Collaborative Downloading in VANET using Network Coding

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Abstract—**Data downloading on the fly is the base of commercial data services in vehicular networks, such as office-on-wheels and entertainment-on-wheels. Due to the sparse special distribution of roadside Base Stations (BS) along the road, downloading through Roadside-to-Vehicle (R2V) connections is intermittent. When multiple vehicles with geographical proximity have a common interest in certain objects to download, they can collaborate to significantly reduce their overall download time. In this paper, we investigate the application of Network Coding (NC) in collaborative downloading (CD). We focus on the R2V part of CD, and analytically derive probability distribution and the expected value of the amount of time necessary to deliver all of the information to the vehicles with and without NC. Our results show that using NC slightly improves the downloading time in addition to removing the need for having any sort of uplink communications from vehicles to the infrastructure.**

I. INTRODUCTION

Recent development and standardization in vehicular ad hoc networks (VANETs) [1] have motivated the increasing interest in data services for in-vehicle consumption, such as 'commerce and entertainment-on-the-wheel' [2]. Hence, in the near future, the number of vehicles equipped with wireless communication devices is poised to increase dramatically, i.e., we are moving closer to an intelligent transportation system. Such a system would provide a wide variety of applications: local information pushed to vehicles (e.g., traffic notification, map updates, location-based advertisements); or specific data pulled from Internet servers (e.g., neighborhood parking, reviews of local restaurants, and video clips of local attractions) [3].

In current intelligent transportation systems, services are provided to vehicles using existing wide-area cellular infrastructure (3G/4G) and/or the roadside infrastructure based on Dedicated Short Range Communications (DSRC) links[1] proposed for VANET networks [2]. However, both of these approaches have their own challenges: the modest data rate of present 3G links and the associated high cost of data downloading on one hand, and the intermittent hotspot-type roadside coverage envisaged with DSRC on the other [4].

The above challenges lead to the following simple premise for content dissemination, called collaborative downloading: if the content available to a subset of vehicles is also desired by (many) others in the network, peer-to-peer content distribution using vehicle-to-vehicle (V2V) ad hoc communications is time and cost efficient.

Collaborative downloading is a data dissemination protocol for distributing information among all nodes inside the network [5], [6], and it has attracted a lot of attention in the VANET community in the past few years [7]. In general, data dissemination in VANET networks consists of two phases [8][9]:

1) Roadside-to-Vehicle (R2V) phase: in this phase, vehicles are communicating with a base station (located in specific location on the road) to receive data.
2) Vehicle-to-Vehicle (V2V) phase: when the vehicles are out of the coverage of the BS, they try to exchange information between each other. If this phase is completed, all the nodes have the same data.

To better understand the concept of collaborative downloading, consider the scenario in Figure 1 in which a group of vehicles equipped with DSRC radios are connected to the Internet via roadside DSRC base stations. Assume that these vehicles have a common interest in a large file on the Internet. The intermittency arises from the sparsity of the roadside DSRC base-stations and the hotspot nature of coverage. As a result, data is downloaded during the (short) intervals of radio connectivity from the infrastructure to the vehicle. Thus each vehicle in a platoon only obtains a fraction of the overall file in each contact duration. In principle, each vehicle could download the whole file through multiple contacts, requiring significant latency due the sparsity of the contacts. A more effective alternative is for the vehicles to form a coalition for sharing their respective pieces after each round of contacts (collaborative downloading) when they are out of coverage (V2V phase). Our focus in this paper is on R2V phase. We assume a perfect V2V transmission, i.e., data dissemination is complete after V2V phase. In other words, we assume that the vehicles have the same information after V2V phase is over.

In the state-of-the-art algorithm for collaborative downloading, nodes need to communicate with the BS to let it know which packets they are interested in. That means a partial time of vehicle to BS connection (in R2V phase) has to be assigned to signaling from the vehicles to the BS[2]. Moreover, this scheme requires a lot of synchronization and handshaking

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[1]DSRC in North America is based on 802.11p wireless link access in 5.9 GHz band, subsequently folded into the IEEE 1609 WAVE standards.

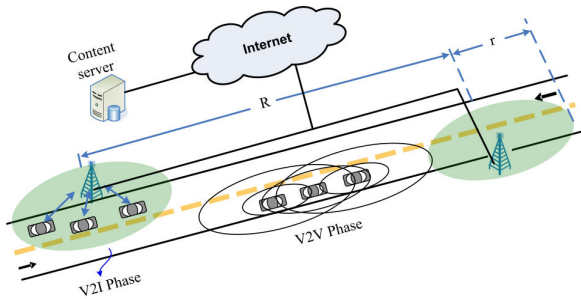[2]R2V links are half-duplex, i.e., the base station can not receive and transmit simultaneously.

Fig. 1. System diagram of collaborative downloading among vehicles on the road.

Fig. 2. In each transmission, BS sends a linear combination of the packets to a vehicle and this transmission can not be seen by any other vehicles.

between the BS and the receiving vehicles. In this paper, we propose to use network coding [10], [11], [12] in downloading data from the infrastructure. Our algorithm omits any kind of signaling from vehicles to the BS. Moreover, we analytically show that the expected time needed to disseminate data from infrastructure to all vehicles is slightly less by using NC. In other words, NC slightly decreases the downloading delay in addition to removing any need for uplink communication.

Network coding has recently been applied to many data dissemination problems in wireless networks. In [13], authors use a gossip-based algorithm (called rumor-spreading) to diffuse information in an ad-hoc network. Their work is continued by [14] who study the performance degradation due to actual MAC schemes.

The rest of the paper is organized as follows: Section II present our system model for collaborative downloading. We derive dissemination delay when NC is not exploited in Section III. Average amount of time needed to deliver information to all nodes in the network with use of NC is derived in Section IV. Our evaluation result is given in Section V. The paper concludes with reflections on future works in Section VI.

Notations: We use bold capitals (e.g. $\mathbf{A}$) to represent matrices and bold lowercase symbols (e.g. $\mathbf{m}$) for vectors. The i-th entry of a vector $\mathbf{m}$ is denoted by m_i and Superscript ^{T} denotes matrix transpose. We use calligraphic capitalized symbols (e.g. $\mathcal{T}$) for random variables.

II. ROADSIDE-TO-VEHICLE PHASE:SYSTEM MODEL

Consider a platoon of N vehicles interested in a certain file comprising of M packets $x_1, x_2, \ldots x_M$. Suppose the network infrastructure (i.e., all the roadside base stations) possesses that common file. The goal is to distribute this file to all vehicles in the network. This is achieved by cycling through a succession of R2V+V2V phases. During each R2V phase, we assume that any of the N vehicles downloads a constant number of $m \ll M$ packets, i.e., the duration and rate of download per contact are constant and independent of the vehicle. For security assurance, we assume that communications between the BS and each vehicle is encrypted. In other words, the BS communicates with only one vehicle at a time and the other nodes can not see the communication link between them. That means, in collaborative downloading sharing is limited to V2V
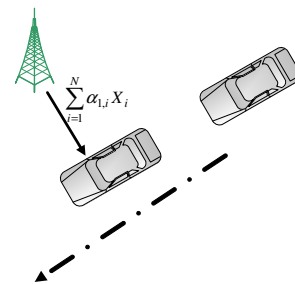
phase and we do not use the advantage of broadcasting in R2V phase for the sake of security.

Obviously, the way the m packets are chosen affects the system performance. We consider the following two ways of selecting m packets in each round:

- *Feedback-based scheme*: The m packets are chosen randomly by the serving BS among the *currently unreceived packets* of the vehicle. It is assumed that in each round, nodes can individually signal to the server the specific indices of packets yet to be received.
- *Network coding aided scheme*: In this scheme the BS uses Random Linear Network Coding (RLNC). In each transmission, it sends a linear combination of the M packets, where the combining coefficients are uniformly chosen over the finite field $\mathbb{F}_{2^q}$. In this scenario no feedback is needed from vehicles to the BS.

When the last vehicle leaves the BS coverage area, all nodes are in V2V phase. In this phase, every node tries to share its information with other nodes inside the network. After V2V phase is complete, all vehicles have the same information. When the vehicles enter the range of the next BS, they try to obtain the remaining missing packets. This continues until every node has the full message of M packets. We call each R2V and its following V2V phase a *round*. The number of rounds required to send the M packets from the infrastructure to the vehicles is the matter of interest. In this paper, we aim to derive probability distribution and expected value of the number of rounds needed to disseminate information to all N vehicles in the network, using either the feedback-based or NC-aided scheme.

For the sake of simplicity, we only consider the case where there are two vehicles in the network, i.e., $N = 2$. Generalizing our result to arbitrary N is considered as one of our future works. For each scheme, we first analyze a simple case of $m = 1$, transmitting only one new packet in each R2V phase of a vehicle. Then, we generalize our result to arbitrary m.

III. FEEDBACK-BASED SCHEME:

Suppose there are $N = 2$ vehicles in the network. As mentioned before, in each R2V phase, a vehicle downloads m packets from the BS. In the following V2V phase, vehicles exchange their uncommon packets. In the i-th round, let $\mathcal{X}_i$ denote the number of *common* packets among the m

downloaded packets by each node. Clearly, $\mathcal{X}_i$ is a random variable in the set $\{0, 1, \ldots, m\}$. Further, let define $\mathcal{S}_i$ as number of packets each node has at the end of i-th round. Clearly, at the end of each round (R2V+V2V) $2m - \mathcal{X}_i$ new packets are added to each node. Thus, It is easy to see the following recursive equation for $\mathcal{S}_i$:

$$\mathcal{S}_i = \mathcal{S}_{i-1} + 2m - \mathcal{X}_i. \tag{1}$$

The dissemination algorithm stops when both cars have all the M packets, i.e., the whole information. Let $\mathcal{T}$ denote stopping time defined as follows:

$$\mathcal{T} = \min_t \{\mathcal{S}_t = M, t \geq 0\}. \tag{2}$$

In this section we aim to calculate probability distribution and expected value of the stopping time for the feedback-based dissemination algorithm.

A. $m = 1$, $N = 2$

lemma 1. *The number of common packets at the i-th round given $\mathcal{S}_{i-1} = s$ has a probability distribution given by*

$$\begin{aligned} P(\mathcal{X}_i = x | \mathcal{S}_{-1} = s_i) &= \frac{\binom{1}{x}\binom{M-s_i-1}{1-x}}{\binom{M-s_i}{1}} \tag{3} \\ &= \frac{\binom{M-s_i-1}{1-x}}{M - s_i} \quad x = 0, 1. \end{aligned}$$

Proof: see [15]

▲

As mentioned before, a vehicle, in each R2V phase, individually signal to the BS the specific indices of packet yet to be received by it. Thus, in each round number of packets each node has, would be increased by at least one when $m = 1$. Hence number of rounds needed to deliver all the M packets is at most M, i.e., $\mathcal{T}$ is bounded from above by M. On the other hand, at each round at most two new packets can be delivered to the vehicles, i.e. $M/2 \leq \mathcal{T}$. By the same argument, it is easy to see that for each $0 \leq t \leq \mathcal{T}$, $\mathcal{S}_t$ is bounded as $t \leq \mathcal{S}_t \leq 2t$.

By definition of $\mathcal{T}$, we have $\mathcal{S}_\mathcal{T} = N$. We aim to calculate probability distribution of stopping time, i.e., $P(\mathcal{T} = t)$. If the stopping time is $\mathcal{T}$, then at $\mathcal{T} - 1$, one and only one of the followings is correct:

- $\mathcal{S}_{\mathcal{T}-1} = M - 1$
- $\mathcal{S}_{\mathcal{T}-1} = M - 2$

By conditioning on the above events and by using the law of total probability, $P(\mathcal{T} = t)$ can be calculated as follows:

$$\begin{aligned} P(\mathcal{T} = t) &= P(\mathcal{T} = t | \mathcal{S}_{t-1} = M - 1)P(\mathcal{S}_{t-1} = M - 1) \tag{4} \\ &\quad + P(\mathcal{T} = t | \mathcal{S}_{t-1} = M - 2)P(\mathcal{S}_{t-1} = M - 2) \\ &= P(\mathcal{S}_t = M | \mathcal{S}_{t-1} = M - 1)P(\mathcal{S}_{t-1} = M - 1) \\ &\quad + P(\mathcal{S}_t = M | \mathcal{S}_{t-1} = M - 2)P(\mathcal{S}_{t-1} = M - 2). \end{aligned}$$

Given $\mathcal{S}_{t-1} = M - 1$, probability of having $\mathcal{S}_t = M$ is the same as probability of $\mathcal{X}_t = 1$, which has been given in Eq. (1). Thus by using Lemma 1, we have:

$$P(\mathcal{S}_t = M | \mathcal{S}_{t-1} = M - 1) = P(\mathcal{X}_t = 1 | \mathcal{S}_{t-1} = M - 1) = 1,$$

because if $\mathcal{S}_{t-1} = M - 1$, it means that the vehicles are one packet short. In the next round, the BS will send them that packet to complete the downloading process.

The above can be summarized to the following lemma:

lemma 2. *Let $\mathcal{S}_t$ be the number of packets each node has at the end of t-th round. Then*

$$\begin{aligned} P(\mathcal{S}_t = s + 1 | \mathcal{S}_{t-1} = s) &= P(\mathcal{X}_t = 1 | \mathcal{S}_{t-1} = s) \tag{5} \\ &= \frac{1}{M - s}, \end{aligned}$$

and

$$\begin{aligned} P(\mathcal{S}_t = s + 2 | \mathcal{S}_{t-1} = s) &= P(\mathcal{X}_t = 0 | \mathcal{S}_{t-1} = s) \tag{6} \\ &= \frac{M - s - 1}{M - s}, \end{aligned}$$

where (as mentioned before)

$$t \leq \mathcal{S}_t \leq 2t.$$

▲

To calculate $P(\mathcal{T} = t)$ in Eq. (4), we need to calculate $P(\mathcal{S}_t = s)$. It can be calculated by conditioning on the two following collectively exclusive events:

- $\mathcal{S}_{t-1} = s - 1$
- $\mathcal{S}_{t-2} = s - 2$

Therefor, we have

$$\begin{aligned} P(\mathcal{S}_t = s) &= P(\mathcal{S}_t = s | \mathcal{S}_{t-1} = s - 1)P(\mathcal{S}_{t-1} = s - 1) \tag{7} \\ &\quad + P(\mathcal{S}_t = s | \mathcal{S}_{t-1} = s - 2)P(\mathcal{S}_{t-1} = s - 2) \\ &= P(\mathcal{X}_t = 1 | \mathcal{S}_{t-1} = s - 1)P(\mathcal{S}_{t-1} = s - 1) \\ &\quad + P(\mathcal{X}_t = 0 | \mathcal{S}_{t-1} = s - 2)P(\mathcal{S}_{t-1} = s - 2) \\ &= \frac{1}{M - s + 1}P(\mathcal{S}_{t-1} = s - 1) \\ &\quad + \frac{M - s + 1}{M - s + 2}P(\mathcal{S}_{t-1} = s - 2). \end{aligned}$$

So, $P(\mathcal{S}_t = s)$ can be recursively calculated using above equation and the following initialization:

$$\begin{aligned} P(\mathcal{S}_1 = 1) &= P(\mathcal{X}_1 = 1) = \frac{1}{M}, \tag{8} \\ P(\mathcal{S}_1 = 2) &= P(\mathcal{X}_1 = 0) = \frac{M - 1}{M}. \end{aligned}$$

We summarize the above findings to the following theorem:

Theorem 1. *Let $\mathcal{T}$ be the stopping time. Then, it has a probability distribution as follows:*

$$P(\mathcal{T} = t) = P(\mathcal{S}_{t-1} = M - 1) + \frac{1}{2}P(\mathcal{S}_{t-1} = M - 2), \tag{9}$$

where $\frac{M}{2} \le t \le M$. $P(\mathcal{S}_t = s)$ can be calculated using the following recursive formulation:

$$P(\mathcal{S}_t = s) = \frac{1}{M-s+1}P(\mathcal{S}_{t-1} = s-1) \quad (10)$$
$$+ \frac{M-s+1}{M-s+2}P(\mathcal{S}_{t-1} = s-2),$$

which has the following initialization expressions:

$$P(\mathcal{S}_1 = 1) = \frac{1}{M}, P(\mathcal{S}_1 = 2) = \frac{M-1}{M}.$$

B. $N = 2$ and arbitrary m

The result in previous section can be readily generalized to the following. Its proof is omitted here for the sake of space limitation.

Theorem 2. *Let BS have M packets to distribute among two vehicles. In each R2V phase it transmits m packets to each car separately. Let $\mathcal{T}$ denote the stopping time. Then, it has the following probability distribution:*

$$P(\mathcal{T} = t) = \sum_{i=1}^{2m} \frac{\binom{m}{2m-i}}{\binom{i}{m}} P(\mathcal{S}_{t-1} = M - i), \quad (11)$$

where $\frac{M}{2m} \le t \le \frac{M}{m}$. Moreover, $P(\mathcal{S}_t = s)$ can be calculated using the following recursive formula:

$$P(\mathcal{S}_t = s) = \sum_{i=m}^{i=2m} \frac{\binom{m}{2m-i}\binom{M-s+i-m}{i-m}}{\binom{M-s+i}{m}} P(\mathcal{S}_{t-1} = s-i), \quad (12)$$

which has initialization expressions as follows:

$$P(\mathcal{S}_1 = i) = \frac{\binom{m}{2m-i}\binom{M-m}{i-m}}{\binom{M}{m}}, \quad i = m, \ldots, 2m. \quad (13)$$

Proof: see [15]

IV. NETWORK CODING AIDED SCHEME

Suppose BS is using linear random network coding. That means, in each transmission it sends a linear combination of the M packets, where combination coefficients are uniformly chosen in finite field $\mathbb{F}_{2^q}$. By the same procedure we did for the feedback-based algorithm, we first derive the probability distribution and expectation of stopping time for a very simple case of $N = 2$ and $m = 1$. Then, we solve the problem in a more general case of $N = 2$ and arbitrary m.

A. $m = 1$, $N = 2$

The BS sends a linear random combination of its M packets in R2V phase. For example, in the first round, it sends the following packet to the first vehicle:

$$\sum_{i=1}^{M} \alpha_{1,i} x_i, \quad \alpha_{1,i} \in \mathbb{F}_{2^q}, \quad (14)$$

where x_i's are the BS information packets, as defined in Section II. The following combination is sent to the second vehicle:

$$\sum_{i=1}^{M} \alpha_{2,i} x_i, \quad \alpha_{2,i} \in \mathbb{F}_{2^q}. \quad (15)$$

At the end of the first round (after one R2V+V2V), both vehicles have the same information, i.e., they both have packets in Eq. (14) and Eq. (15). The above can be written as $\mathbf{Ax}$ where $\mathbf{x} = [x_1 \, x_2 \, \ldots \, x_M]^T$ is an $M \times 1$ vector of BS packets and $\mathbf{A}$ is a $2 \times M$ matrix containing NC coefficients. Clearly, at the end of second round two rows are added to matrix $\mathbf{A}$ and $\mathbf{Ax}$ is a 4×1 vector representing 4 packets in each vehicle. In general at the end of each round, two rows are added to $\mathbf{A}$. Therefore, at the end of round t, matrix $\mathbf{A}$ has dimension $2t \times M$. Clearly, nodes are able to recover BS information whenever matrix $\mathbf{A}$ is invertible or equivalently when $rank(\mathbf{A}) = M$. Thus, we can define the stopping time when NC is applied, $\mathcal{T}_{NC}$, as follows:

$$\mathcal{T}_{NC} = \min_t \{rank(A_{2t \times M}) = M\}. \quad (16)$$

Obviously $\mathcal{T}_{NC} \ge M/2$. As mentioned before, our goal is to calculate the probability distribution and expected value of $\mathcal{T}_{NC}$. The following lemma gives the probability of rank of matrix $\mathbf{A}_{t \times M}$ with random entries in a finite field.

lemma 3. *Let $\mathbf{A}_{t \times n}$ be a random matrix over finite field $\mathbb{F}_{2^q}$ such that each entry $a_{i,j}$ is picked uniformly from $\mathbb{F}_{2^q}$. suppose $t \ge n$. Then, probability of $\mathbf{A}$ having rank n is given as follows:*

$$P(rank(\mathbf{A}_{t \times n}) = n) = \prod_{i=1}^{n}(1 - \frac{1}{q^{-(t-n+i)}}) \quad (17)$$
$$\approx 1 - \frac{1}{q^{-(t-n+1)}},$$

where the approximation is valid for sufficiently large q.

Proof: See [16].

▲

Using the above result, an upper-bound on the probability of the stopping time can be calculated as given in following lemma. Its proof is omitted here for the sake of space limitation.

lemma 4. *Let $\mathcal{T}_{NC}$ be stopping time defined in Eq. (16). Then,*

$$P(\mathcal{T}_{NC} = t) \le (1 - \frac{1}{q})(1 - P(rank(\mathbf{A}_{2t-1 \times M}) = M)) \quad (18)$$
$$\le (1 - \frac{1}{q})\frac{1}{q^{-(2t-M)}},$$

where the last inequality is valid for large q.

Proof: See [15].

▲

Finally, the following theorem gives an upper-bound for the expected value of the stopping time when q is large enough.

Theorem 3. *Let $\mathcal{T}_{NC}$ be stopping time defined in (16). Suppose q is large enough such that the approximation results in equations (17) and (18) are valid. Then the expected value of stopping time is bounded from above with the following:*

$$\mathbb{E}[\mathcal{T}_{NC}] \le \frac{M}{2} + \frac{1}{q-1}. \quad (19)$$

Proof: See [15].

B. $N = 2$ and arbitrary m

Results in previous section can be readily generalized to arbitrary m, number of packets transmitted from BS to vehicles in each R2V phase. Its proof is omitted here for the sake of space limitation.

Theorem 4. *Let $\mathcal{T}_{NC}$ be stopping time defined in (16). Suppose q is large enough such that the approximation results in equations (17) and (18) are valid. Then the expected value of stopping time is bounded from above with the following:*

$$\mathbb{E}[\mathcal{T}_{NC}] \leq \frac{M}{2m} + \frac{1}{q-1} \qquad (20)$$

.

V. EVALUATION RESULTS

In this section, we present results from simulations conducted using MATLAB R2008b. We conduct simulations to confirm the analytical results in Section III and Section IV, describing data dissemination in R2V phase with and without network coding.

Figure 3 depicts the average number of rounds needed to disseminate information from infrastructure to the vehicles. As explained before, we are assuming complete information exchange in V2V phase. In the simulation, the BS contains M packets and in each round it sends m packets to each vehicle separately. As we mentioned before, in this paper we limit ourselves to $N = 2$ number of vehicles.

As one can see in Figure 3, $\mathcal{T}$ linearly increases with M, the number of total information packets the BS possesses. There is a very small benefit in using NC when the matric is number of rounds. In fact, for only two vehicles, the feedback-based algorithm needs almost one round more than NC aided scheme (note that NC almost achieves minimum number of rounds). Therefore, for dissemination delay, NC has a small advantage compared to feedback-based algorithm. However, NC scheme removes any need for signaling. In the NC aided scheme the BS keeps sending network coded data to the vehicles without any knowledge of what information they possess. On the other hand, for the feedback-based algorithm vehicles are required to communicate with the BS before receiving any data. They need to inform the BS regarding the packets in which they are interested.

VI. CONCLUSION

In this paper, we consider amount of time necessary (in R2V+V2V rounds) to distribute an object of M packets to two vehicles in a VANET network. We show that using NC has the advantage of removing any need for signaling. In other words, to transmit the data from infrastructure to the vehicles there is no need for the BS to know which packets the vehicles already possess. This advantage of using NC comes with no cost. In fact, the data dissemination latency is slightly better when NC is applied. However, our analysis here is limited to two vehicles. Generalizing the result to arbitrary number of vehicles in the network is one of our future research area.
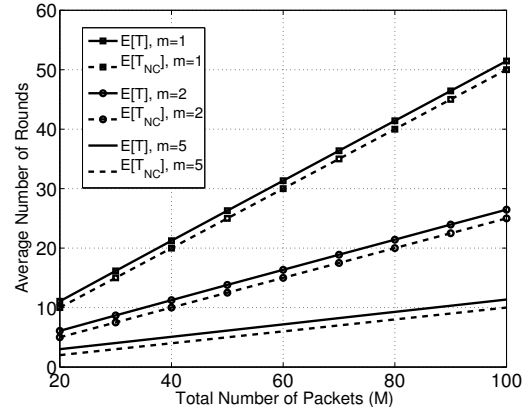

Fig. 3.   Average stopping time for $N = 2$, $m = 1, 2, 5$ and q=8

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