Following the standard Aloha analysis, consider a completely connected network of infinite users where aggregate traffic follows a Poisson distribution with mean offered load $G = g/p_l$. where g is the packets transmitted per second and p_l is the transmit time of each packet in seconds.

Nodes use a pure aloha access scheme where any node with a packet to send will immediately transmit the packet. For the case where any overlap of packets is considered a collision, the event of a successful packet becomes the event that only one packet is delivered over a time period of $2p_l$, so the probability of a successful transmission is then

$$P\{Success\} = e^{2G}$$

from the definition of a poisson distribution where the number of events per 'unit time' is 0. The probability of transmitting a packet is G, so the average throughput, S, is given by

$$S = Ge^{2G}$$

Which yields the standard result of maximum throughput of 0.184 at G=0.5.

If nodes are using frequency hopping, all nodes are using the same hopping pattern, and nodes are not capable of receiving multiple simultaneous packets, the assumption of a collision occuring from any overlap is invalid. Two packets may overlap for a significant period of time without ever overlapping in frequency. The contention times for a transmitted packet in this case will still be stretched over a time period of $2p_l$, but now in spread out intervals as illustrated in Figure 1. Looking at the packet transmission over a single frequency, it is apparent that the contention window is now several intervals. If a potentially interferring transmission occurs such that it begins its hopping pattern while the initial transmission is transmitting on a different frequency, the two packets will never collide as they are using the same hopping pattern.

The length of time spent transmtting on each frequency before hopping is denoted t_{dwell}. The number of times cycled through each frequency per packet is n_{cyc}, and the average delay spread of the arriving signal at a receiver is t_{del}. The contention time for each signal, t_{scon}, is give by

$$t_{scon} = 2t_{dwell} + t_{del}$$

All of the contention intervals summed up, t_{con} is

$$\begin{aligned} t_{con} &= 2n_{cyc}(t_{scon}) \\ &= 2n_{cyc}(2t_{dwell} + t_{del}) \end{aligned}$$

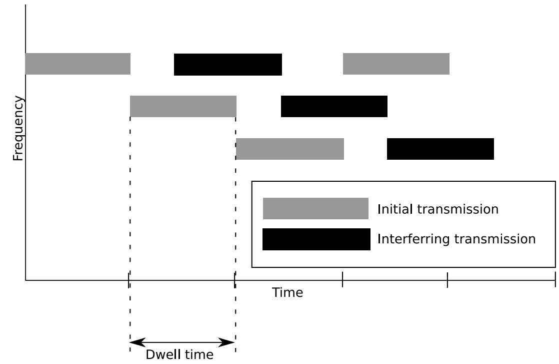

Fig. 1. Frequency Vs. Time

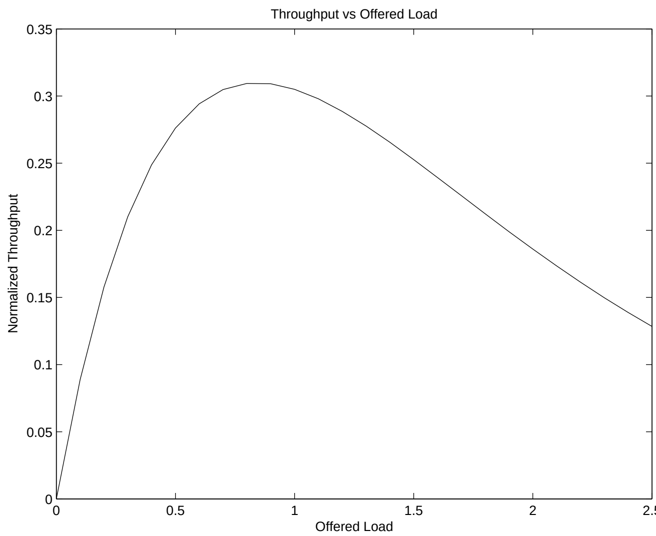

Fig. 2. Throughput vs Offered Load for Micromodem Aloha

TABLE I

PARAMETERS USED IN WHOI MICROMODEM

Paramater	Value
# Bins	13
Dwell time (ms)	12.5
Delay spread (ms)	70

Because of the memoryless property of the Poisson distribution, we can assume the probability of transmitting in any one of the intervals is the same as transmitting in one long interval with length equivalent to the sum of the lengths of all of the shorter intervals. The probability of a transmission being received successfully then becomes

$$P\{Success\} = e^{2Gn_{cyc}(2t_{dwell}+t_{del})/p_l}$$

and the throughput

$$S = Ge^{2Gn_{cyc}(2t_{dwell}+t_{del})/p_l}$$

The theoretical throughput using the parameters given in Table I are shown in Figure 2. Simulation output

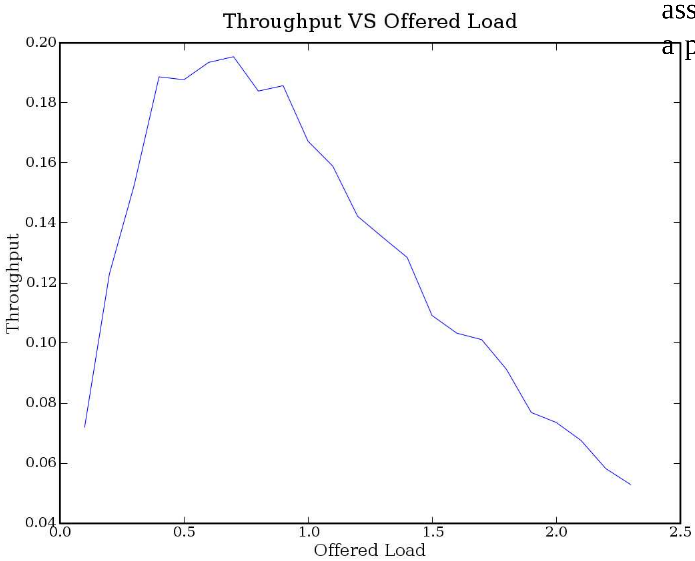

Fig. 3. Baseline Aloha. Bins=1, t_{sym}=0

!t]

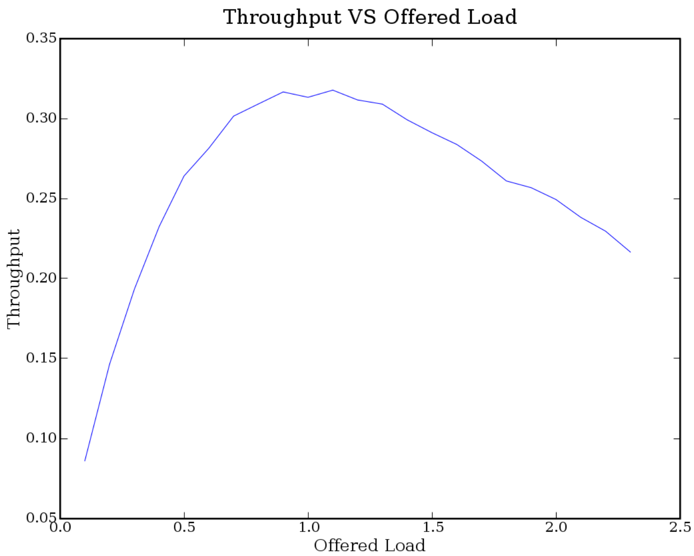

Fig. 4. NS UAN FH-FSK Simulation output

using the NS2 UAN module with the same parameters are shown in Fig. 4. The throughput results are slightly higher in simulation do to laxed collision requirements, i.e. It is possible for two symbol intervals to overlap in time and frequency but have SINR such that the originially acquired transmission can still be received. For contrast, the baseline throughput seen with a single center frequency (a simple FSK) network is shown in Fig. 3. Again in the non frequency hopping case, a slight improvement in throughput is seen over the previously mentioned theoretical maximum throughput due to the assumption that a packet is receivable until it falls below a predefined SINR threshold.