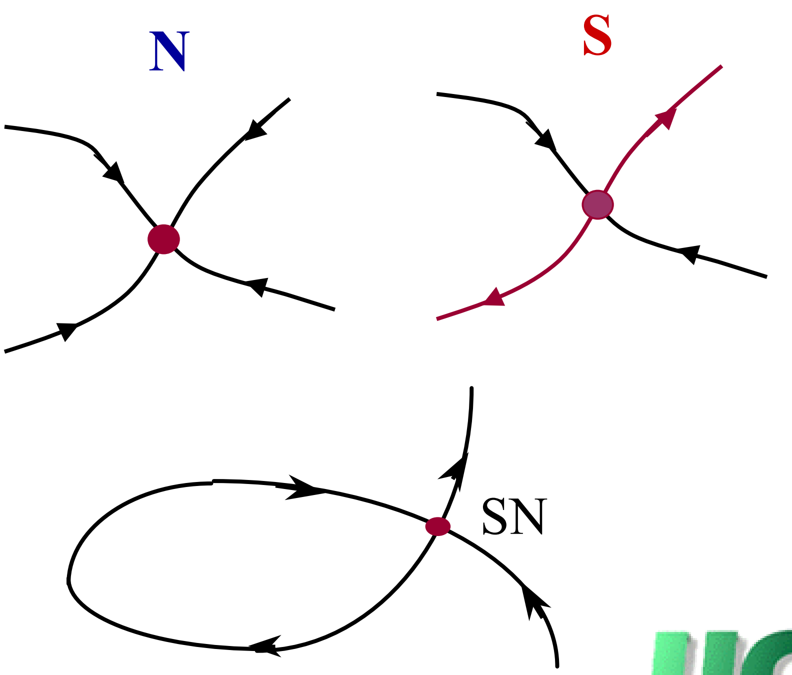
Global bifurcations

- Saddle connection
- Mode-locking at saddle-node : Synchronization / synchronization loss

Bifurcation diagram



Poincaré map



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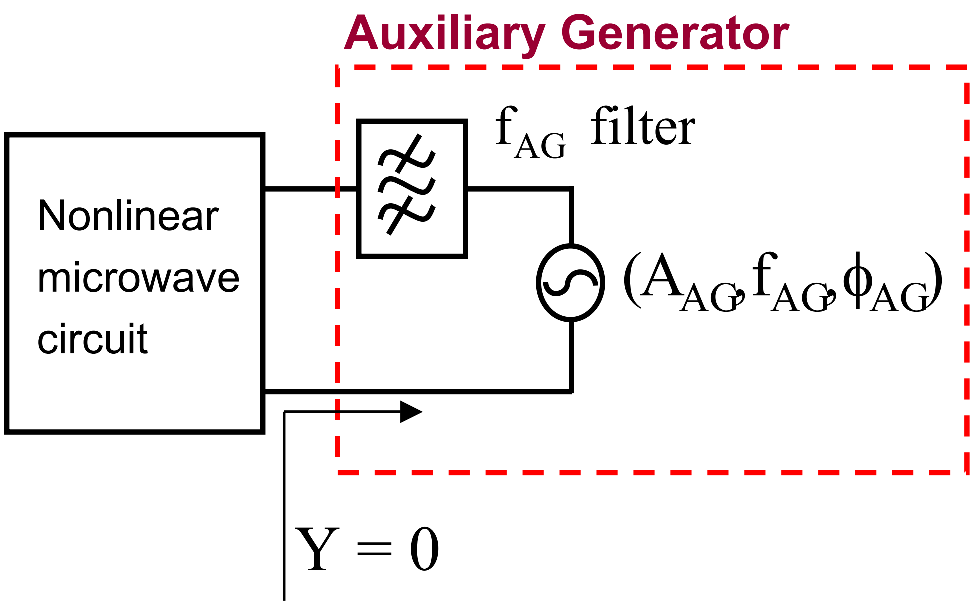
Autonomous and subharmonic solutions:

$$\overline{X}^{o} = [Ag]\,\overline{G} \longrightarrow$$ Leads to trivial mathematical solutions



- 2 more unknowns:

$$\begin{cases} A_{AG}, f_{AG} & \text{Autonomous} \\ A_{AG}, \phi_{AG} & \text{Synchronized} \end{cases}$$

- 2 more equations:

$$\begin{cases} Y_r = 0 \\ Y_i = 0 \end{cases}$$

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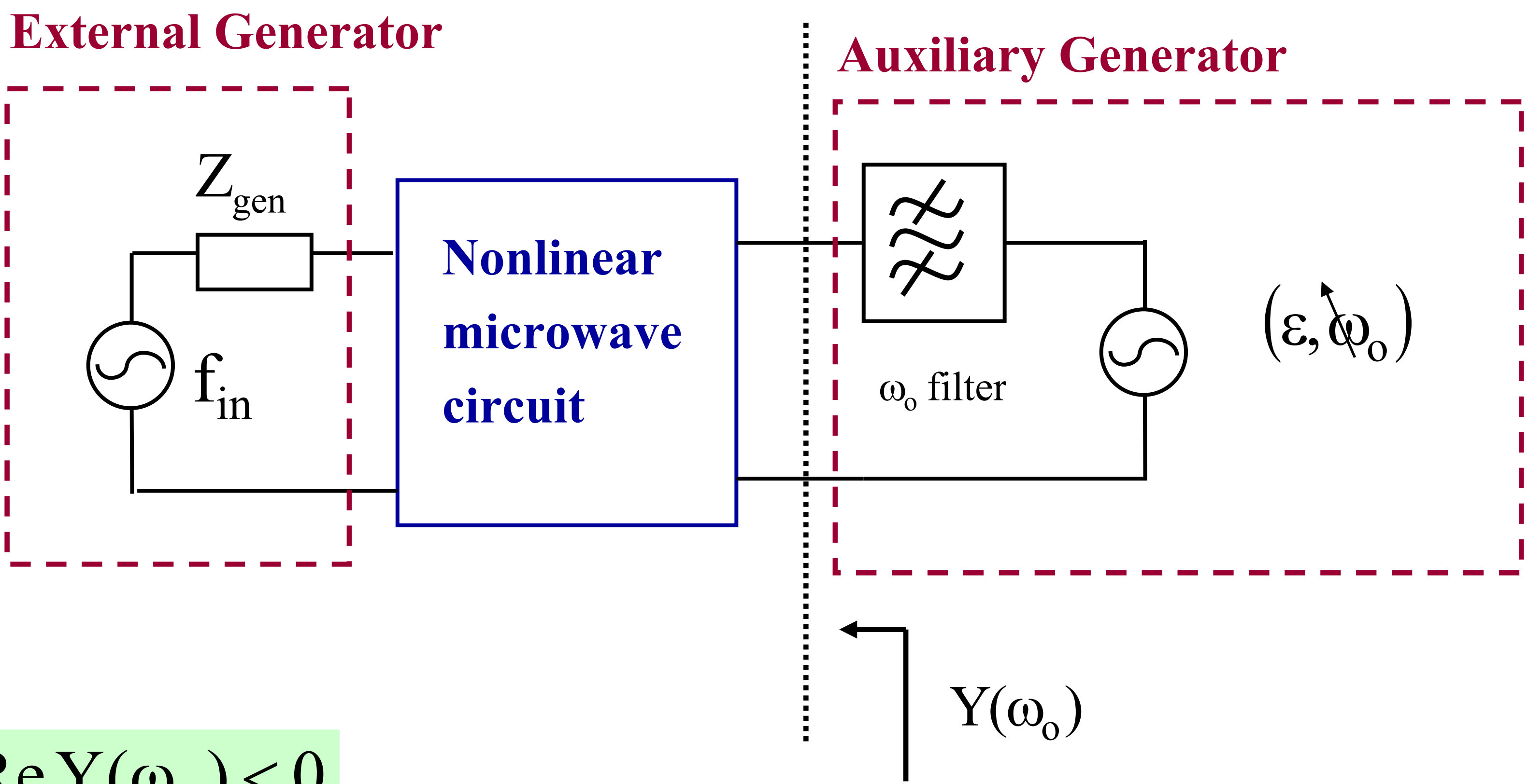
- Stability analysis of steady-state regimes



$$\mathrm{Re}\,Y(\omega_o) < 0$$
$$\mathrm{Im}\,Y(\omega_o) = 0$$

Combined with zero-pole identification techniques

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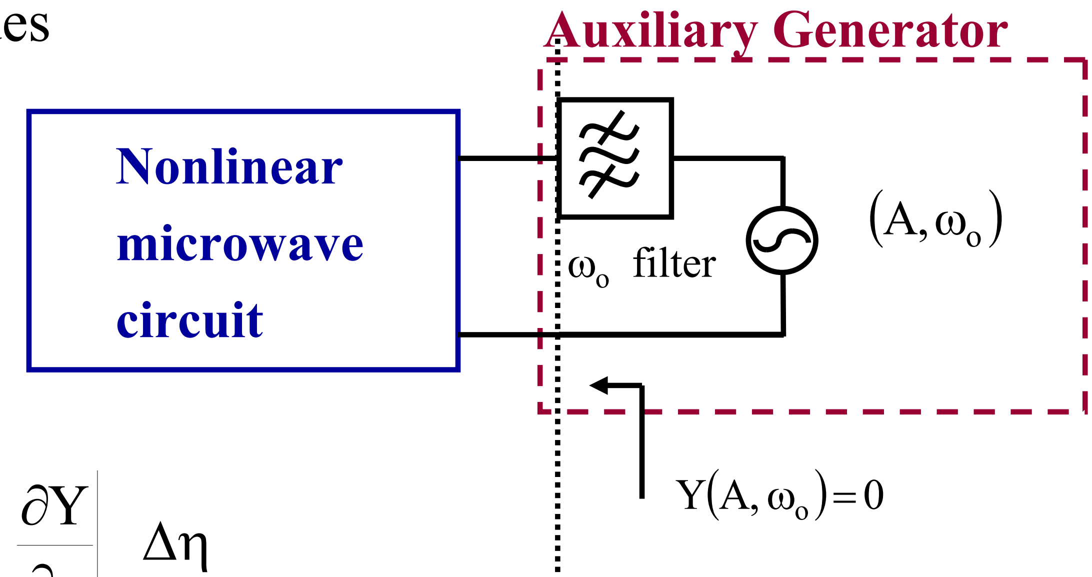
- Continuation techniques



$$\begin{bmatrix} A^{n+1} \\ \omega_o^{n+1} \end{bmatrix} = \begin{bmatrix} A^{n} \\ \omega_o^{n} \end{bmatrix} - [JY]_n^{-1} \left.\frac{\partial Y}{\partial \eta}\right|_n \Delta\eta$$

AG jacobian matrix:

$$[JY] = \begin{bmatrix} \frac{\partial Y_r}{\partial A} & \frac{\partial Y_r}{\partial \omega_o} \\ \frac{\partial Y_i}{\partial A} & \frac{\partial Y_i}{\partial \omega_o} \end{bmatrix}$$

HB
Inner tier

Comparison of increments $\{\Delta\eta,\ \Delta A,\ \Delta\omega_o\}$ $\longrightarrow$ Parameter switching

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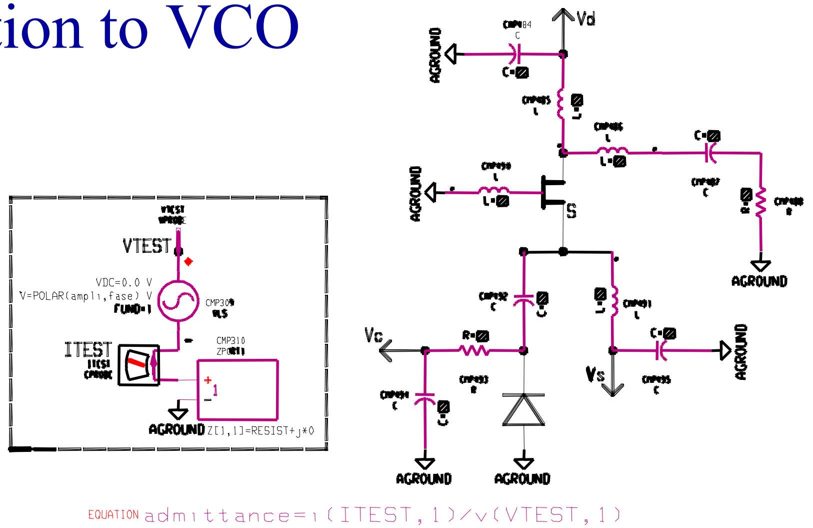
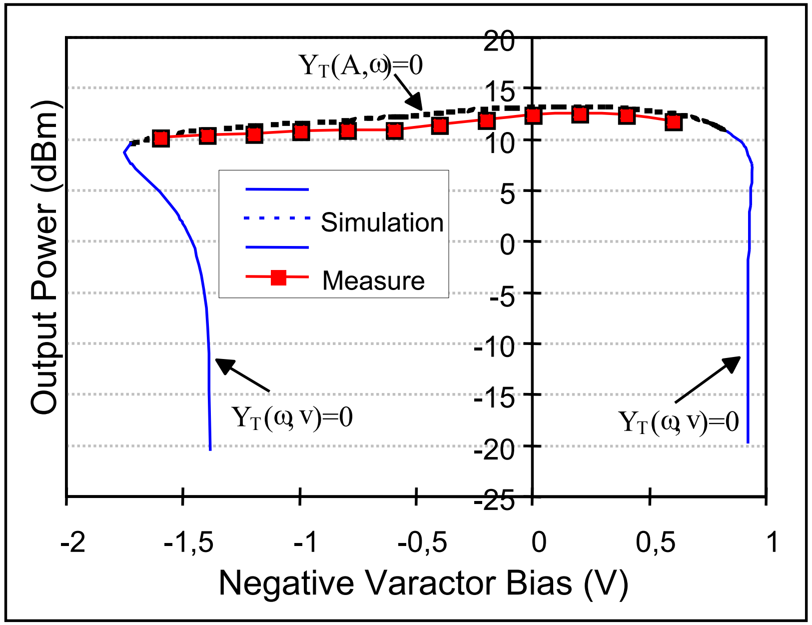
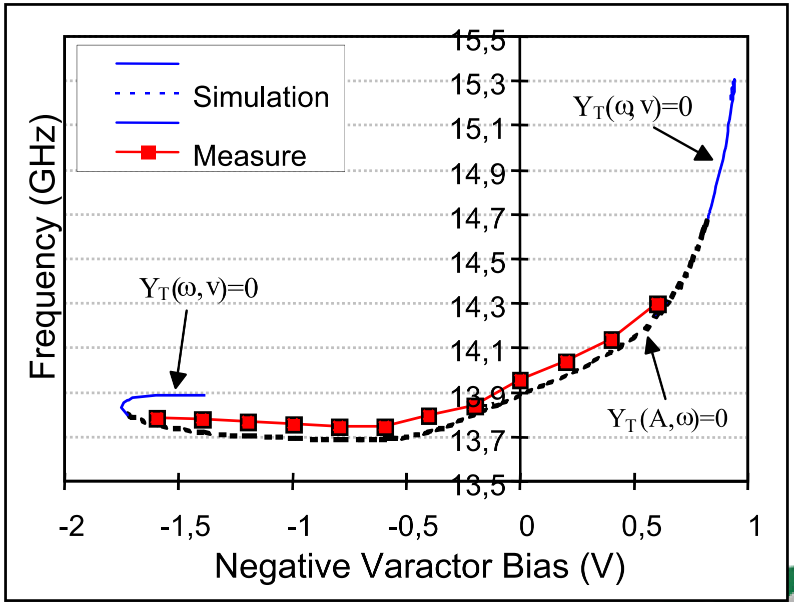
Continuation technique   Application to VCO

Y = 0   Optimization goal







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- Simple bifurcation conditions

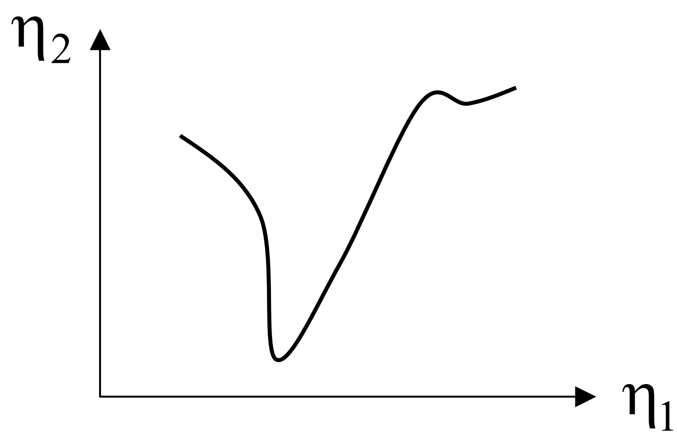
AG jacobian matrix: $$[JY] = \begin{bmatrix} \frac{\partial Y_r}{\partial A} & \frac{\partial Y_r}{\partial \omega_o} \\ \frac{\partial Y_i}{\partial A} & \frac{\partial Y_i}{\partial \omega_o} \end{bmatrix}$$

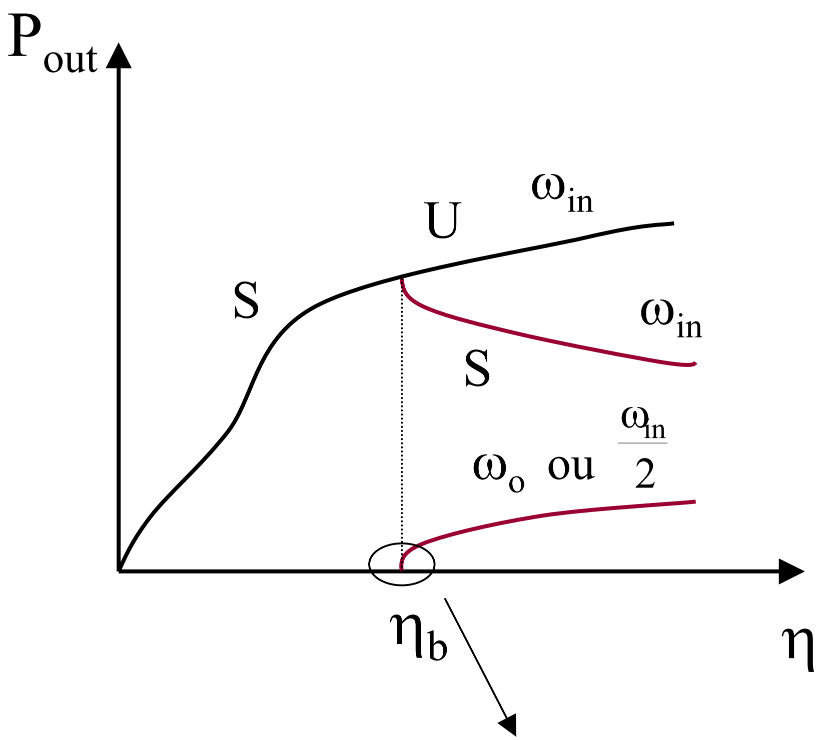
$$\begin{bmatrix} A^{n+1} \\ \omega_o^{n+1} \end{bmatrix} = \begin{bmatrix} A^{n} \\ \omega_o^{n} \end{bmatrix} - [JY]_n^{-1} \left.\frac{\partial Y}{\partial \eta}\right|_n \Delta\eta$$

Sadde-node bifurcation locus

$$\det[JY] = 0$$

$$\begin{cases} \det(A, \omega_o, \eta_1, \eta_2) = 0 \\ Y_r(A, \omega_o, \eta_1, \eta_2) = 0 \\ Y_i(A, \omega_o, \eta_1, \eta_2) = 0 \end{cases}$$





New branch starts at zero amplitude

Hopf-type bifurcation locus

$$A = \varepsilon \ , \quad \phi = 0$$

$$\begin{cases} Y_r(\omega_o, \eta_1, \eta_2) = 0 \\ Y_i(\omega_o, \eta_1, \eta_2) = 0 \end{cases}$$

Flip- type bifurcation locus

$$A = \varepsilon \quad , \quad \omega_o = \frac{\omega_g}{2}$$

$$\begin{cases} Y_r(\phi, \eta_1, \eta_2) = 0 \\ Y_i(\phi, \eta_1, \eta_2) = 0 \end{cases}$$

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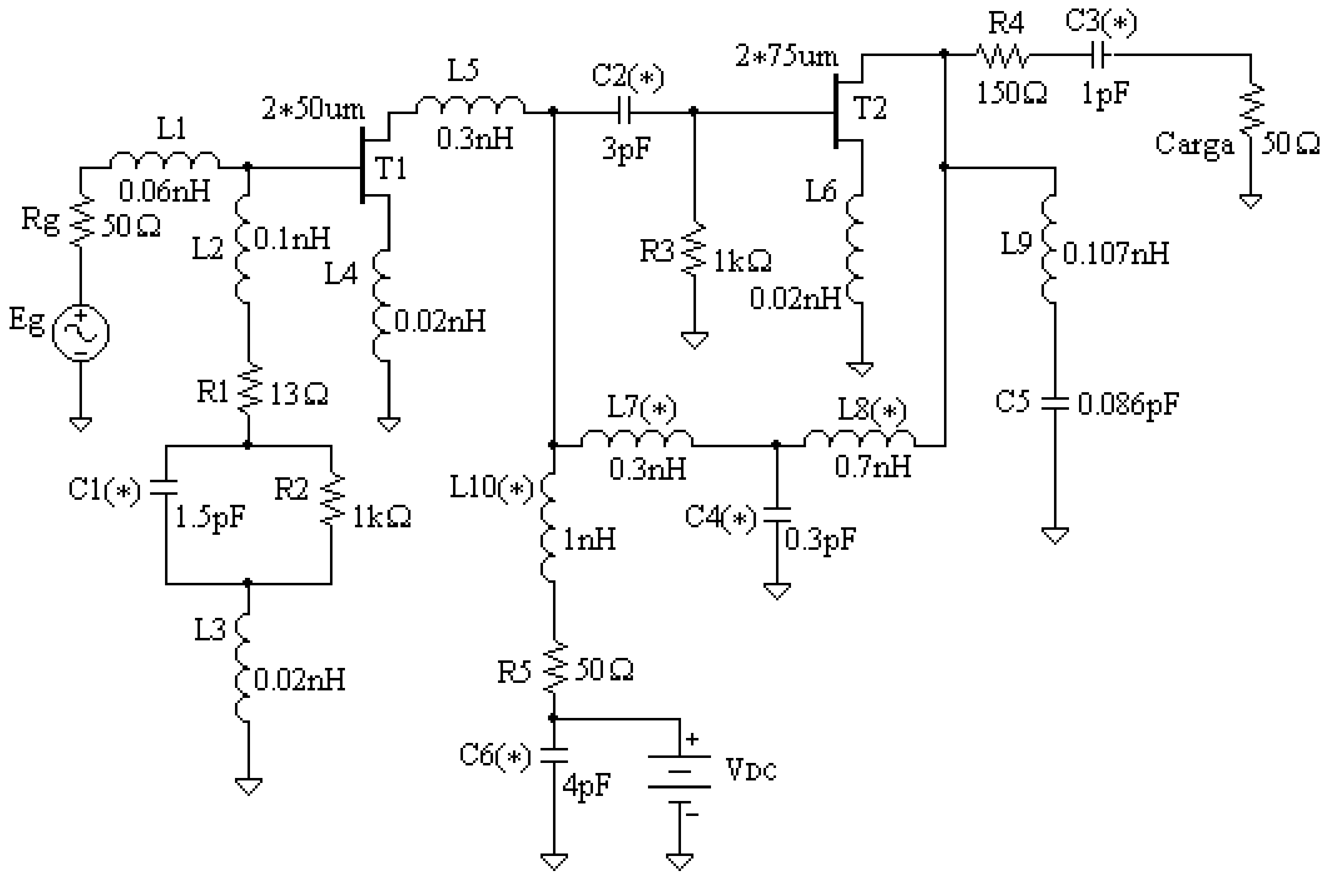
Bifurcation loci

Frequency divider 28-14 GHz



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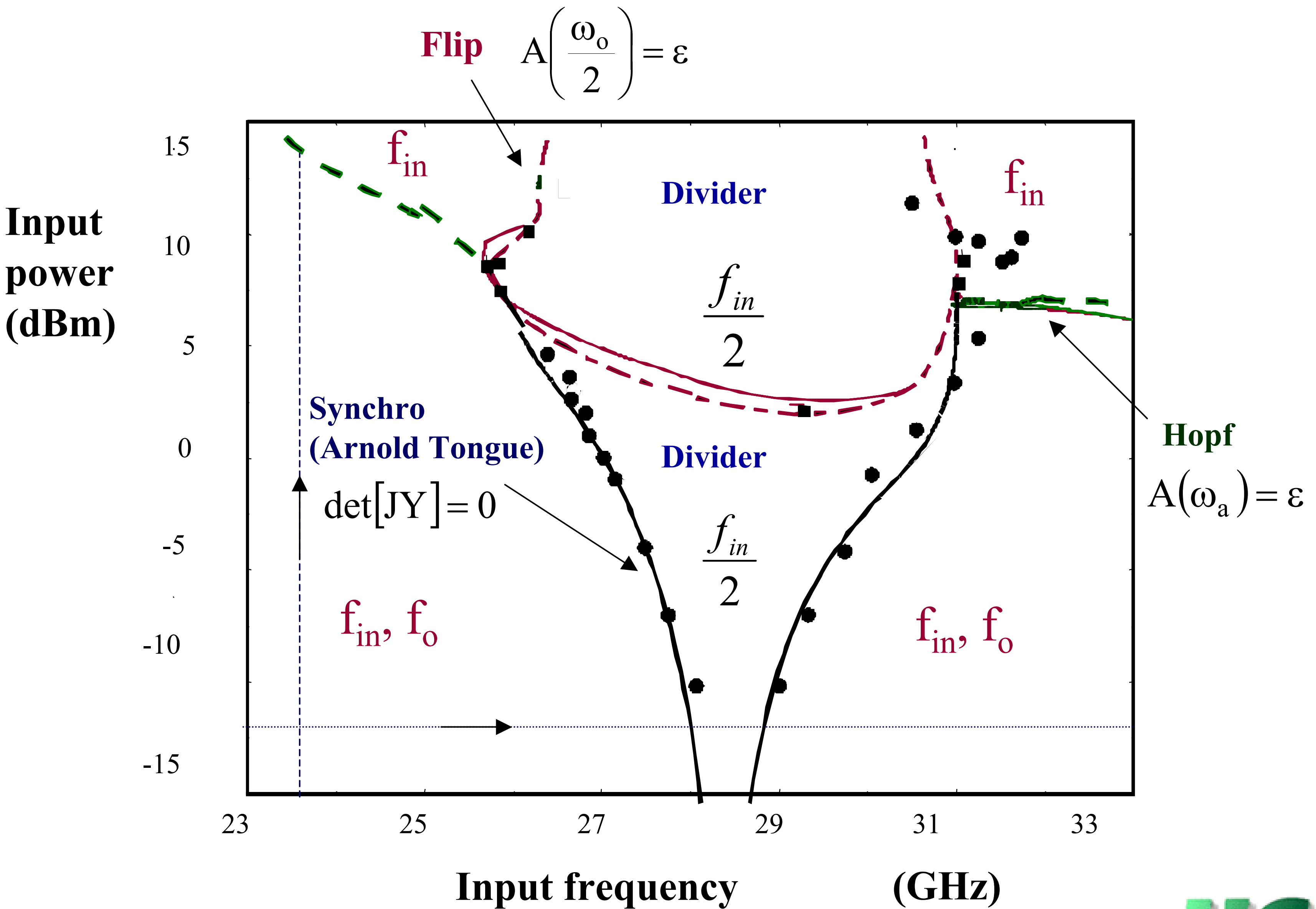

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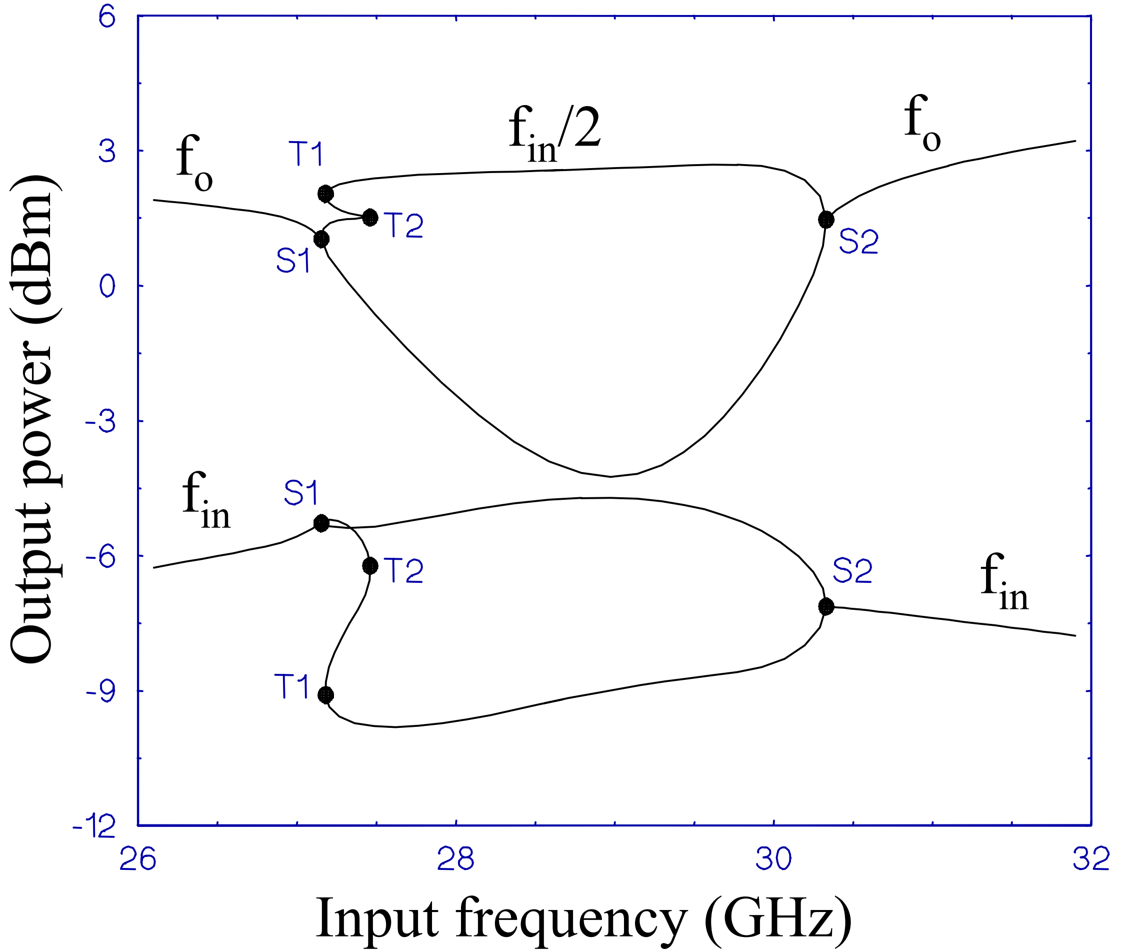
Bifurcation diagram

S : Synchronization

T : Turning point (jump)



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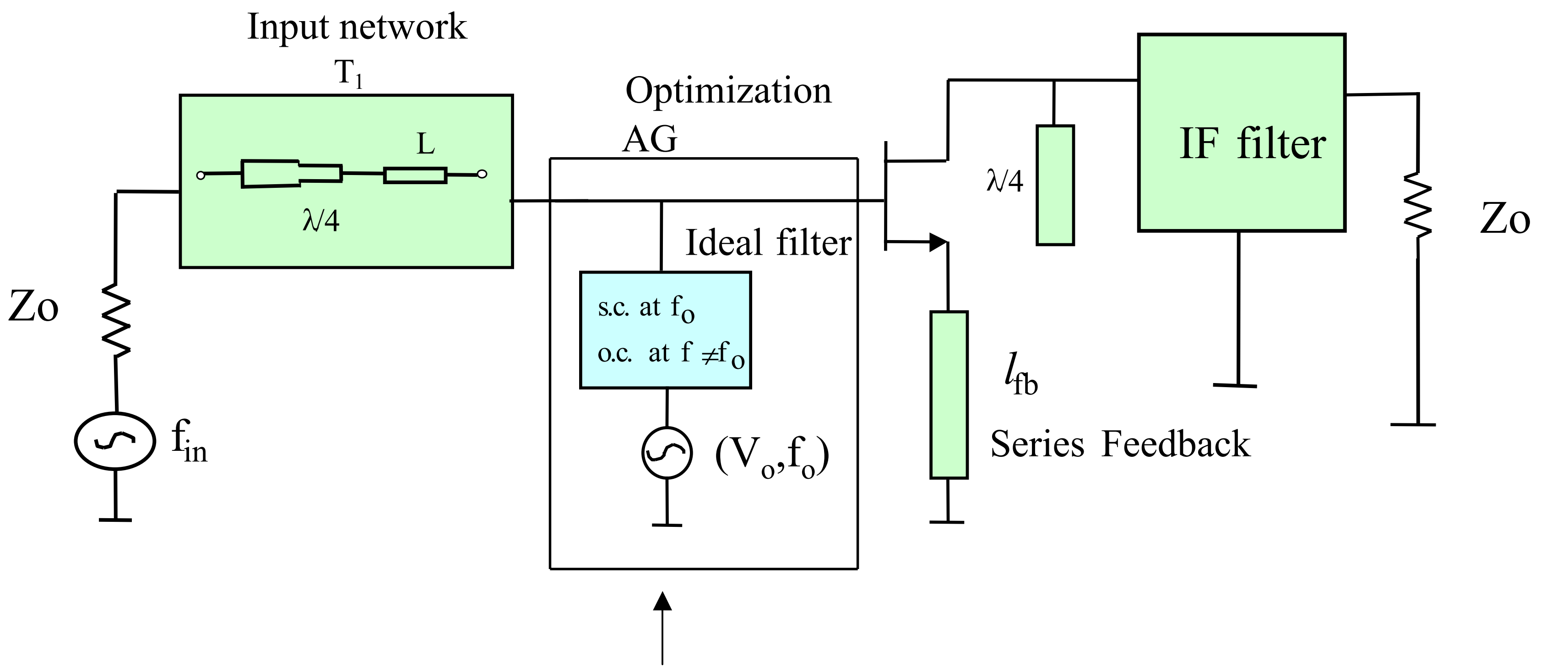
4 - NOVEL CIRCUIT DESIGN

4.1 - SELF-OSCILLATING MIXERS



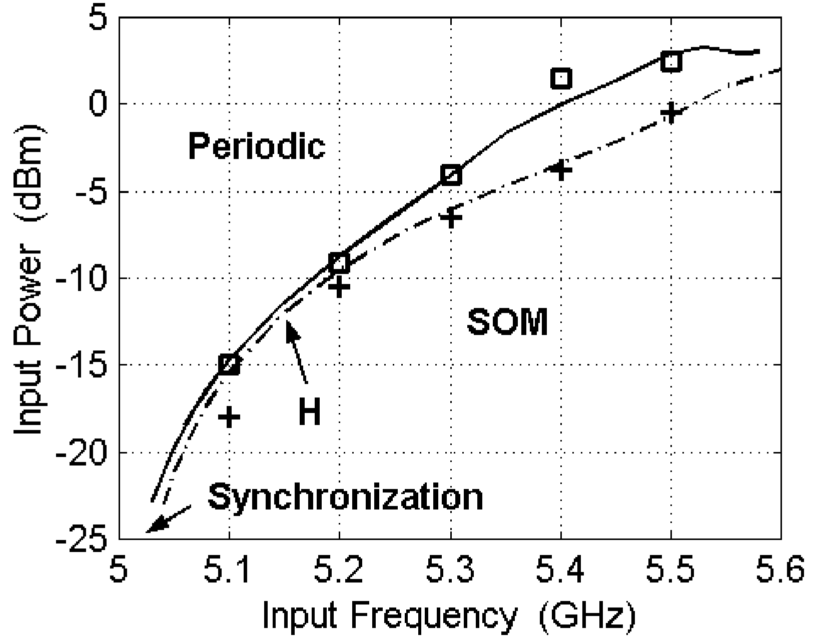
Fixed self-oscillation frequency and amplitude

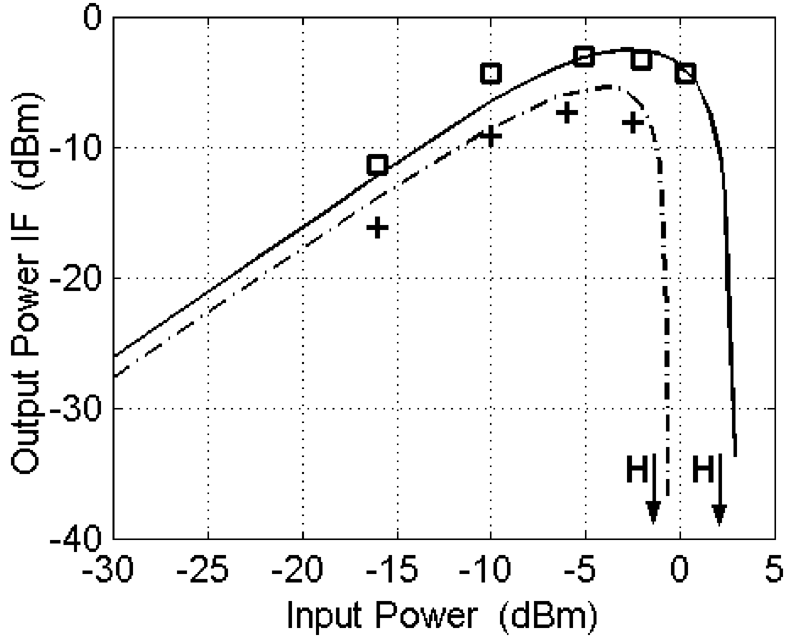
Optimization of line length and width

Shift of inverse-Hopf bifurcation

$Y_T(\gamma_1, \gamma_2) = 0$ with $V_o = 10^{-2}$ V, $f_o = 5$ GHz, $P_{in} = P^k_{inH}$ (Inverse Hopf)

$Y_T(\gamma_1', \gamma_2') = 0$ with $V_o = 0.8$ V, $f_o = 5$ GHz, $P_{in} = 0$ W (Free-running oscillator)



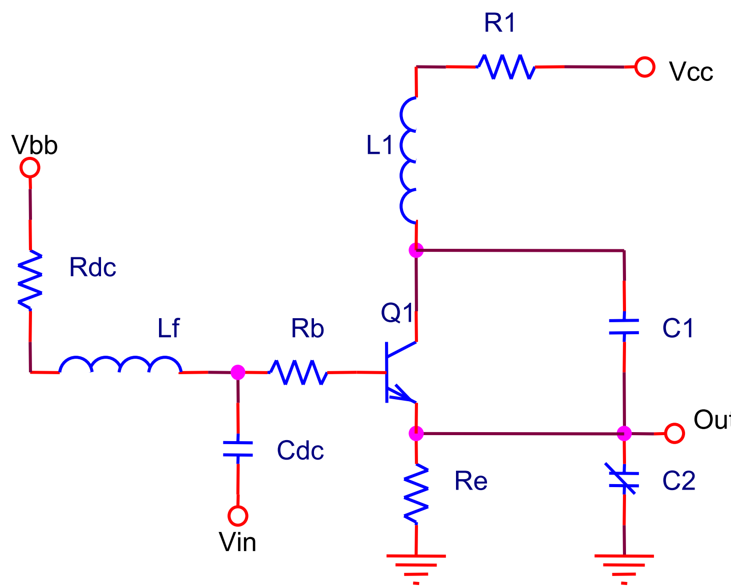


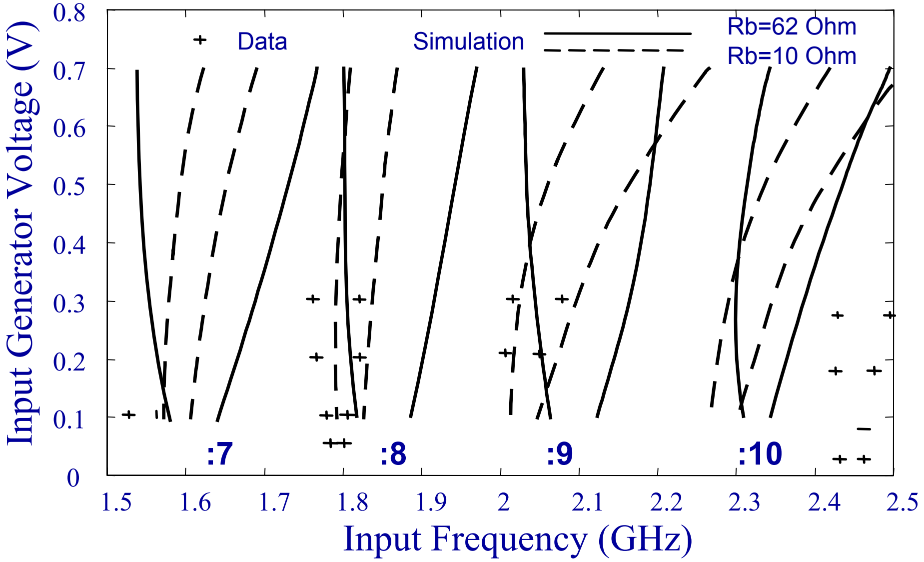
4.2 - ANALOG FREQUENCY DIVIDER BY VARIABLE ORDER 6 TO 9

Arnold tongues of very nonlinear oscillators

Period-adding routes to chaos

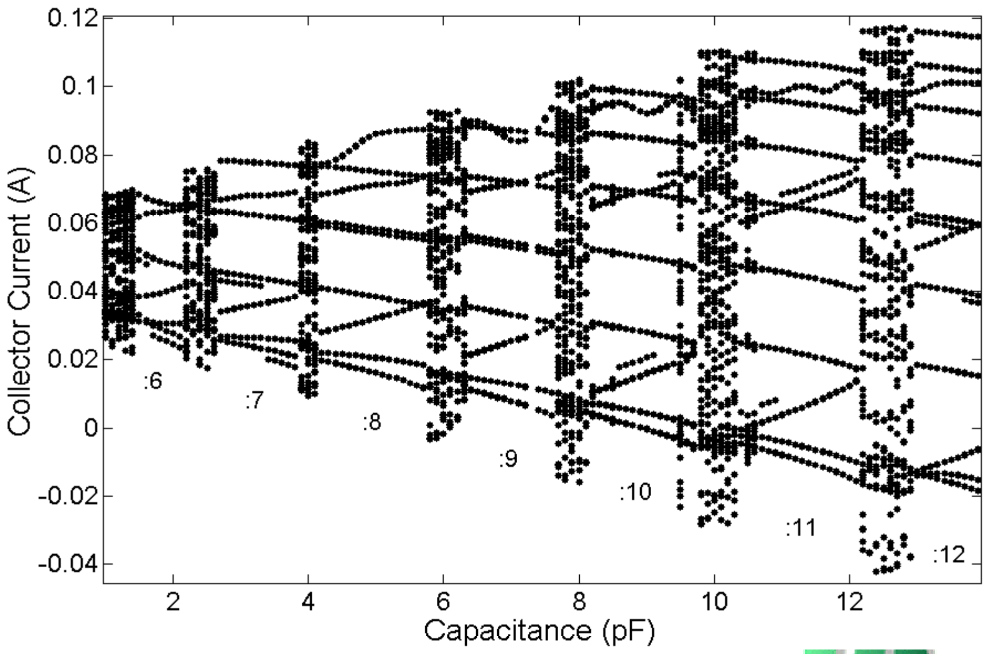
Colpitts-type oscillator in very nonlinear regime





Arnold tongues

Period-adding sequence



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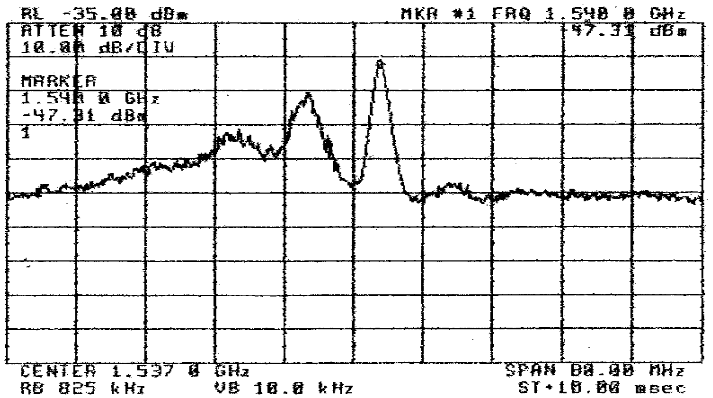
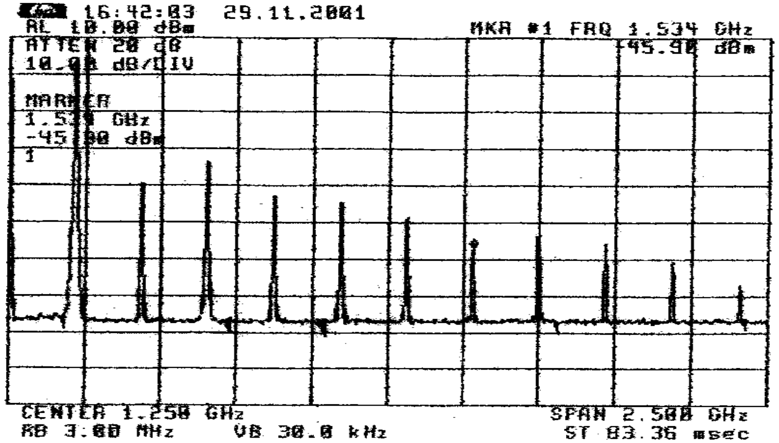
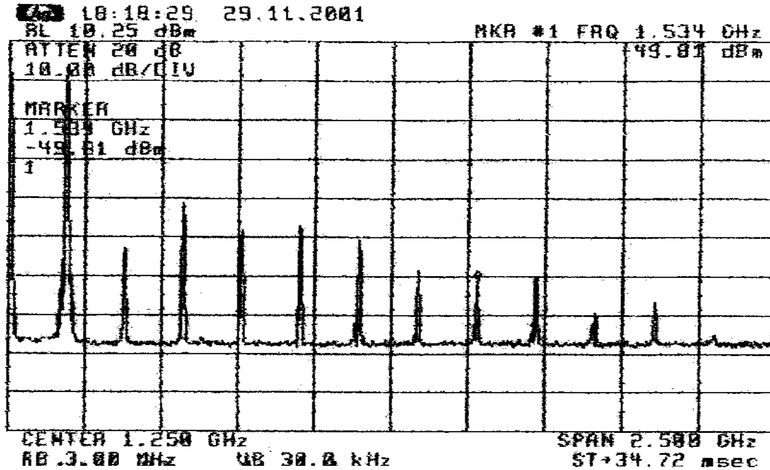
Experimental results



Near-synchronization spectrum



Division by 7, f_{in}= 1.534 GHz (C_1=2.9 pF)



Division by 8, f_{in}= 1.534 GHz (C_1=4.7 pF)

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Conclusions

Nonlinear microwave circuits ⟶ Coexistence of solutions
Bifurcations

Bifurcations delimit operation borders. They precede chaotic behavior

Continuation techniques ⟶ Easily combined with harmonic balance

Difficulty with autonomous and synchronized regimes:
solved through auxiliary generators

Auxiliary generators enable simple techniques for bifurcation detection

Novel circuit design:

- Fixing of oscillation point
- Fixing of bifurcation point (SOM)
- Use of Arnold tongues and period-adding routes to chaos

Variable-order frequency dividers

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