

# Delay Analysis of OFDMA-Aloha

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**Abstract**—OFDMA is the basis of future broadband access, due to its many inherent advantages such as scalability and fine granularity for multi-user access. OFDMA-Aloha combines the flexibility of OFDMA with basic Aloha’s collision resolution mechanism over sub-carriers, in an attempt to reduce packet collisions and achieve faster retransmission. However, this comes at the expense of a larger slot size, due to lower channel rates per subcarrier. The above gives rise to a fundamental question: whether to use a single wide-band Aloha channel and retransmit via random back-off in next  $K$  time slots, or to retransmit immediately in one of  $K$  narrow-band sub-channels which are each  $1/K$  slower (OFDMA-Aloha)? We answer this question, by analyzing the two protocols: Aloha and OFDMA-Aloha under the same total bandwidth and load conditions. We first derive the exact distribution of the packet access delay of OFDMA-Aloha in the saturated case. Then, we extend the analysis to the unsaturated case and derive the mean queue length and packet delay by decomposing the system of interfering queues into multiple independent queues utilizing the symmetry in our system. Our results show that if the network is already saturated, channelization does not bring substantial reduction in the collision rate to the point where it outweighs the effect of expanded slot size. In this case the single channel Aloha performs better than OFDMA-Aloha especially when the gap between the number of channels and the number users is large. On other hand, when the network is lightly loaded, OFDMA-Aloha enjoys smaller packet delays, but not for long as it saturates faster than the single channel Aloha.

**Index Terms**—OFDMA, Aloha, multi-channel MAC, fast re-trial algorithm.

## I. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) is the radio access technology used in new cellular systems such as 3GPP Long Term Evolution (LTE) and IEEE 802.16 (WiMax). In OFDMA systems, the total bandwidth is divided into many channels by clustering OFDM sub-carriers into sub-bands (sub-channels). Then, multiple users are allowed to access different sub-bands simultaneously. An example of this channelization is done in LTE by grouping 12 OFDM sub-carriers into a 180 kHz sub-channel which gives about 100 sub-channels in a 20 MHz channel. This is in contrast to IEEE 802.11a/g wireless LANs which are also based on OFDM, but only give access to the whole 20 MHz channel to one user at a time. Furthermore, switching between different sub-channels is instantaneous due

to the nature of OFDMA. This instantaneous channelization plus the fact that all sub-channels are orthogonal provide a new degree of freedom to the MAC layer.

Typically in OFDMA-based cellular networks, the base station controls access to all sub-channels within the cell and coordinates with adjacent cells in order to limit inter-cell interference. However, this centralized approach is not suitable for emerging heterogeneous networks where small cells like femtocells are expected to play an important role in offloading traffic and increasing the capacity of the network. Femtocells consist of a small access point designed to serve few users in a small indoor area. Since they are deployed in ad-hoc locations by end users and appear/disappear frequently, centralized frequency planning and coordination becomes a challenging task. A femtocell serves a single user most of the time and a cluster of adjacent femtocells can be essentially thought of as multiple users competing for all available sub-channels in an OFDMA network. Random access protocols are “natural” choices in such scenarios and are expected to play an important role in future dense femtocell deployments.

Several random access protocols have been proposed for OFDMA networks, see for example [1]–[3]. In [1], the authors proposed an opportunistic multi-channel Aloha in which the transmission probability is adapted based on the channel state information in each sub-channel. The objective here is to exploit the multi-user diversity in a distributed manner and compare it to the centralized sub-channel allocation. OFDMA-Aloha was proposed in [2] as a direct extension of the single-channel Aloha, where instead of waiting for a random backoff period when a collision occurs in one sub-channel, the node tries another (randomly selected) sub-channel immediately subject to a maximum retry limit. Recently [3] extended the basic idea of [2] to CSMA/CA systems. In OFDMA CSMA/CA, a node first senses all sub-channels, randomly selects one sub-channel from the set of idle sub-channels and then backoffs for a random Collision Avoidance (CA) period in a way similar to the standard procedure in IEEE 802.11. The difference is that it decrements the backoff counter by the number of sensed idle sub-channels in every slot, thus resulting in faster channel access time.

In this paper, we focus on an OFDMA-Aloha system which attempts to exploit the flexibility of OFDMA by extending the basic time-domain backoff procedure employed for standard ALOHA using a single channel, to the frequency-domain in the multi-channel scenario. The idea of OFDMA-Aloha was originally proposed in [2] as a fast-retrial algorithm which is explained in detail in Section I-A. The authors assumed a finite network with  $N$  users and approximated all new arrivals plus fast-retransmissions from frequency- and time-domain retransmissions by an aggregate Poisson model. They derived

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the distribution of the access delay for a fixed number of channels  $K$ . However, their results do not allow a fair comparison between the single channel Aloha versus OFDMA-Aloha for different values of  $K$ . It is not clear from their analysis whether or not the total system bandwidth is fixed for both Aloha and OFDMA-Aloha, as would be necessary for a fair comparison.

The primary contributions of this paper are as follows. First, we provide a delay analysis of OFDMA-Aloha in the saturated case which is helpful to study the limits of the protocol under various system settings. Here, the *exact distribution of the packet access delay* is derived in terms of the number of channels  $K$ , number of users  $N$  and other system parameters. Our results allow us to study the scalability of the OFDMA-Aloha protocol with varying number of channels and compare it to the single channel Aloha under fixed system bandwidth and load conditions. Second, we derive the mean queue length and mean packet delay in the unsaturated case using an approximate queuing analysis. For this, we utilized the symmetry in the network to decompose the system of interfering queues into multiple independent queues which can be studied using standard queuing theory. The analysis reveals an interesting phase transition in the performance of OFDMA-Aloha (compared to Aloha) when going from the unsaturated to the saturated region. The major results are organized into two main sections- the analysis of the saturated case is presented in Section II and the queuing model and analysis of the unsaturated case are presented in Section III.

#### A. System Model and Assumptions

The basic operation of OFDMA-Aloha is depicted in Fig. 1. The total system bandwidth is divided equally into  $K$  channels (or sub-channels in OFDMA terms). We assume a *collision channel* model where transmission errors can only occur because of collisions, i.e. we ignore noise and other channel imperfections. A node with a packet to transmit, selects a channel randomly from the  $K$  channels and transmits at the beginning of the next slot. If a collision occurs, the node tries to retransmit the packet on a different channel immediately in the next slot. If the packet collides again, the node persists in retransmission until a maximum number of retrials  $M$  is reached, at which time the node backs off for a random amount of time and resets its fast-retry counter. This fast retrial feature is what differentiates OFDMA-Aloha from standard multi-channel Aloha.

For a consistent and fair comparison between OFDMA-Aloha with  $K$  channels and the single channel Aloha, we analyze the two systems under the same total channel bandwidth and net arrival rates. We use the subscript 1 to denote Aloha and subscript 2 to denote OFDMA-Aloha. The system model in both cases is depicted in Fig. 2. We assume a (usual) noise-free collision channel with total bandwidth  $B$  (total channel rate  $R$  bits/second) and fixed packet size  $L$ . The network consists of  $N$  users, each with an infinite buffer. Both MAC protocols are slotted and packet transmission is only allowed at the slot boundary. Time is discretized into *mini-slots* of duration  $\tau$  corresponding to the packet transmission time over the full bandwidth, i.e.,  $\tau = L/R$  seconds. All time related

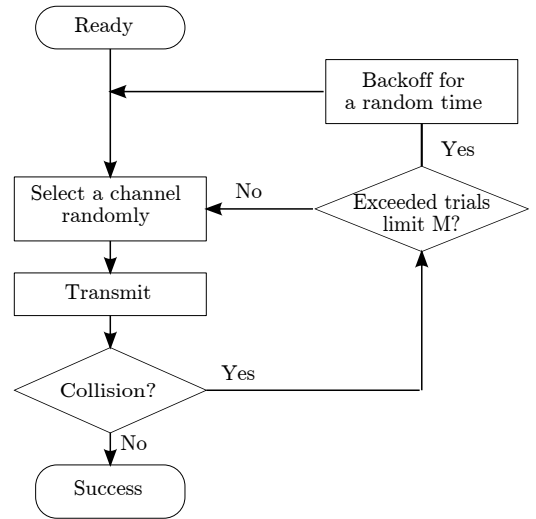


Fig. 1. OFDMA-Aloha MAC Algorithm.

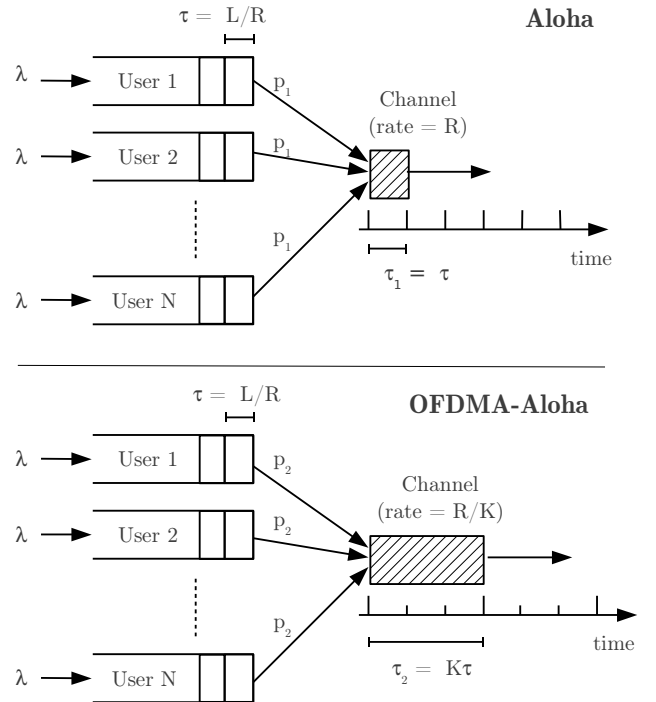


Fig. 2. System model.

quantities are defined in multiples of the mini-slot  $\tau$ . With this, the slot duration becomes  $\tau_1 = \tau$  seconds in Aloha and  $\tau_2 = K\tau$  seconds in OFDMA-Aloha because the rate of each sub-channel is  $R/K$ . The queuing model is described in more detail in Section III.

For tractability, we employ the Delayed First Transmission (DFT) model which is widely used in the literature [4]. Under this model, the same transmission probability is used for all packets, whether new or old. That is, a newly arriving packet is forced to wait for the same random backoff period as any other retransmitted packet. Fig. 3 and Fig. 4 show the two MAC protocols under the DFT model. The consequence of this model is that when all users are busy, the transmission probability of each user is fixed in all time slots. Using

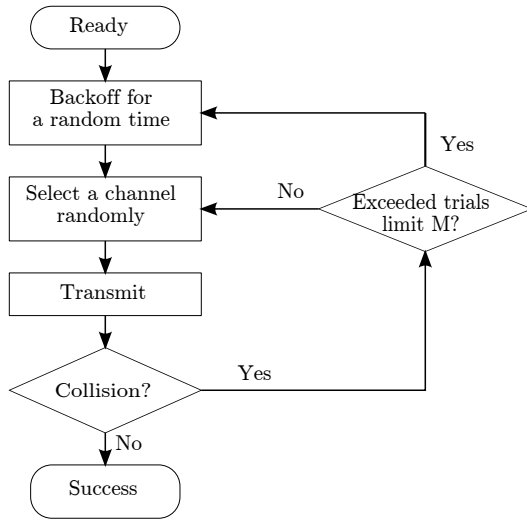


Fig. 3. OFDMA-Aloha MAC flow chart under the DFT model.

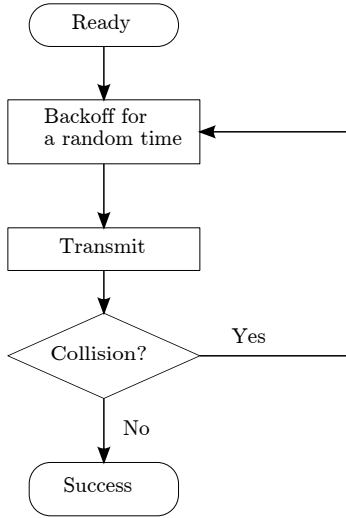


Fig. 4. Aloha MAC flow chart under the DFT model.

this DFT assumption, upon a new arrival, the node moves immediately into the backoff mode. In the backoff mode, the node transmits (or retransmits) the Head of Line (HOL) packet with probability  $p$  or delays it to the next slot with probability  $1 - p$ . The transmission probabilities are  $p_1$  and  $p_2$  for Aloha and OFDMA-Aloha respectively. Finally, we assume the number of users  $N$  is strictly greater than the number of channels  $K$  in OFDMA-Aloha; otherwise, the bandwidth becomes underutilized.

## II. SATURATED CASE ANALYSIS

In the saturated case, we assume all buffers are always full, such that every successful packet is replaced immediately with a new packet. In this case, the quantity of interest is the throughput or equivalently its reciprocal: the packet access delay  $d$ . One approach to the analysis of such system is to construct a Markov chain representing the state of the system in any slot and finding the state transition probability matrix. While the system state for Aloha in the saturated case is trivial, we need an  $M + 1$  dimensional Markov chain to

represent the state of the system in OFDMA-Aloha. If  $m_i$  is the fast-retry counter at user  $i$ , then the system state is given by  $\mathbf{n} = (n_1, n_2, \dots, n_M, n_{bk})$  where  $n_j$ ,  $j = 1, \dots, M$  represents the number of users with fast-retry counter  $m = j$  and  $n_{bk}$  represents the number of users in the backoff state, i.e. their fast-retry counters have exceeded  $M$ . This Markov chain is difficult to analyze even for the simplest case of  $N = 2$ .

By utilizing the symmetry in the system, we carry out a much simpler analysis using state flow graph techniques [5]. Since all users are statistically identical, we derive the quantity of interest by analyzing the state flow graph of the Markov process representing packet transmission in a typical, *tagged* user. The steady state distribution and moments of the process are derived using transform analysis of the state flow graph described in [5]. This approach has been used previously, for example for the analysis of Tone Sense Multiple Access in [6] and for an approximate queuing analysis of Aloha in [7] and CSMA/CD in [8]. We start by illustrating this technique for Aloha in order to facilitate the exposition of the more complicated OFDMA-Aloha system.

### A. Aloha Saturation Analysis

The state transition diagram of the packet transmission process in a typical user in Aloha is shown in Fig. 5. The state flow graph is constructed from the original state transition diagram of the process by multiplying each branch between any two states by the Probability Generating Function (PGF) of the time (or number of steps) required for the transition. Under the DFT assumption, a ready packet moves to the backoff state immediately. In every subsequent slot, the user transmits with probability  $p_1$  and defers transmission with probability  $1 - p_1$ . If we denote the probability of successful transmission by  $q_1$ , then the packet moves into the success state with probability  $p_1 q_1$  and stays in the backoff state with probability  $(1 - p_1) + p_1(1 - q_1)$ . Each of these two transitions require one time slot, and hence the PGF is just  $z$  times these probabilities. Upon successful packet transmission, a new packet moves to the ready state immediately and the process repeats again.

From the state flow graph, the access delay  $d_1$  seen by each packet in Aloha is the total time required to move from the "Ready" state to the "Success" state. The PGF of this delay  $D_1(z)$  is the transfer function from state  $S_r$  to state  $S_s$  in the flow graph.  $D_1(z)$  can be obtained using flow graph reduction methods or directly using Mason's rule, see [9]:

$$D_1(z) = \frac{p_1 q_1 z}{1 - [(1 - p_1)z + p_1(1 - q_1)z]} \quad (1)$$

$$= \frac{p_1 q_1 z}{1 - (1 - p_1 q_1)z}$$

where the average success probability  $q_1$  of the tagged user in Aloha is given by:

$$q_1 = (1 - p_1)^{N-1}$$

By differentiating  $D_1(z)$  and evaluating at  $z = 1$  we get the mean access delay  $d_1$  (in mini-slots):

$$d_1 = D_1'(1)$$

$$= \frac{1}{p_1 q_1} = \frac{1}{p_1(1 - p_1)^{N-1}} \quad (2)$$

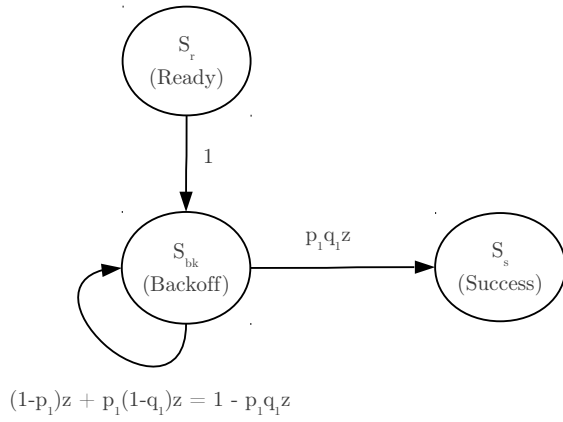


Fig. 5. State flow graph of packet transmission process in Aloha.

### B. OFDMA-Aloha Saturation Analysis

Similarly, we show the state flow graph of the packet transmission process in OFDMA-Aloha in Fig. 6, where  $p_2$  and  $q_2$  denote the transmission probability and the success probability respectively. In this case we have  $M$  additional states representing each stage of the fast retry mode. As in Aloha, a ready packet moves to the backoff state  $S_{bk}$  immediately. In  $S_{bk}$ , the user transmits with probability  $p_2$  and defers transmission to the following slot with probability  $1 - p_2$ . When a collision occurs for the first time, the retry counter is incremented to  $m = 1$ , and the packet moves to state  $S_1$ . When a second collision occurs in the following slot, the packet moves to state  $S_2$ , continuing this way until the maximum retry limit  $m = M$  is reached in state  $S_M$ . If a collision occurs in state  $S_M$ , the user "gives up" retrying and falls back into the backoff state  $S_{bk}$  and resets  $m = 0$ . Note that in all the fast retry states:  $S_1, S_2, \dots, S_M$ , the user transmits with probability 1, whereas in the backoff state he transmits with probability  $p_2$ .

As before, we find the PGF of the access delay  $D_2(z)$  in OFDMA-Aloha by writing down the transfer function from state  $S_r$  to state  $S_s$  in the state flow graph using Mason's rule:

$$D_2(z) = \frac{p_2 q_2 z [1 - (1 - q_2)^{M+1} z^{M+1}]}{[1 - (1 - p_2)z - p_2(1 - q_2)^{M+1} z^{M+1}]} \times \frac{1}{[1 - (1 - q_2)z]} \quad (3)$$

We could differentiate  $D_2(z)$  and evaluate at  $z = 1$  to get the mean access delay  $d_2$  in OFDMA-Aloha. However, we follow an easier approach which will prove useful later in finding the success probability  $q_2$ . For this, we apply the transient Markov process analysis described in [5, ch. 4]. Note that from the perspective of a newly arrived packet, the states:  $S_{bk}, S_1, \dots, S_M$  are transient states and state  $S_s$  is a trapping, or an absorbing state. Therefore, the delay seen by the packet is the total time spent in the transient process which is the sum of the time spent in the backoff state and all fast retry states.

To find the average time spent in the transient backoff state,  $T_{bk}$ , we could find the transfer function from the input state

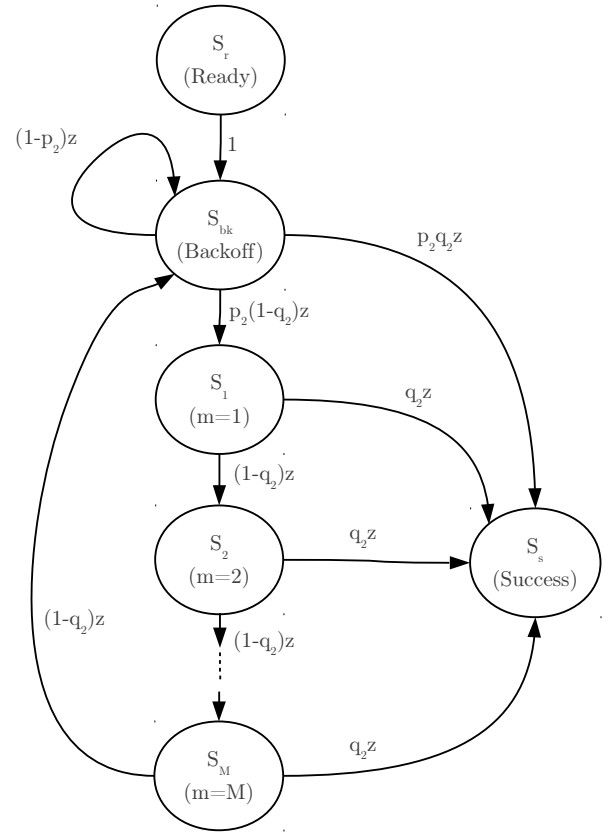


Fig. 6. State flow graph of OFDMA-Aloha.

$S_r$  to state  $S_{bk}$  to obtain the PGF of  $T_{bk}$  and then differentiate to get  $\mathbb{E}[T_{bk}]$ . Alternatively, since we are interested in the first moment, we could find the path transmission gain from  $S_r$  to  $S_{bk}$  using a modified state flow graph with  $z = 1$ . Applying Mason's rule between state  $S_r$  and state  $S_{bk}$  in the modified flow graph, we get the average time spent in the backoff state:

$$T_{bk} = \frac{1}{p_2 - p_2(1 - q_2)^{M+1}} = \frac{1}{\Delta}$$

where  $\Delta \equiv p_2 - p_2(1 - q_2)^{M+1}$ . Similarly, we use the modified flow graph to find the average time spent in the fast retry states:

$$\begin{aligned} T_1 &= \frac{p_2(1 - q_2)}{\Delta} \\ T_2 &= \frac{p_2(1 - q_2)^2}{\Delta} \\ &\vdots \\ T_M &= \frac{p_2(1 - q_2)^M}{\Delta} \end{aligned}$$

The mean access delay is the total time spent in the transient process (in mini-slots):

$$\begin{aligned} d_2 &= (T_{bk} + T_1 + T_2 + \dots + T_M) \times K \\ &= \left( \frac{1}{\Delta} + \sum_{m=1}^M \frac{p_2(1 - q_2)^m}{\Delta} \right) \times K \\ &= \frac{q_2 - p_2 q_2 + p_2 [1 - (1 - q_2)^{M+1}]}{p_2 q_2 [1 - (1 - q_2)^{M+1}]} \times K \quad (4) \end{aligned}$$

where the multiplier  $K$  signifies that the slot size in OFDMA-Aloha is  $K$  mini-slots.

Next, we find the probability of success  $q_2$  for the tagged user in OFDMA-Aloha. Denote by  $p_t$  the average transmission probability of any user in any slot and the set of all states by  $\mathcal{S} = \{S_{bk}, S_1, \dots, S_M\}$ :

$$\begin{aligned} p_t &= \sum_{i \in \mathcal{S}} \Pr[\text{user transmits} \mid \text{state } i] \Pr[\text{state } i] \\ &= p_2 \times p_{bk} + 1 \times p_f \end{aligned} \quad (5)$$

where  $p_{bk}$  and  $p_f$  are the probabilities of being in the backoff mode (state  $S_{bk}$ ) and the fast retry mode (states  $S_1, \dots, S_M$ ) respectively. To find  $p_{bk}$  and  $p_f$ , we utilize the above transient process analysis.  $p_{bk}$  is the proportion of time spent in the backoff mode and is given by:

$$\begin{aligned} p_{bk} &= \frac{\mathbf{E}\{\text{time spent in the backoff state}\}}{\mathbf{E}\{\text{total time spent in the transient process}\}} \\ &= \frac{T_{bk}}{d_2} \\ &= \frac{q_2}{q_2 - p_2 q_2 + p_2 [1 - (1 - q_2)^{M+1}]} \end{aligned} \quad (6)$$

Similarly,  $p_f$  is the proportion of time spent in the fast retry mode and is given by:

$$\begin{aligned} p_f &= \frac{\mathbf{E}\{\text{time spent in all fast retry states}\}}{\mathbf{E}\{\text{total time spent in the transient process}\}} \\ &= \frac{\sum_{m=1}^M T_m}{d_2} \\ &= \frac{p_2 [1 - (1 - q_2)^{M+1}] - p_2 q_2}{q_2 - p_2 q_2 + p_2 [1 - (1 - q_2)^{M+1}]} \end{aligned} \quad (7)$$

Now, suppose that the tagged user selects channel  $c$  for transmission. Since all remaining  $N-1$  users are statistically identical and channel selection is uniform with probability  $1/K$ , the probability of success of the tagged user on this channel is given by:

$$q_2 = \left(1 - p_t \times \frac{1}{K}\right)^{N-1} \quad (8)$$

Substituting  $p_{bk}$  and  $p_f$  in (5) and (8), we get the following non-linear relation between the packet success probability and the system parameters in OFDMA-Aloha:

$$q_2 = \left[1 - \frac{1}{K} \left(\frac{p_2 [1 - (1 - q_2)^{M+1}]}{q_2 - p_2 q_2 + p_2 [1 - (1 - q_2)^{M+1}]}\right)\right]^{N-1} \quad (9)$$

which can be solved numerically for  $q_2$  to derive the mean access delay  $d_2$  in (4).

In the analysis above, we assumed the DFT model in Fig. 3 and Fig. 4. For Aloha in the saturated case, this assumption does not pose any significant difference, and hence we can use the result for  $d_1$  in (2) without this assumption. From this we can state the following result for the original OFDMA-Aloha in Fig. 1. If we allow unlimited number of fast retrials ( $M = \infty$ ), the access delay in OFDMA-Aloha is always larger than the access delay in Aloha regardless of the number of channels, as quantified in the following proposition:

**Proposition 1.** *Let  $d_2^\infty$  be the mean access delay in OFDMA-Aloha with  $K$  channels and with  $M = \infty$ , i.e. unlimited*

*number of fast retrials. Let  $d_1$  be the mean access delay in Aloha with the same total bandwidth and with transmission probability  $p_1 = 1/N$  where  $N$  is the number of users. If all users are saturated, then:*

$$d_2^\infty \geq d_1 \quad \forall \quad K > 1 \quad (10)$$

*with the equality satisfied only when  $K = N$ .*

*Proof:* Note that when  $M = \infty$  in OFDMA-Aloha without the DFT assumption (Fig. 1), every user transmits with probability  $1/K$  in every slot until the packet is successfully transmitted. This means no time-domain backoff will be encountered. In this case, the success rate of the tagged user in each slot is  $q_2 = (1 - 1/K)^{(N-1)}$  and the mean access delay is  $1/q_2$  slots. In terms of mini-slots, the mean access delay is given by:

$$\begin{aligned} d_2^\infty &= \frac{1}{q_2} \times K \\ &= \frac{K}{\left(1 - \frac{1}{K}\right)^{(N-1)}} \end{aligned} \quad (11)$$

From the first and second derivatives of (11) with respect to  $K$ , we see that  $d_2^\infty$  is strictly convex on the interval  $(1, \infty)$  and attains its minimum value at  $K = N$ . At this value of  $K$ ,  $d_2^\infty = d_1$ , the mean access delay of Aloha with  $p_1 = 1/N$  given by (2):

$$d_1 = \frac{N}{\left(1 - \frac{1}{N}\right)^{(N-1)}} \quad \blacksquare$$

### C. Numerical and Simulation Results

In order to verify our analytical results, we simulated the two MAC algorithms using the flow charts in Fig. 3 and Fig. 4. The simulation environment represents the abstract system model described in I-A. We present numerical and simulation results for OFDMA-Aloha with  $K = 5, 10, 15$  and  $M = 10, 20$ . In all cases, the transmission probabilities of all users were fixed and the load line is varied by changing the number of users. Since we are studying a ‘‘symmetric’’ system of  $N$  homogeneous users, we set the transmission probability (attempt rate) of Aloha to  $p_1 = 1/N$ . For a fair comparison, we set the transmission probability in OFDMA-Aloha to  $p_2 = K/N$  which is  $K$  times the attempt rate of Aloha because the slot size is  $K$  times larger. Also,  $p_2 = K/N$  gives identical performance to Aloha during the initial startup phase when all users are in the backoff state. For the numerical evaluation, we used the `fzero` routine in MATLAB to compute the numerical solution of  $d_2$  in (4). In all cases of interest ( $K < N$ ), the numerical results agree very well with the simulation results.

The mean access delay in the saturated case is shown in Fig. 7 for  $M = 10$ . The results show that the single channel Aloha always performs better than OFDMA-Aloha when all users are saturated. This can be interpreted as follows; the key idea behind OFDMA-Aloha is to reduce the retransmission time by reducing the collision probability. This is achieved at the expense of an expanded slot duration due to the lower channel rate. The reduction in the collision rate has to be

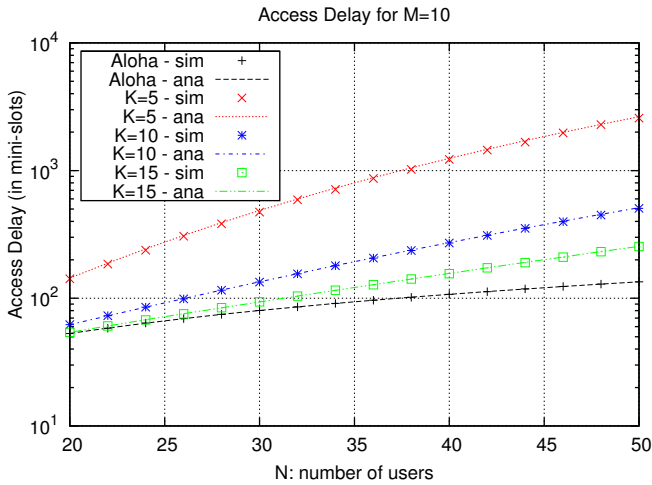


Fig. 7. Mean access delay in the saturated case for  $M = 10$ .

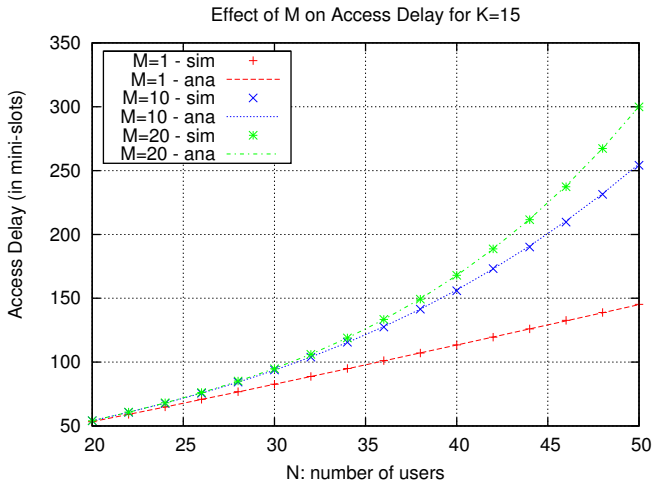


Fig. 8. Effect of  $M$  (Max retry limit) on access delay in the saturated case for  $K = 15$ .

significant to overcome this expense and this is only achieved with larger number of channels, i.e  $K$  close to  $N$ . For example at  $N = 20$ , the access delay of OFDMA-Aloha with  $K = 15$  is very close to that of Aloha. However, as the gap between  $K$  and  $N$  increases, the reduction in the collision rate decreases and the impact of the expanded slot duration becomes more dominant. This suggests that OFDMA-Aloha might be helpful for lightly loaded network with low collision rates. We will study this case in the next section.

To investigate the effect of the fast retry limit  $M$ , we compare the access delay of OFDMA-Aloha with 3 different values of  $M$  in Fig. 8 for  $K = 15$ . Clearly, higher values of  $M$  perform worse because they permit more fast retrials which result in higher number of collisions in the saturated case.

### III. UNSATURATED CASE ANALYSIS

In the unsaturated case, some user buffers may be empty, and hence will not contend for channel access. This necessitates a queuing analysis of the system. As depicted in Fig. 2, our system model consists of  $N$  queues served by one server (channel). Due to channel contention, the service time of each

queue depends on the state of all other queues. The state of the system is thus an  $N$ -dimensional vector  $\mathbf{q} = (q_1, q_2, \dots, q_N)$ , where  $q_n$  is the queue size of the  $n$ th queue. A direct brute-force analysis of this system is intractable due to the large state space, and approximate analysis is necessary. This problem of interfering or interacting queues in multiple access networks has been studied extensively over the past 30 years with various approximation techniques proposed in the literature, see for example [10] and the references therein.

A notable approximation technique is based on decomposing the multi-queue system into  $N$  independent queuing systems and capturing the interaction between the interfering queues in the service time distribution using some simplifying assumption. One approach is to assume that each user operates at its steady state independently of all other users and is busy with probability  $\rho < 1$ . In this case, the distribution of the service time of a typical, “tagged” user can be expressed as a function of the busy probabilities of all other users. Because of the symmetry and fairness of the protocol, the busy probabilities of all other users is identical to that of the tagged user. Therefore, the service time distribution of the tagged user can be expressed as a function of its own busy probability. Then, by applying classical queuing models on the tagged user’s queue, we can obtain a relation between the busy probability and the service time distribution. These two relations can be used to solve the system for the quantities of interest. This approach has been used for a queuing analysis of Aloha in [7] and CSMA/CD in [8].

Utilizing the general idea above, we proceed to compare the two systems: Aloha and OFDMA-Aloha with unsaturated load. Particularly, we derive the mean packet delay which is the sum of the queuing delay and the access delay. For clarity of exposition, we split the analysis into two parts: a contention analysis and a queuing analysis and then we combine the results to solve for the quantity of interest. First, we state our queuing model and assumptions.

#### A. Queuing Model and Assumptions

In addition to the general system assumptions stated earlier, we further invoke the following in our queuing analysis:

- 1) A discrete-time queuing model is used to model each of the two systems: Aloha and OFDMA-Aloha with respective slot sizes  $\tau_1 = \tau$  second and  $\tau_2 = K\tau$  seconds as illustrated in Fig. 2.
- 2) The common arrival process is Bernoulli with parameter  $\lambda$  with respect to the mini-slot  $\tau$ , i.e. in every mini-slot a packet arrives with probability  $\lambda$  and does not arrive with probability  $1 - \lambda$ .
- 3) The service discipline is First Come First Serve (FCFS) with late arrival mode. This means a packet arriving to an empty queue must wait for the next slot to get service.

Next, we define several quantities needed in the analysis. All discrete-time related quantities are expressed in slots where the slot size is defined for each system accordingly. We use subscript  $i = 1$  for Aloha and  $i = 2$  for OFDMA-Aloha.

- 1)  $T_i$ : the total time spent by a packet in the system.
- 2)  $X_i$ : the service time of the Head of Line (HOL) packet in the queue; from the instant it reaches the HOL until

it is successfully transmitted. The probability generating function (PGF) of  $\mathbf{X}_i$  and its first two moments are defined as follows:

$$X_i(z) \equiv \sum_{k=0}^{\infty} \Pr[\mathbf{X}_i = k]z^k$$

$$x_i = \mathbb{E}[\mathbf{X}_i] = X_i'(1)$$

$$x_i^{(2)} = \mathbb{E}[\mathbf{X}_i^2] = X_i''(1) + X_i'(1)$$

- 3)  $\mathbf{Q}_i$ : the number of packets in the queuing system (including the HOL packet) at the boundary of an arbitrary slot and its PGF is defined as follows:

$$Q_i(z) \equiv \sum_{k=0}^{\infty} \Pr[\mathbf{Q}_i = k]z^k$$

- 4)  $\mathbf{\Lambda}_i$ : the number of packets that arrive in a slot and its PGF and first two **factorial moments** are defined as follows::

$$\Lambda_i(z) \equiv \sum_{k=0}^{\infty} \Pr[\mathbf{\Lambda}_i = k]z^k$$

$$\lambda_i = \mathbb{E}[\mathbf{\Lambda}_i] = \Lambda_i'(1)$$

$$\lambda_i^{(2)} = \mathbb{E}[\mathbf{\Lambda}_i(\mathbf{\Lambda}_i - 1)] = \Lambda_i''(1)$$

With the above assumptions, the queuing models for the two systems are described below.

- **Queuing Model for Aloha:** Because the slot duration in Aloha matches with the mini-slot ( $\tau_1 = \tau$ ) in the common arrival process, no more than one packet can arrive in a slot, hence the arrival process in this system is also Bernoulli with parameter  $\lambda_1 = \lambda$ . This system can be modeled as **Geo/G/1** queuing system because the packet inter-arrival time is geometrically distributed with mean  $1/\lambda_1$ .
- **Queuing Model for OFDMA-Aloha:** The slot size in OFDMA-Aloha is ( $\tau_2 = K\tau$ ). The number of packets that can arrive in one slot is Binomially distributed with parameters  $(K, \lambda)$ . To simplify the analysis, we assume that all packets arriving in a slot arrive in a batch at the end of the last mini-slot of this slot (late arrival model), see Fig. 9. This approximation is needed because we will derive the mean packet delay of this discrete queuing system in terms of the slot time  $\tau_2$  and later convert it to mini-slots by multiplying by  $K\tau$ . With the batch arrival model, the inter-arrival time is still geometrically distributed and we have a **batch arrival Geo/G/1** system which is denoted by **Geo<sup>X</sup>/G/1**, see [11]. The PGF and the first two factorial moments<sup>1</sup> of this arrival process in this case are given by:

$$\Lambda_2(z) = (1 - \lambda + \lambda z)^K$$

$$\lambda_2 = K\lambda$$

$$\lambda_2^{(2)} = (1 - \frac{1}{K})\lambda_2^2$$

<sup>1</sup>The first factorial moment of a random variable  $\mathbf{X}$  with a PGF  $X(z)$  is defined as  $x = \mathbb{E}[\mathbf{X}] = X'(1)$  and the second factorial moment is defined as  $x^{(2)} = \mathbb{E}[\mathbf{X}(\mathbf{X} - 1)] = X''(1)$ .

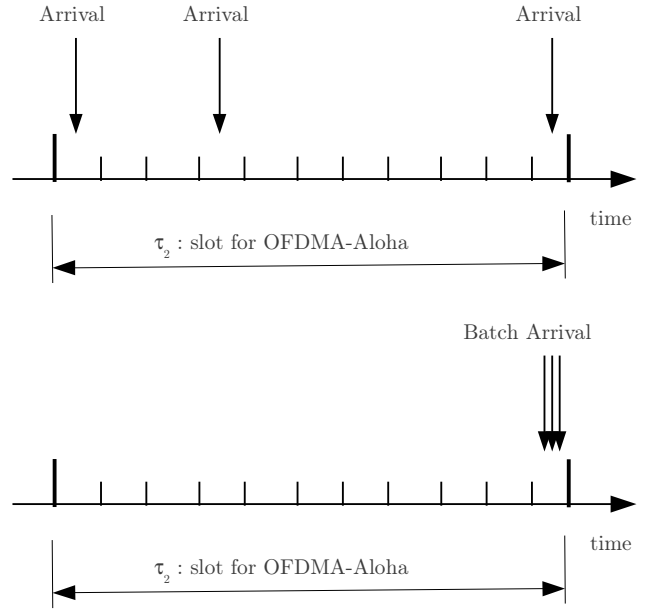


Fig. 9. Bernoulli arrivals vs. batch arrivals.

### B. Contention Analysis

We first start with the contention analysis of Aloha to obtain  $X_1(z)$ , the service time distribution, of the tagged user in terms of the busy probability:  $p_b$ , or equivalently the idle probability  $p_0 = 1 - p_b$ . Using the fact that only busy users contend for channel access, we repeat the state flow graph analysis given in the previous section, but this time multiplying the transmission probability of each user by its busy probability  $(1 - p_0)$ . Following the same approach we used to derive the access delay in (1), we get

$$X_1(z) = \frac{p_1 q_1 z}{1 - (1 - p_1 q_1)z} \quad (12)$$

and by differentiating and setting  $z = 1$  we get the mean service time in Aloha:

$$x_1 = \frac{1}{p_1 q_1} \quad (13)$$

The success probability  $q_1$  in this case is given by:

$$q_1 = [1 - (1 - p_0)p_1]^{N-1} \quad (14)$$

where  $p_1$  is the transmission probability in the Aloha system.

Similarly, we use the state flow graph in Fig. 6 to derive the service time distribution  $X_2(z)$  of OFDMA-Aloha:

$$X_2(z) = \frac{p_2 q_2 z [1 - (1 - q_2)^{M+1} z^{M+1}]}{[1 - (1 - p_2)z - p_2(1 - q_2)^{M+1} z^{M+1}]} \times \frac{1}{[1 - (1 - q_2)z]} \quad (15)$$

and its mean service time:

$$x_2 = \frac{q_2 - p_2 q_2 + p_2 [1 - (1 - q_2)^{M+1}]}{p_2 q_2 [1 - (1 - q_2)^{M+1}]} \quad (16)$$

Note that this is the same as the mean access delay  $d_2$  given in (4) in the saturated case. However, the probability of success

here is given by:

$$q_2 = \left[ 1 - (1 - p_0) \frac{1}{K} \times \left( \frac{p_2 [1 - (1 - q_2)^{M+1}]}{q_2 - p_2 q_2 + p_2 [1 - (1 - q_2)^{M+1}]} \right) \right]^{N-1} \quad (17)$$

From equations (13-16), we see that the mean service time of each system can be expressed as a function of the idle probability  $p_0$  of the tagged user in addition to the system parameters:  $M$ ,  $K$ ,  $N$ ,  $p_1$  and  $p_2$ .

### C. Queuing Analysis

Second, we utilize known results from discrete-time queuing theory to obtain another relation between the service time and the idle probability  $p_0$  in each system. In both Geo/G/1 and Geo<sup>X</sup>/G/1 systems, the PGF of the queue size at the boundary of an arbitrary slot is given by, see [11]:

$$Q_i(z) = \frac{(1 - \rho_i) X_i[\Lambda_i(z)]}{X_i[\Lambda_i(z)] - z} \quad (18)$$

where  $\rho_i = \lambda_i x_i$  is the load or the utilization of the server. This relation is only valid for  $\rho < 1$ . From this, we get:

$$\begin{aligned} p_0 &= \Pr[\mathbf{Q}_i = 0] \\ &= 1 - \rho_i \\ &= 1 - \lambda_i x_i \end{aligned} \quad (19)$$

Therefore, we have two independent relations which can be used to solve for the success probability  $q_i$  in each system in terms of the system parameters and the arrival rate. We substitute (13) and (19) into (14) to get the success probability in Aloha:

$$q_1 = \left[ 1 - \frac{\lambda_1}{q_1} \right]^{N-1} \quad (20)$$

Similarly, we substitute (16) and (19) into (17) to the get success probability in OFDMA-Aloha:

$$q_2 = \left[ 1 - \frac{\lambda_2}{K q_2} \right]^{N-1} \quad (21)$$

Once the success probability  $q_i$  is obtained for each respective system, the first and second moments of the service time, can be derived from their PGFs in (12) and (15) for Aloha and OFDMA-Aloha respectively. Once the service time distribution is determined, the queuing system is actually solved. From (18), we get the mean queue size:

$$\mathbb{E}[\mathbf{Q}_i] = \frac{\lambda_i^2 x_i^{(2)} - \lambda_i \rho_i + \lambda_i^{(2)} x_i}{2(1 - \rho_i)} + \rho_i \quad (22)$$

Applying the discrete-time version of Little's theorem, we get the mean packet delay:

$$\mathbb{E}[\mathbf{T}_i] = \frac{\mathbb{E}[\mathbf{Q}_i]}{\lambda_i} \quad (23)$$

Finally, we stress that this approximate queuing analysis is based on the assumption that every user operates at steady state independently of all other users. This assumption is violated in two cases. The first is when the number of users is small because in this case the interaction among the users

is strong and the above approximation is poor. The second is when the mean number of arrivals in a slot exceeds the mean service rate, i.e.  $\rho > 1$ . In this case, the steady state probability distribution does not exist as all queues saturate and all users will be busy with probability 1. When this happens, no solution can be found for the probability of success in (20) and (21) and the above analysis cannot be used. The maximum admissible arrival rate before the system saturates defines the stability of the system as discussed in the following subsection.

### D. Stability Analysis

A queuing system is said to be *stable* if  $\rho = \lambda x < 1$ , where  $\lambda$  is the mean arrival rate and  $x$  is the mean service time. In buffered multiple access systems, the mean service time  $x$  depends on the interaction among all competing queues which makes their stability very difficult to characterize. The general stability region of even the simplest multiple access systems like Aloha has not been fully understood except for few simple cases and several approximate bound. However, for Aloha with homogeneous users, i.e. all users having the same arrival rate  $\lambda_1$  and the same transmission probability  $p_1 = 1/N$ , then the necessary and sufficient condition for system stability was derived by Tsybakov and Mikailov [12] as follows:

$$\lambda_1 < \lambda_{\text{aloha}}^{\max} = \frac{1}{N} \left(1 - \frac{1}{N}\right)^{N-1} \quad (24)$$

This stability result was generalized in [13] for any arrival vector  $\boldsymbol{\lambda}$  and any transmission probability vector  $\mathbf{p}$  as

$$\lambda_i < p_i \prod_{\substack{j \neq i \\ j=1}}^N (1 - p_j), \quad \forall \quad i = 1, \dots, N$$

The stability bound of OFDMA-Aloha is difficult to express algebraically because it involves the unknown zeros of (21) which must be found numerically. However, for the settings assumed in this analysis, it can be shown that the stability region of OFDMA-Aloha is strictly smaller than the corresponding region for Aloha. This indicates that OFDMA-Aloha saturates faster than Aloha under the same arrival rate as highlighted in the following proposition.

**Proposition 2.** *In OFDMA-Aloha with  $N$  homogeneous users,  $K$  channels, a finite number of fast retrials  $M$  and a transmission probability  $p_2 = \frac{K}{N}$ , the maximum admissible arrival rate  $\lambda_{\text{ofdma}}^{\max}$  for which the system does not saturate, satisfies:*

$$\lambda_{\text{ofdma}}^{\max} < \lambda_{\text{aloha}}^{\max} = \frac{1}{N} \left(1 - \frac{1}{N}\right)^{N-1} \quad (25)$$

for all  $1 < K < N$  and  $M \geq 1$ .

*Proof:* The proof hinges on using the dominant system approach introduced in [14] and extended in [13]. Denote by S1 our original  $N$ -queue OFDMA-Aloha system with the queuing model described before. Define another OFDMA-Aloha system S2 which is identical to S1, with the same arrival rate  $\lambda_2$  and the same transmission probability  $p_2 = K/N$ . The only difference is that when a queue becomes empty in S2, it continues to transmit "dummy" packets with the same



transmission probability  $p_2$ . Dummy packets can result in collisions, but the successful transmission of a dummy packet does not reduce the queue size. Therefore, S2 always has larger queue size than S1 if both start from the same initial conditions, i.e. S2 dominates S1. Clearly, if S2 is stable then our original system S1 is also stable.

The mean service time of the dominant system is given by (16), but its success probability  $q_2$  is given by (9) because it is indistinguishable from OFDMA-Aloha under saturation, i.e.  $p_0 = 0$ . Substituting (16) in (9), we get:

$$q_2 = \left[ 1 - \frac{1}{Kq_2x_2} \right]^{N-1}$$

or equivalently,

$$x_2 = \frac{1}{Kq_2(1 - q_2^{1/(N-1)})}$$

For a stable queuing system, we must have  $\lambda_2x_2 < 1$ , giving the following stability bound for  $\lambda$  under OFDMA-Aloha:

$$\begin{aligned} \lambda_2 &< Kq_2(1 - q_2^{\frac{1}{N-1}}) \\ K\lambda &< Kq_2(1 - q_2^{\frac{1}{N-1}}) \\ \lambda &< q_2(1 - q_2^{\frac{1}{N-1}}) \end{aligned} \quad (26)$$

where  $q_2$  is the solution of (9). Define

$$h(x) = x(1 - x^{\frac{1}{N-1}}), \quad 0 < x < 1$$

It can be shown that  $h(x)$  is maximized at  $x_0 = (1 - \frac{1}{N})^{N-1}$ . In addition,  $h'(x) > 0$  for  $0 < x < x_0$  and  $h'(x) < 0$  for  $x_0 < x < 1$  and hence:

$$\begin{aligned} \max_{0 < x < 1} h(x) &= h(x_0) \\ &= \frac{1}{N}(1 - \frac{1}{N})^{N-1} \end{aligned}$$

It is sufficient for our purpose to prove that  $x_0 = (1 - \frac{1}{N})^{N-1}$  is not a solution of (9). In the Appendix, we show that if  $p_2 = K/N$  then there exist a solution  $q_2^*$  of (9) and it is strictly less than  $x_0$ , hence

$$\lambda_{\text{ofdma}}^{\max} = h(q_2^*) < h(x_0) = \lambda_{\text{aloha}}^{\max}$$

■

### E. Numerical and Simulation Results

For the unsaturated case, we compare Aloha versus OFDMA-Aloha with  $K = 10, 20$  and  $M = 10$ . We fix the number of users at  $N = 30$  and vary the load line by increasing  $\lambda$  which gives  $\lambda_1 = \lambda$  for Aloha and  $\lambda_2 = K\lambda$  for OFDMA-Aloha. We use the same transmission probability as in the saturated case:  $p_1 = 1/N$  for Aloha  $p_2 = K/N$  for OFDMA-Aloha.

The mean packet delay is plotted in Fig. 10. When the load is very low, there is no queuing delay and very few collisions occur in both systems. In this case, the mean packet delay is just the initial backoff time under the DFT assumption. The expected backoff period is the same in both systems:  $1/p_1 = N$  mini-slots in Aloha and  $1/p_2 = N/K \times K = N$

mini-slots in OFDMA-Aloha. For relatively small  $\lambda$ , OFDMA-Aloha enjoys smaller packet delays as compared to Aloha. Also, within OFDMA-Aloha, smaller values of  $K$  perform better than larger values. This is because the collision rate is relatively low and fewer channels are sufficient to absorb the load and reduce the number of collisions. The fast retrieval feature of OFDMA-Aloha works best in this region and achieves faster retransmission time. However, increasing the number of channels increases the slot duration without any further reduction in the number of collisions, and hence results in slightly larger delay.

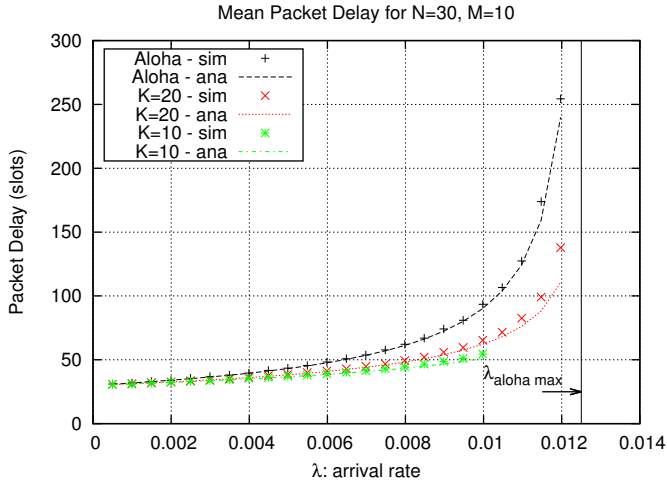
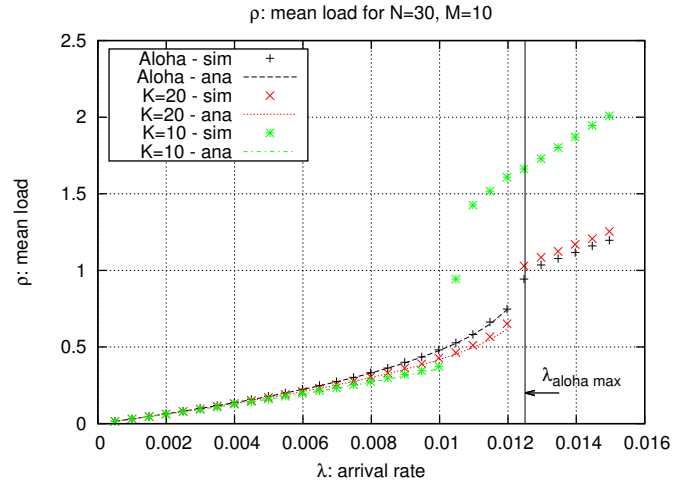
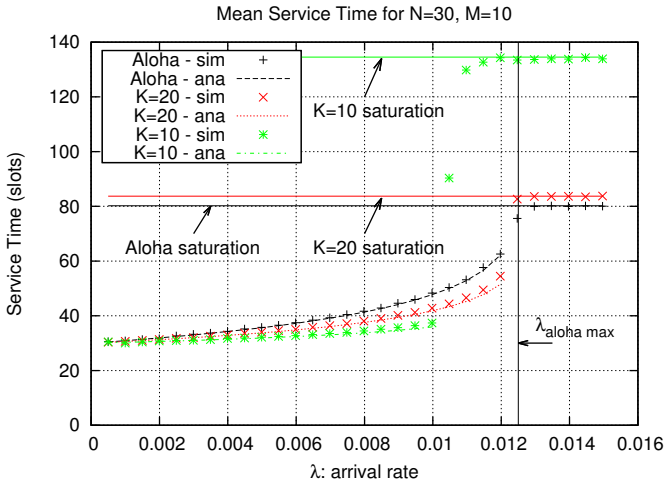
When the load increases beyond Aloha's saturation line at  $\lambda_{\text{aloha}}^{\max}$ , both systems become saturated and the delay grows without bound. However, we note that for smaller values of  $K$ , OFDMA-Aloha saturates faster than larger values and also faster than Aloha, a result expected from proposition 2. When the system saturates, we cannot find a numerical solution and the simulation returns very large numbers. We omit these large numbers from the plot in order not to obscure the other results. This behavior near the saturation line is somehow expected because we have seen from the analysis in section I-A that OFDMA-Aloha with smaller values of  $K$  performs the worst in saturation. To verify this, we plot the mean service time of the two systems in Fig. 11 and the average load  $\rho$  in Fig. 12. Note that in these two figures, no analytical results are shown for values of  $\lambda$  near Aloha's saturation line because no solutions could be found here as explained before. This is particularly evident in Fig. 12 where the analytical results are missing when  $\rho$  approaches 1.

In all cases, the mean service time beyond Aloha's saturation line converges to a constant which equals the mean access delay that was derived in the saturated case, see (2) for Aloha and (4) for OFDMA-Aloha as shown by the solid horizontal lines in Fig 11. Therefore, the behavior of the two MAC protocols in the unsaturated case is exactly opposite to that of the saturated case. This confirms our earlier reasoning that the reduction in the collision rate in OFDMA-Aloha has to be significant enough to overcome the effect of the expanded time scale. When the collision rate is relatively low, a small number of channels is sufficient to absorb the load and improve the retransmission time. However, when the collision rate is very high, e.g. in saturation, channelization only helps to expand the time scale and hence performs worse than the single channel.

From the above, we see that the performance of the two MAC protocols undergoes a phase transition when going from the unsaturated region to the saturated region. This phenomena was also reported in the past in the analysis of multi-channel Aloha by Yue and Matsumoto in [15, chapter 2]. Multi-channel Aloha allows a collided node to switch to another random channel, but only after a random back-off period, so it does not have the fast retrieval feature of OFDMA-Aloha. We note though, that their analysis is for nodes with single-packet buffer size and is substantially different from our approach.

## IV. CONCLUSION

OFDMA-Aloha is a new MAC protocol that promises to exploit the channel switching flexibility of OFDMA. By

Fig. 10. Mean packet delay in the unsaturated case for  $N = 30$  and  $M = 10$ .Fig. 12. Mean load  $\rho$  in the unsaturated case for  $N = 30$  and  $M = 10$ .Fig. 11. Mean service time in the unsaturated case for  $N = 30$  and  $M = 10$ .

allowing collision resolution over the frequency as well as time domains, the protocol attempts to reduce the packet retransmission time. However, this comes at the great expense of expanded time scale, or larger slot size due to lower channel rates. We showed that when the network is already saturated, channelization does not bring substantial reduction in the collision rate to the point where it outweighs the effect of expanded slot size; single channel Aloha performs better than OFDMA-Aloha especially when the gap between the number of channels and the number users is large. On other hand, when the network is lightly loaded, OFDMA-Aloha enjoys smaller packet delays, but not for long as it saturates faster than the single channel Aloha. This suggests the need for further study on the stability region of OFDMA-Aloha as it may help develop practical adaptive algorithms for the future.

#### APPENDIX

Here we show that all solutions of eq (9) in the interval  $(0, 1)$  are strictly less than  $x_0 = (1 - 1/N)^{N-1}$  when  $p_2 = K/N$ .

*Proof:* The proof is tedious, but straightforward. Substi-

tute  $p_2 = K/N$  and rewrite (9) as follows:

$$0 = (N - K)q_2(1 - q_2^{\frac{1}{N-1}}) + [1 - (1 - q_2)^{M+1}](K - Kq_2^{\frac{1}{N-1}} - 1)$$

Define  $z = q_2^{\frac{1}{N-1}}$ , and for convenience define  $n = N - 1$ ,  $m = M + 1$ , and  $a = N - K$ . Then, we have the following polynomial:

$$g(z) = az^n(1 - z) + K - Kz - 1 - (1 - z^n)^m(K - Kz - 1)$$

Note that  $n > 1$ ,  $m > 1$ ,  $1 < K \leq n$  and  $1 \leq a \leq n$  from the assumptions in this paper. For our purpose, it is sufficient to show that  $g(z)$  has at least one real root in the interval  $(0, z_0)$  and no real roots in  $(z_0, 1)$  where  $z_0 = 1 - 1/N$ .

First, we show the existence of the solution and in particular, we show that there is a real root in  $(0, z_0)$ . Let  $z_1 = 1/N$ .

$$g(z_1) = a\left(\frac{1}{N}\right)^n\left(1 - \frac{1}{N}\right) + \left(K - \frac{K}{N} - 1\right)\left[1 - \left(1 - \left(\frac{1}{N}\right)^n\right)^m\right]$$

Since  $\frac{K}{N} < 1$  and  $K \geq 2$ , all above terms are positive and  $g(z_1) > 0$ .

$$g(z_0) = \left(1 - \frac{K}{N}\right)\left(1 - \frac{1}{N}\right)^n - \left(1 - \frac{K}{N}\right)\left[1 - \left(1 - \left(1 - \frac{1}{N}\right)^n\right)^m\right] = -\left(1 - \frac{K}{N}\right)(1 - w)$$

where  $w$  is the term:

$$w = \left(1 - \frac{1}{N}\right)^n + \left[1 - \left(1 - \frac{1}{N}\right)^n\right]^m$$

For  $m \geq 2$ ,  $w < 1$  and  $g(z_0) < 0$ . Hence, by the Intermediate Value Theorem,  $g(z)$  must have a zero in  $(z_1, z_0)$  or equivalently  $g(z)$  has at least one real root in  $(0, z_0)$ .

Next, we show that there are no real roots of  $g(z)$  in  $(z_0, 1)$ . Taking the first derivative of  $g(z)$ :

$$g'(z) = anz^{n-1} - a(n+1)z^n + mn(K - Kz - 1)z^{n-1}(1 - z^n)^{m-1} + K(1 - z^n)^m - K$$

Defining  $w_1 = (1 - z^n)^{m-1}$  and  $w_2 = (1 - z^n)^m$  and after some algebra, we can write  $g'(z)$  as:

$$g'(z) = -z^{n-1}(ah_1 + mnw_1h_2) - K(1 - w_2)$$

where:

$$h_1 = (n+1)z - n$$

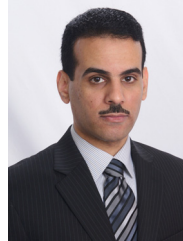
and

$$h_2 = Kz + 1 - K$$

It can be shown that  $h_1$  and  $h_2$  are always positive for  $z \in [z_0, 1]$  and  $K \in (1, n]$ . Since  $0 < w_1, w_2 < 1$  and  $a, n, m$  are all positive,  $g'(z) < 0$  and hence  $g(z)$  is decreasing on  $[z_0, 1]$ . Since  $g(z_0) < 0$  (from above analysis) and  $g(1) = -1$ ,  $g(z)$  has no real roots in  $(z_0, 1)$ . ■

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