Structured Channel Estimation Methods for Cooperative Underwater Communication

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ABSTRACT

This paper examines structured methods to perform channel estimation for cooperative underwater acoustic communication networks. A simplified channel model based on a geometric ray-tracing model is proposed. For this new model, an iterative structured channel estimation method is developed. The optimal training sequence for the method is determined. The proposed method exploits the sparse nature of underwater acoustic channels and inter-channel relationships, providing performance improvements over unstructured methods in the modest to low SNR region and robustness above that of structured single channel estimators. The efficacy of the proposed method is evaluated via simulations and compared to Cramer-Rao bounds.

Categories and Subject Descriptors

C.3.4 [**Computer Systems Organization:Special Purpose and Application-Based Systems**]: Signal processing systems

General Terms

Algorithms

Keywords

cooperative systems, multipath channels, sparse channels, multi-channel equalization, underwater acoustic communication

1. INTRODUCTION

Cooperative communication for terrestrial sensor networks enabling power savings and improved fidelity is widely studied. Recent work has shown the gains of cooperative communication for underwater acoustic communications [1, 2] – these works presumed perfect channel state information at the destination.

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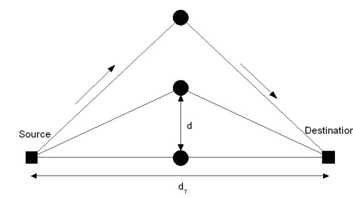

Figure 1: Topology for two-hop cooperative communications network with four cooperating nodes.

Given the promised gains of cooperative communications, we seek to develop pragmatic channel estimation methods to support such schemes. By making some topology assumptions, we exploit the ray-tracing model of [4] to develop an approximate multi-channel model between cooperating relays and the destination. In particular, we determine conditions under which the multipath *profile* is common between each cooperating node and the receiver (modulo a leading delay and associated attenuation). Our methods also exploit the sparseness of underwater acoustic channels. Methods for sparse channel estimation in underwater acoustic environments have been explored in [3, 5]. Our overall approach shares several key features with [5] (tap detection and least squares estimation); however, our model (multi-channel) differs and we explicitly exploit features of the permutation matrices which result from our modeling.

Our model ignores some subtleties regarding underwater acoustic propagation such as ray-bending, surface scattering and non-white noise – however, we numerically justify this model and establish the regimes in which it is accurate. We develop maximum-likelihood based profile estimators coupled with structure least-squares estimation. This paper subsumes [6], but provides the following additions: development of estimation algorithms of decaying, rather than uniform, multipath profiles; analysis of such methods; derivation of the optimal training sequence; comparisons with a structured single channel estimator; and practical algorithm approximations and simplifications.

This paper is organized as follows. In Section 2, the signal model for cooperative underwater acoustic communication is introduced. Section 3 reviews approximations supporting geometry-based multi-channel models. In Section 4, channel estimation algorithms are derived, followed by comments on performance bounds in Section 5. Lastly, simulation results are provided in Section 6.

2. SIGNAL MODEL

Our signal model is motivated by approximations made on the ray-tracing model for the multipath profile provided in [4]. We consider the topology depicted in Figure 1. A single source communicates to a set of cooperating relays, which then transmit to a common destination node. The channel estimation problem for the first hop is essentially a set of single channel estimation problems. This is in contrast to more classical multi-channel estimation for connected vertical arrays. We assume pulse matched filtering and sampling at the destination node. For simplicity of exposition, we assume that the maximal delay spread of the farthest node is known at the receiver. The discrete time vector equivalent signal corresponding to the channel output of a common single sequence from all cooperating nodes is given by,

$$\underline{\tilde{r}} = B\mathcal{C}\underline{h} + \underline{n}, \tag{1}$$

where B is the $M \times N_h$ lower triangular Toeplitz training data matrix, h $\underline{b}$ as the first column, and $\underline{b}$ is the $M \times 1$ common transmitted sequence from the K cooperating nodes (we assume BPSK), N_h is an upper bound on the overall channel delay spread and $M \geq N_h$ is the number of samples transmitted by each node. $\mathcal{C} = [D^{\tau_1}S_1, D^{\tau_2}S_2, \cdots, D^{\tau_K}S_K]$ where D^{τ} is a $N_h \times N_h$ downshifting matrix with shift τ and S_k is a $N_h \times N_h$ scaling selection matrix with the zero-padded $N_p \times 1$ multipath profile, $\underline{p}_k$, along its diagonal. To be specific,

$$p_{k,i} = \sum_{l=1}^{L} \frac{1}{\sqrt{2^{\gamma_l} A(\lambda_l)}} e^{-j2\pi f_c \tau_{k,l}} \delta(i - \lfloor \tau_{k,l} \rfloor + \lfloor \tau_{k,1} \rfloor)$$

where $A(\lambda_l) = \lambda_l^{1.5}\left[a(f_c)\right]^{\lambda_l}$, f_c is the carrier frequency and $\tau_{k,l}$ is the propagation delay of path l from the k'th user, calculated from the path length, λ_l,and the speed of sound. $\mathcal{C}$ parameterizes the channel structure, and thus contains the key channel profile parameters. In the sequel, we will state and justify the common multipath profile assumption. We assume, without loss of generality, that $1 < \tau_1 < \tau_2 < ... < \tau_K < N_h - N_p$, thus each delay is distinct and nodes are not synchronized. We use the convention that $D^0 = \mathbf{I}$. We assume fast fading, $\underline{h} = \left[\underline{h}_1^H, \underline{h}_2^H, \cdots, \underline{h}_K^H\right]^H$, where $\underline{h}_k$ is modeled as $\mathcal{CN}(0, \mathbf{I})$. The ambient channel noise is $\underline{n} \sim \mathcal{CN}(0, \sigma^2\mathbf{I})$ where $\underline{n}$ is an $M \times 1$ vector. Since B is comprised of the known training data, we consider the post-processed received signal

$$\underline{r} = B_{inv}\underline{\tilde{r}} = \mathcal{C}\underline{h} + B_{inv}\underline{n} \tag{2}$$

where B_{inv} is the left-inverse of B and $\underline{r}$ is of dimension N_h. This preprocessing facilitates the algorithm description for profile estimation.

3. MULTICHANNEL APPROXIMATION

The goal of this section is to show that for a given range of topologies, we can assume that (modulo leading delays and associated attenuation): $\underline{p}_1 \cong \underline{p}_2 \cong \cdots \cong \underline{p}_K \cong \underline{p}$, and therefore that we can let $S_1 = S_2 = \cdots = S_K$. Note that the amplitude and phase differences of cooperating nodes will be handled by the final tap value estimates. We exploit the highly sparse nature of the channels and the fact that the gain of a multipath component decays as a function of the number of reflections along the path and the length of the path. In our simulations, we normalize these gains such that the path gain associated with the smallest overall path length has unit magnitude. *Our signal-to-noise ratio (SNR) is the ratio of the power of the first arrival to the power of the noise.*

Given the topology of Figure 1 and assuming that the ocean floor is uniform across the cooperating links, then the only parameter varying the individual multipath profiles is

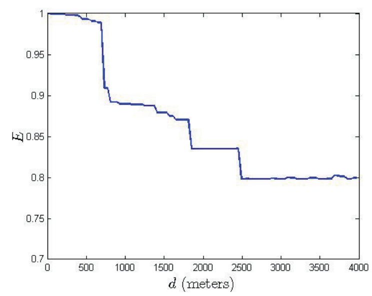

Figure 2: Energy capture for approximate model: $K = 3$**;** $f_c = 10$ **kHz; ocean depth 30 m; node depth 10m;, receiver-node array distance 4km;** $T_s = 22\mu$**s.**

the transmission distance. Figure 2 shows the normalized correlation between the true channel profile, $\underline{p}_k$ and the approximated common profile $\underline{p}_c$ ($\frac{1}{K}\sum_{k=1}^{K} \underline{p}_k^H \underline{p}_c / \|\underline{p}_k\|$). As can be seen from Figure 2, the common profile $\underline{p}_c$ is able to capture a significant amount of the overall energy, even with internode separations that are a significant fraction of the distance between the receiver and the horizontal array. Although not presented herein for brevity, we can bound the loss in energy capture due to the approximate model.

4. CHANNEL ESTIMATION

4.1 Multipath profile estimation

Given the channel model previously outlined, we define $\underline{c}$ as

$$\underline{c} \doteq \mathrm{diag}\left(\mathbf{E}\{\underline{r}\underline{r}^H\}\right) = \mathrm{diag}\left(\mathcal{C}\mathcal{C}^T + \sigma^2 B_{inv}B_{inv}^H\right) \tag{3}$$

$$= \left(\sum_{k=1}^{K} \tilde{D}^{\tau_k}\right)\underline{p} + \sigma^2 \mathrm{diag}(B_{inv}B_{inv}^H) \tag{4}$$

where $\tilde{D}^{\tau}$ is the $N_h \times N_p$ downshifting matrix of shift τ. With this signal model, we see that the noise depends on the transmitted sequence. To minimize the noise energy, we optimize the transmitted sequence such that

$$B_{inv}^{\star} = \arg\min_{B_{inv}} \left\{ \mathrm{Tr}\left(B_{inv}B_{inv}^H\right)\right\}. \tag{5}$$

Proposition 1: The training sequence optimizing Equation 5 is the constant sequence *i.e.* $b_i = 1$ (or -1).

For brevity the proof is omitted. This choice optimizes tap **detection** versus channel estimation. From here forward, we consider $B_{inv} = B_{inv}^{\star}$.

In practice, $\underline{c}$ will be approximated by a sample average,

$$\underline{\hat{c}} = \frac{1}{N}\sum_{j=1}^{N} \underbrace{\left[\mathcal{C}\underline{h}_j + B_{inv}\underline{n}_j\right]}_{\underline{r}_j} \odot \left[\mathcal{C}\underline{h}_j + B_{inv}\underline{n}_j\right]^*.$$

Since $\mathcal{C}$ is the horizontal concatenation of K shifted diagonal matrices, all having elements whose magnitude is bounded by unity, the i'th element of $\underline{\hat{c}}$ is a chi-square random variable with distribution

$$f_{\hat{c}_i}(x) = \left(\frac{N}{\tilde{\sigma}_i^2}\right)^N \frac{x^{N-1}}{(N-1)!} \exp\left\{-xN/\tilde{\sigma}_i^2\right\} \tag{6}$$

where $\tilde{\sigma}_i^2 = |\mathrm{row}_i(\mathcal{C})|^2 + 2\sigma^2$, $|\mathrm{row}_i(\mathcal{C})|^2 \in [0, K)$ and $B_{inv} = B_{inv}^{\star}$. [1] We will adopt the notation $\hat{c}_i \sim \chi_N(\tilde{\sigma}_i^2)$ to denote $\hat{c}_i$ having the above distribution.

[1] In reality, the i'th component of $\underline{\hat{c}}$ should use the i'th com-

Motivating our iterative multi-channel estimation scheme is the fact that we have two simple, but alternative characterizations of the vector $\underline{c}$,

$$\underline{c} = Q(\underline{a})\underline{p} + 2\sigma^2\underline{1} = R(\underline{p})\underline{a} + 2\sigma^2\underline{1},$$

where $Q(\underline{a}) = \sum_{k=1}^{K} \tilde{D}^{\tau_k}$, $R(\underline{p}) = \left[\tilde{D}^0\underline{p}, \tilde{D}^1\underline{p}, \cdots, \ \tilde{D}^{N_h - N_p}\underline{p}\right]$, and $a_i = \sum_{k=1}^{K} \delta(i - \tau_k)$. Our procedure will be to alternate between estimating $\underline{p}$ and $\underline{a}$ using the descriptions above.

To initialize the algorithm, first observe that nodes at the same depth cause surface-bottom and bottom-surface multipath to have the same arrival time. Additionally, if they are located at half the ocean depth, then the surface-surface and bottom-bottom multipath will have the same arrival time. Depending on the reflection loss, the average power of these combined multipath may be greater than the direct path. In our simulations, we assume a reflection loss of $\sqrt{2}$, so the weakest direct path is at worst the $3K$'th strongest path. We also make the safe assumption that these $3K$ strongest multipath exist in K disjoint clusters, one from each cooperating node. Therefore, we initialize the algorithm by finding the $3K$ strongest taps in $\underline{\hat{c}}$, classifying K clusters around them, and using the first tap of the k'th cluster as $\hat{\tau}_k$, *i.e.* the initial estimate for $\underline{a}$, *i.e.*

$$\hat{a}_\tau = 1, \quad \tau \in \{\hat{\tau}_k\}_{k=1}^{K} \quad \text{and} \quad 0 \quad \text{else}.$$

With $\underline{\hat{a}}$, the unconstrained least squares estimate of $\underline{p}$ is

$$\underline{p}' = \arg\min_{\underline{\tilde{p}}} \left\|\underline{\hat{c}} - Q(\underline{\hat{a}})\underline{\tilde{p}}\right\|^2 = \left[Q^T(\underline{\hat{a}})Q(\underline{\hat{a}})\right]^{-1} Q^T(\underline{\hat{a}})\underline{\hat{c}}.$$

The vector $\underline{\hat{c}}$ has colored noise, whose coloration is dependent on $\underline{a}$, which we are trying to estimate; thus we cannot perform whitening. The vector $\underline{p}$ is sparse where the number of non-zero components and associated magnitudes are unknown *a priori*. However, we assume knowledge of the noise statistics and can thus utilize a Neyman Pearson test to threshold $\underline{p}'$ to create the constrained estimate $\underline{\hat{p}}$. Let us assume that $|\tau_k - \tau_i| \geq N_p$ for all $i \neq k$; this is true for the topology in Figure 1 if $\sqrt{d_T^2 + d^2} - d_T \geq \nu N_p T_s$ where d is the internode spacing, ν is the speed of sound and T_s is the sample time. This simplifies $\underline{p}' = \frac{1}{K}Q^T(\underline{\hat{a}})\underline{\hat{c}}$. Given the form of $B_{inv}^\star$, it can be found that $p_i' \sim \frac{1}{K}\chi_{KN}(|p_i|^2 + 2\sigma^2)$, where p_i is the i'th component of $\underline{p}$. The noise-only case is found by setting $p_i = 0$ in the above. Given a target probability of false alarm, P_{FA}, one can numerically find the threshold Δ_p corresponding to this P_{FA}. One can then find $\underline{\hat{p}}$ via:$\hat{p}_i = p_i' u(p_i' - \Delta_p)$, where $u(t)$ is the unit-step function.

Having $\underline{\hat{p}}$, the unconstrained least squares estimate of $\underline{a}$ is

$$\underline{a}' = \arg\min_{\underline{\tilde{a}}} \left\|\underline{\hat{c}} - R(\underline{\hat{p}})\underline{\tilde{a}}\right\|^2 = \left[R^T(\underline{\hat{p}})R(\underline{\hat{p}})\right]^{-1} R^T(\underline{\hat{p}})\underline{\hat{c}}.$$

Note that $\underline{a}$ is a zero-one vector with exactly K ones. We can thus constrain $\underline{a}'$ by setting the K largest values of $\underline{a}'$ to one and set the remaining terms to zero. Therefore, the final estimate of $\underline{a}$ is

$$\hat{a}_\tau = 1, \quad \tau \in \{\hat{\tau}_k\}_{k=1}^{K} \quad \text{and} \quad 0 \quad \text{else},$$

where $\hat{\tau}_k$ are the indices of the K largest taps.

One can now iterate between $\underline{\hat{p}}$ and $\underline{\hat{a}}$. These parameters can then be used directly to form the estimate of $\mathcal{C}$, denoted $\hat{\mathcal{C}}$. The calculation of $\underline{a}'$ and $\underline{p}'$ can be burdensome due to the matrix inversions; we have practical low complexity approximations, the discussion of which is omitted for brevity.

ponent of $\sigma^2 [1 \quad 2 \quad 2 \cdots 2]^T$ instead of purely $2\sigma^2$. Since we assume that $1 < \tau_1$, the first component of $\underline{\hat{c}}$ is only noise, so we will ignore it.

4.2 Channel tap estimation

We consider: unstructured, structured multi-channel and structured single channel estimation. We first note that the channel output in (1) can be rewritten as

$$\underline{\tilde{r}} = B\sum_{k=1}^{K} \underline{h}_k^s + \underline{n} = B\underline{h}^s + \underline{n}$$

The unstructured least-squares estimate of $\underline{h}^s$ is given by $\underline{\hat{h}}^u = \arg\min_{\underline{\tilde{h}}} \left\{\|\underline{\tilde{r}} - B\underline{\tilde{h}}\|^2\right\} = B_{inv}\underline{\tilde{r}} = \underline{r}$. In contrast, if we know the multipath profile, we form a structured multi-channel estimate of $\underline{h}^s$. This is achieved by utilizing Equation (1) and performing the least-squares optimization

$$\underline{\hat{h}}^m = \hat{\mathcal{C}} \arg\min_{\underline{\tilde{h}}} \left\{\|\underline{\tilde{r}} - B\hat{\mathcal{C}}\underline{\tilde{h}}\|^2\right\} \tag{7}$$

$$= \hat{\mathcal{C}}\left[\hat{\mathcal{C}}^T B^T B\hat{\mathcal{C}}\right]^\dagger \hat{\mathcal{C}}^T B^T \underline{\tilde{r}} = B_{inv}\left[B\hat{\mathcal{C}}\right]^P \underline{\tilde{r}}, \tag{8}$$

where $[\cdot]^P$ denotes the projection into the column space of the argument and $\dagger$ corresponds to taking the pseudo-inverse. It can be observed that $B\hat{\mathcal{C}}$ is column-sparse. We form,$\underline{\psi}_l$, a zero-one vector that is one in the locations from the location of the l-th arrival to, but not including, the location of the $l+1$-th arrival. Since Ψ has mutually orthogonal columns, and it is column-space equivalent to $B\hat{\mathcal{C}}$, Equation (8) can be written as

$$\underline{\hat{h}}^m \;=\; B_{inv}[\Psi]^P\,\underline{\tilde{r}} = \sum_{l=1}^{KL}\left(\frac{\underline{\psi}_l^T\underline{\tilde{r}}}{\left\|\underline{\psi}_l\right\|} - \frac{\underline{\psi}_{l-1}^T\underline{\tilde{r}}}{\left\|\underline{\psi}_{l-1}\right\|}\right)\underline{e}_{\psi_l}$$

where $\underline{e}_{\psi_l}$ is a zero-one vector with a one in the first location that $\underline{\psi}_l$ is one and where $\underline{\psi}_0 \doteq \underline{0}$ and $\left\|\underline{\psi}_0\right\| \doteq 1$. Analogous to our multi-channel approach, we can form a sparse channel estimator which does not exploit the approximate multi-channel model, by forming a Neyman Pearson detector for each channel tap and then forming a structured single channel estimator. This results in: $\underline{\hat{h}}^s = B_{inv}\left[B\mathrm{diag}(\underline{\hat{s}})\right]^P \underline{\tilde{r}}$.

5. PERFORMANCE BOUNDS

Our performance metric of interest is the mean-squared error, $\mathbf{MSE} = \mathbf{E}\left\{\|\underline{\hat{h}}^x - \underline{h}^s\|^2\right\}$, where $x = \{u, s, m\}$ correspond to the unstructured, structured single channel, and structured multi-channel estimates, respectively. Given the respective channel models and deterministic deterministic multipath profile and sensor delays all estimator strategies are unbiased. It is also straightforward to calculate the Cramer-Rao bounds:

$$\mathbf{CRB}_U \;=\; \sigma^2 B_{inv}B_{inv}^H$$
$$\mathbf{CRB}_{S,M} \;=\; [I - H]\,\mathcal{C}\mathcal{C}^H[I - H] + \sigma^2 HB_{inv}B_{inv}^H H,$$

where, H is the zero-one diagonal matrix such that

$$[H]_{i,i} = u\left(\|\mathrm{row}_i(\mathcal{C})\|^2 - 2\sigma^2\right).$$

Due to the form of $B_{inv}^\star$, we then have $\mathrm{Tr}\left[\mathbf{CRB}_U\right] = 2N_h\sigma^2$. Due to the form of H, we have

$$\mathrm{diag}\left(\mathbf{CRB}_{S,M}\right) = H\mathrm{diag}\left(\mathbf{CRB}_U\right) + \mathrm{r.c.e},$$

where r.c.e. denotes residual channel energy, *i.e.* the channel energy which exists beneath the noise floor. We can show that $\mathrm{Tr}\left[\mathbf{CRB}_{S,M}\right] \leq 2Kw(\underline{p})\sigma^2$. Given the sparsity of $\underline{p}$ and the typical small values of K, $Kw(\underline{p}) \ll N_h$. Therefore, significant gains are achievable over the unstructured estimator using the either structured estimator.

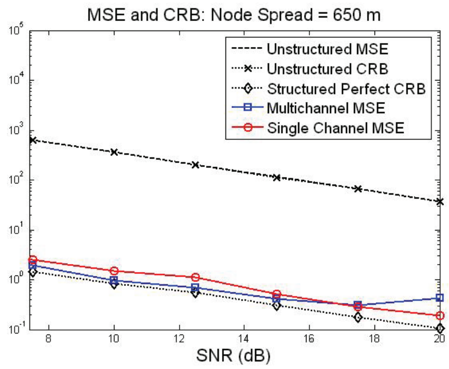

Figure 3: MSE and CRB for structured and unstructured channel estimators for matched topology.

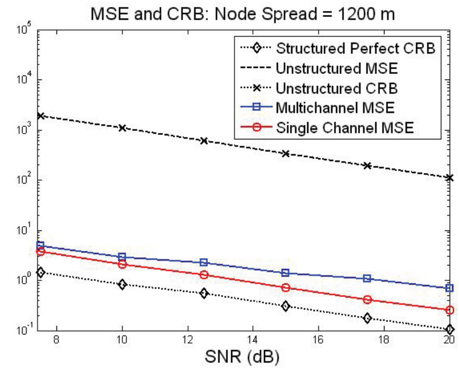

Figure 4: MSE and CRB for structured and unstructured channel estimators for mismatched topology.

6. SIMULATION RESULTS

Simulations were conducted to analyze the relative performances of the three estimators studied. In these simulations, the following parameter values were used: ocean depth of 30 meters, node depth of 20 meters, speed of sound of 1500 m/s, carrier frequency of 10kHz and the topology of Figure 1 with the horizontal array of 3 nodes located 4km from the receiving node. 15 realizations are used to obtain our sample averaged $\underline{\hat{c}}$. In the simulations, we consider the cooperating nodes equally spaced over a total distance of 650 and 1200 meters. A target false alarm probability of 0.01 is used with the structured estimators to obtain the corresponding thresholds, Δ_c and Δ_p, for the single channel and multi-channel estimators, respectively.

We can first observe that for the 650 meter node spread (see Figure 3), the multi-channel estimator outperforms the single channel estimator, which outperforms the unstructured estimator at low to moderate SNR – gains on the order of 2dB. The reason for this performance difference is that the multi-channel estimator only needs to detect multipath arrivals over the delay spread of the *individual* channels, which is of length N_p, whereas the single channel estimator needs to detect multipath arrivals over the delay spread of the *overall* channel, which is of length N_h. The ratio N_h/KN_p is in general quite large, on the order of tens to thousands, for low SNR. Therefore, the expected number of false alarms for the single channel estimator can be much larger than that for the multi-channel estimator. At high SNR, the noise floor is decreasing, which allows weaker multipath arrivals to be resolvable. This has two effects: (1) N_p increases since these multipath have a longer relative delay, causing the expected number of false alarms for the two structured estimation methods are essentially the same and (2) our common multipath profile model breaks down. This breakdown of our common multipath profile model causes noise-only taps to be treated as channel-plus-noise taps, which degrades the performance of the multi-channel estimator. Since the single channel estimator is unaffected by profile structure, it is able to detect the overall channel structure with greater accuracy in the high SNR regime.

In the second set of simulations, we use the same parameters as the first simulation, but with the cooperating nodes equally spaced over a total distance of 1200 meters. We expect poorer performance for the multi-channel approach as we have a topology that violates the assumptions underlying our approximate model – as seen in Figure 4 The multi-channel estimator is no longer able to achieve its CRB at low SNR. This is because our common multipath profile model has broken down for even the most dominant taps.

7. CONCLUSIONS

In this paper, an investigation of structured channel estimation schemes for a cooperative underwater acoustic link is conducted. A simple iterative multi-channel estimation scheme which exploits the sparseness and similarity of channels from cooperating nodes is derived, as well as a simple single channel estimation scheme. It is demonstrated via simulations and performance bounds that large performance gains can be achieved using these structured LS estimates over an unstructured LS estimate. Although not shown, we also have robustness to the noise variance estimate with the structured LS multi-channel estimator versus the structured LS single channel estimator. Practical operating regimes are found for the two structured estimators.

8. REFERENCES

[1] C. Carbonelli, S.-H. Chen, and U. Mitra "Error Propagation Analysis for Underwater Cooperative Multihop Communications," *Elsevier Journal on Ad Hoc Networks*, invited submission, to appear 2008.

[2] M. Vajapeyam, S. Vedantam, U. Mitra, J. Preisig, and M. Stojanovic, "Distributed Space-Time Cooperative Schemes for Underwater Acoustic Communications," *IEEE JOE*, accepted October 2007.

[3] W. Li and J. Preisig, "Estimation of Rapidly Time-Varying Sparse Channels", *IEEE JOE*, vol.32 (4), Oct '07, pp.927-939.

[4] A. Zielinski, Y-H. Yoon and L. Wu, "Performance Analysis of Digital Acoustic Communication in a Shallow Water Channel," *IEEE JOE*, 20(4), pp. 293-299, October 1995.

[5] T. Eggen, J. Preisig and A. Baggeroer, "Communication over Doppler Spread Channels - II: Receiver Characterization and Practical Results", *IEEE JOE*, vol.26 (4), Oct '01, pp.612-621.

[6] N. Richard and U. Mitra, "Sparse Channel Estimation for Cooperative Underwater Communications: A Structured Multichannel Approach," *IEEE ICASSP*, April '08, invited.