Optimum Transmitter/Receiver Design for a Narrowband Overlay in Noncoordinated Subscriber Lines

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***Abstract*—Transmitter/receiver optimization for a generic *narrowband overlay* communication scenario is addressed. The overlay and existing legacy systems are assumed to occupy the same frequency bands for spectral efficiency, thus introducing cochannel interference; however, the new and legacy systems are assumed to be noncooperative, as is appropriate for some pragmatic scenarios. A composite figure of merit is used consisting of a weighted sum of the mean-squared error (MSE) of the (new) overlay system plus the excess MSE in the legacy system caused by the introduction of the overlay system. Necessary conditions on the transmitter and receiver that jointly optimize the above metric are derived. The effects of varying key parameters such as the loop length (range) and transmitter power are investigated via computational examples.**

***Index Terms*—Crosstalk, digital subscriber lines (DSL), mean square error (MSE) methods.**

I. Introduction

THE DEMAND for higher-rate services (typically driven by emerging data and video applications) for home and office users motivates continuing evolution for next-generation network products. However, scarcity of available bandwidth typically necessitates *spectrum sharing* between legacy and new systems. Such *overlay* scenarios are increasingly commonplace in wireless applications, where future third-generation (3G) cellular systems must coexist with present-day second-generation (2G) (or 2.5 G) networks, for example. The immediate context of our work is, however, *wireline* digital subscriber line (DSL) networks which provide users high-speed data connectivity over subscriber loops (universal twisted pairs) that may be rate asymmetric or symmetric (see [9] and [11]). Recent advances in DSL technology has resulted in very-high-speed DSL (VDSL) service that offers downstream rates of 52 Mb/s in asymmetric mode,[1] and symmetric rates of 13 Mb/s over a single twisted-pair copper loop. VDSL uses up to 20 MHz of bandwidth to support data, voice, and video services simultaneously, while coexisting in the same spectrum with earlier DSL services that provide basic access at 1.544 Mb/s for two digitized voice plus one data channels only.

In DSL topology, twisted-copper pairs from the central office (CO) reach out to the subscriber's location. Typically, multiple competing DSL service providers use separate head-end equipment, but their cable bundles at the CO are proximate. This physical proximity of lines leads to far-end crosstalk (FEXT) between adjacent cable pairs of uncoordinated transmitters as new providers add service from the same CO in the presence of existing (legacy) ones. The resulting crosstalk (between uncoordinated cochannel transmissions in the same direction) is the primary factor limiting maximum DSL range, which is the most important current limitation to DSL network rollout for new customers. As a result, new and effective cross-talk cancellation techniques are a key component to enhancing the range of future DSL systems.

Crosstalk mitigation in the context of DSL systems has, and continues to, receive considerable attention. The available strategies at the transmitter include shaping of the transmit power spectrum. For example, [15] proposed an algorithm for transmit spectrum optimization, based on dividing the available spectrum into sufficiently small bins so that the channel response over each bin may be viewed as being constant. For each bin, the optimal response shape based on the direct and cross-channel response is then selected. Applying this optimization to all bins results in a transmit spectrum that theoretically achieves the channel capacity. In [13], a game-theoretic formulation was applied to the problem of multiuser power control in cross-talk-impaired channels. A distributed waterfilling algorithm that implicitly takes into account the loop transfer functions and cross couplings was developed, and shown to reach a competitively optimal power allocation by allowing loops to negotiate the best use of power and frequency with each other. Yet another paper [14] (which assumed coordination at the CO) investigated a BLAST-like algorithm whereby the QR decomposition was used to precode the transmitted data, which was then iteratively decoded at the receiver, resulting in large performance gains in systems with particularly strong interference.

Our paper contributes to an understanding of coexistence between *like* DSL systems.[2] A sensible design approach suggests that the performance degradation to existing providers must be

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[1]In asymmetric service, more bandwidth is allotted to downstream (central office to subscriber) than upstream (subscriber to central office) communication, whereas the downstream and upstream directions are allotted equal bandwidth in symmetric mode.

[2]By like systems, we imply identical data rates. There is, of course, natural interest in extending the present analysis to multirate systems that effectively model the coexistence problem between older and newer generations of DSL services; this is, however, deferred to the future.

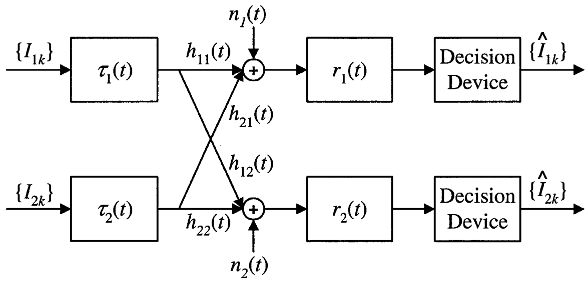

Fig. 1. System model.

bounded by the additional interference caused by the new system, subject to meeting the new system's own design objectives in the presence of the old system.

The main contribution of our paper is an investigation of the impact of joint transmitter-receiver pulse-shape optimization as a means toward improved cross-talk cancellation in DSL systems. Restated, such joint optimization would unveil more efficient usage of the common spectrum that could extend the range for a given average transmit power budget.

A. Problem Statement

A generic system model is shown in Fig. 1. The lower branch represents the generic legacy system (subscript 2) with respective transmitter $\tau_2(t)$ and receiver $r_2(t)$ operating over the channel $h_{22}(t)$. Note that $\tau_2(t), r_2(t)$ are fixed and may not be changed to mitigate interference to/from the presence of the new overlay system (subscript 1). *Our objective is to jointly optimize the new system transmitter $\tau_1(t)$ and receiver $r_1(t)$ for its optimized performance, while simultaneously mitigating interference into system 2, subject to the constraint that the legacy system remains fixed.* A composite mean-square error (MSE) (to be defined in Section I-B) which accounts for the performance of the new system, as well as the performance degradation of the legacy system due to interference from the new system, is used as a figure of merit. The assumption of identical rates for both the overlay and legacy systems imply that they occupy the same bandwidth, thus causing a complete overlay of the frequency band. Further, it is assumed for analytical tractability that the cross-talk transfer function between any cable pairs is symmetric.

B. Literature Review

This contribution relates to the body of analytically inspired work on joint transmitter-receiver optimization in multiuser communications such as [1], [2], and [5]–[7]. We assume a *coexistence* viewpoint where the transmitters do not cooperate; in terms of the problem formulation, this implies that an average transmit power constraint per user is more appropriate than a total power constraint in the case of coordinating users. This problem, which represents many real-world scenarios, appears to have merited less attention by way of analytical exploration, as compared with those where full transmitter coordination was assumed. A possible explanation is that the latter scenario was shown to be analytically tractable and was solved for reasonable generality in [10]; however, the uncoordinated-transmitters case has proved to be more challenging, with partial results available in [2] and [5] *et al.*. This paper is another contribution in this direction; we adopt the familiar MSE criterion-based approach, but with an interesting difference, as described below.

For single-user systems with additive noise as the only impairment, [1] proved that the optimal minimum MSE (MMSE) system, in general, has support that exceeds the basic Nyquist zone, but this is nevertheless limited to occupying only one Nyquist *set*.[3] Unlike single-user systems, the performance of multiuser systems is limited by crosstalk. [7] first investigated joint transmitter/receiver optimization under the MMSE criterion for coordinated users (full transmitter and receiver matrices), but the analysis assumed that the system was strictly limited to one Nyquist zone. The generalization to systems occupying multiple Nyquist zones was achieved in [6]. An investigation of fully uncoordinated users (i.e., both diagonal transmitter and receiver matrices) was first conducted in [2] for two users, and subsequently generalized in [5] for an arbitrary number of users. Both [2] and [5] assumed symmetry in the direct and cross-talk channel responses seen by each user. In contrast to the results in [1] for the single-user system, the optimal N-user system in this case was shown to require support on, at most, $2N-1$ points in the Nyquist set.

The paper is organized as follows. In Section II, we describe the system model and formulate the problem. Section III derives the necessary conditions for the optimum transmitter-receiver filter pairs. Section IV contains computational results for commonly used cables.

II. Problem Formulation

In the system model in Fig. 1, assume that the existing (legacy) system has independent and identically distributed (i.i.d.) input $\{I_{2k}\}$ and $h_{22}(t)$ as its channel, with fixed transmitter and receiver filters whose responses are $\tau_2(t)$ and $r_2(t)$. A new overlay system that transmits a mutually independent i.i.d. sequence $\{I_{1k}\}$ over a channel given by $h_{11}(t)$ is to be designed with the transmitter/receiver filters $\tau_1(t)$, $r_1(t)$ as variables. The interference (cross-talk) between the two systems is modeled by $h_{21}(t)$ and $h_{12}(t)$ as shown, respectively; in the computational examples, these are assumed identical for reasons explained above.

Given the constraint of the legacy system that cannot be further modified, we propose a *composite* MSE metric given by

$$\mathrm{MSE} = \mathrm{MSE}_1 + \alpha \mathrm{MSE}_2^e \tag{1}$$

where

$$\begin{aligned}\mathrm{MSE}_1 &\equiv \text{MSE in channel 1 (overlay MSE)}\\ &= E\left[\left|I_{1k} - \hat{I}_{1k}\right|^2\right] \qquad (2)\\ \mathrm{MSE}_2^e &\equiv \text{Excess MSE in channel 2 (excess legacy MSE)}\\ &= E\left[|e_{2k}|^2\right] \qquad (3)\end{aligned}$$

and where

$$e_{2k} = e_2(kT) \tag{4}$$

$$e_2(t) = \tau_1(t) \otimes h_{12}(t) \otimes r_2(t). \tag{5}$$

[3]Consider the sets $\mathcal{I} \triangleq \{f : f_0 + k/T\}$, $k \in \mathbb{Z}$, where T is the symbol period, and $\mathcal{I}_0 \triangleq \{-1/2T, 1/2T\}$. $\mathcal{I}_0$ is denoted the first Nyquist *zone*, and $\mathcal{I}$ is the Nyquist *set*.

$\tau_i(t) \leftrightarrow T_i(f)$ form a Fourier pair where $\tau_i(t)$, $i = 1, 2$ represent the pulse shaping at the respective transmitters. Note that MSE_2^e represents the *excess* MSE introduced into the legacy system by the introduction of the overlay, and $1 \geq \alpha > 0$ denotes a *relative* weight in the optimization between the two components $\mathrm{MSE}_1, \mathrm{MSE}_2^e$ that may be suitably chosen to de-weight MSE_2^e (by using $\alpha \ll 1$) as desired.

The signal at the receiver input in the presence of additive white Gaussian noise (AWGN) $n_1(t)$, $n_2(t)$ is processed by the receiver filters $r_1(t)$ and $r_2(t)$ as shown to produce outputs $y_1(t)$, $y_2(t)$ that are input to the sampler (decision device) to produce the final estimates

$$y_1(t) = \sum_{k=-\infty}^{\infty} I_{1k} g_{11}(t-kT) + \sum_{k=-\infty}^{\infty} I_{2k} g_{21}(t-kT) + \nu_1(t) \quad (6)$$

where

$$\nu_1(t) = n_1(t) \otimes r_1(t) \quad (7)$$

with power spectra

$$S_{\nu_1}(f) = N_0 |R_1(f)|^2 \quad (8)$$

and

$$g_{11}(t) = \tau_1(t) \otimes h_{11}(t) \otimes r_1(t) \quad (9)$$

$$g_{21}(t) = \tau_2(t) \otimes h_{21}(t) \otimes r_1(t). \quad (10)$$

Assume that both the overlay and legacy system occupy a bandwidth of M/T, where T is the symbol period and $M \in \mathbb{Z}$. Let us define the kth Nyquist translate of any frequency-domain entity as

$$X_k = X\left[f + \frac{k-1}{T} - \frac{M-1}{2T}\right], \quad k \in \mathbb{Z} \quad (11)$$

where the above definition applies to $H_{i,j}(f)$, $T_i(f)$, $R_i(f)$, $1 \leq i, j \leq 2$. Thus, T_{1k} represents the kth Nyquist translate of the overlay transmitter in the sequel.

III. Derivation of Optimal Design Conditions for Overlay System

Suppose $y_1(t)$ is synchronously sampled at instants kT; the sample at instant $t = 0$ is denoted y_{10}. Let σ^2 be the input symbol variance, and let N_0 be the additive noise level. Then

$$y_{10} = \sum_{k=-\infty}^{\infty} I_{1k} g_{11}(kT) + \sum_{k=-\infty}^{\infty} I_{2k} g_{21}(kT) + \nu_{10} \quad (12)$$

$$\Rightarrow E[|y_{10} - I_{10}|^2] = \sigma^2 \sum_{k=-\infty}^{\infty} |g_{11}(k) - \delta_{k0}|^2 + \sigma^2 \sum_{k=-\infty}^{\infty} |g_{21}(k)|^2 + N_0 \int_{-\infty}^{\infty} |r_1(t)|^2 dt \quad (13)$$

$$= \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \left| \sum_{k=1}^{M} T_{1k} H_{11k} R_{1k} - T \right|^2 df + \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \left| \sum_{k=1}^{M} T_{2k} H_{21k} R_{1k} \right|^2 df + N_0 \int_{-1/2T}^{1/2T} \sum_{k=1}^{M} |R_{1k}|^2 df \quad (14)$$

$$\text{Excess MSE } = \mathrm{MSE}_2^e = E|y_{2n}^e|^2 = \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \left| \sum_{k=1}^{M} T_{1k} H_{12k} R_{2k} \right|^2 df \quad (15)$$

assuming that $\{I_{1k}\}$, $\{I_{2k}\}$, $n_1(t)$, and $n_2(t)$ are mutually independent. For the optimization to be well posed, we impose an average transmitter power constraint on each user, and include the excess MSE resulting from the overlay within the constraint. Introducing the Lagrange multiplier λ

$$\Rightarrow \mathrm{MSE} = \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \left| \sum_{k=1}^{M} T_{1k} H_{11k} R_{1k} - T \right|^2 df + \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \left| \sum_{k=1}^{M} T_{2k} H_{21k} R_{1k} \right|^2 df + N_0 \int_{-1/2T}^{1/2T} \sum_{k=1}^{M} |R_{1k}|^2 df + \lambda \left[\alpha \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \left| \sum_{k=1}^{M} T_{1k} H_{12k} R_{2k} \right|^2 df + \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \sum_{k=1}^{M} |T_{1k}|^2 df \right]. \quad (16)$$

Define the overlay system's transmitter and receiver as

$$\mathbf{r}_1 = [R_{1,1}, \ldots, R_{1,M}]^T \quad (17)$$

$$\boldsymbol{\tau}_1 = [T_{1,1}, \ldots, T_{1,M}]^T. \quad (18)$$

To facilitate analytical tractability, we will write (16) explicitly in terms of $\mathbf{r}_1$ and $\boldsymbol{\tau}_1$

$$\mathrm{MSE}(\mathbf{r}_1, \tau_1) = \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \|\mathbf{A}_1 \mathbf{r}_1 - T\mathbf{b}\|^2 df + N_0 \int_{-1/2T}^{1/2T} \|\mathbf{r}_1\|^2 df + \lambda \alpha \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \left| \sum_{k=1}^{M} T_{1k} H_{12k} R_{2k} \right|^2 df + \lambda \frac{\sigma^2}{T} \int_{-1/2T}^{1/2T} \|\tau_1\|^2 df \quad (19)$$

$$= \frac{\sigma^2}{T}\int_{-1/2T}^{1/2T}\|\mathbf{A}_2\boldsymbol{\tau}_1 - T\mathbf{b}\|^2 df + N_0\int_{-1/2T}^{1/2T}\|\mathbf{r}_1\|^2 df + \frac{\sigma^2}{T}\int_{-1/2T}^{1/2T}\left|\sum_{k=1}^{M} T_{2k}H_{21k}R_{1k}\right|^2 df + \lambda\frac{\sigma^2}{T}\int_{-1/2T}^{1/2T}\|\boldsymbol{\tau}_1\|^2 df \quad (20)$$

where we have (21)–(23), shown at the bottom of the page. From (19) and (20), using the Kuhn–Tucker conditions for (first-order) necessity, we obtain the following coupled equations for optimal choice of $T_1(f)$ and $R_1(f)$ using standard variational techniques:

$$\mathbf{r_1} = T\left[\mathbf{A}_1^\dagger\mathbf{A}_1 + \eta^{-1}\mathbf{I}\right]^{-1}\mathbf{A}_1^\dagger\mathbf{b} \quad (24)$$

$$\boldsymbol{\tau_1} = T\left[\mathbf{A}_2^\dagger\mathbf{A}_2 + \beta^{-1}\mathbf{I}\right]^{-1}\mathbf{A}_2^\dagger\mathbf{b} \quad (25)$$

where

$$\eta = \frac{\sigma^2}{N_0 T}, \quad \beta = \frac{1}{\lambda}. \quad (26)$$

Note that in accordance with intuition, the optimum $\mathbf{r}_1$ is dependent on the SNR parameter η, and tries to minimize the MSE (for a given $\boldsymbol{\tau}_1$ of the desired data in the presence of the legacy-system transmission (from H_{21}). Similarly, the optimum $\boldsymbol{\tau}_1$ depends on transmit power via β, and optimizes the MSE of the desired data for a given R_1 while minimizing leakage into the legacy system (from H_{12}, R_2).

Let us rewrite $\mathbf{A}_1$, $\mathbf{A}_2$ as

$$\mathbf{A}_1 = \begin{bmatrix}\mathbf{H}_{11}\mathbf{T}_d \\ \mathbf{K}_1\end{bmatrix} \quad (27)$$

$$\mathbf{A}_2 = \begin{bmatrix}\mathbf{H}_{11}\mathbf{R}_d \\ \mathbf{K}_2\end{bmatrix} \quad (28)$$

where

$$\mathbf{R}_d = \mathrm{diag}\left[R_{11}, R_{12}, \ldots, R_{1M}\right] \quad (29)$$

$$\mathbf{T}_d = \mathrm{diag}\left[T_{11}, T_{12}, \ldots, T_{1M}\right] \quad (30)$$

$$\mathbf{H}_{11} = \left[H_{11,1}, H_{11,2}, \ldots, H_{11,M}\right] \quad (31)$$

$$\mathbf{K}_1 = \left[T_{21}H_{21,1}, T_{22}H_{21,2}, \ldots, T_{2M}H_{21,M}\right] \quad (32)$$

$$\mathbf{K}_2 = \sqrt{\lambda\alpha}\left[R_{21}H_{12,1}, R_{22}H_{12,2}, \ldots, R_{2M}H_{12,M}\right]. \quad (33)$$

Thus, from (27) and (28)

$$\Rightarrow \mathbf{A}_1^\dagger\mathbf{A}_1 = \mathbf{T}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{T}_d + \mathbf{K}_1^\dagger\mathbf{K}_1 \quad (34)$$

$$\mathbf{A}_2^\dagger\mathbf{A}_2 = \mathbf{R}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{R}_d + \mathbf{K}_2^\dagger\mathbf{K}_2. \quad (35)$$

Substituting the above in (24) and (25) yields

$$\mathbf{r}_1 = T\left[\mathbf{T}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{T}_d + \mathbf{K}_1^\dagger\mathbf{K}_1 + \eta^{-1}\mathbf{I}\right]^{-1}\mathbf{T}_d^\dagger\mathbf{H}_{11}^\dagger \quad (36)$$

$$\boldsymbol{\tau_1} = T\left[\mathbf{R}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{R}_d + \mathbf{K}_2^\dagger\mathbf{K}_2 + \beta^{-1}\mathbf{I}\right]^{-1}\mathbf{R}_d^\dagger\mathbf{H}_{11}^\dagger. \quad (37)$$

We can rewrite (36) and (37) as

$$\mathbf{T}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{T}_d\mathbf{r}_1 + \mathbf{K}_1^\dagger\mathbf{K}_1\mathbf{r}_1 + \eta^{-1}\mathbf{r}_1 = T\mathbf{T}_d^\dagger\mathbf{H}_{11}^\dagger \quad (38)$$

$$\mathbf{R}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{R}_d\boldsymbol{\tau}_1 + \mathbf{K}_2^\dagger\mathbf{K}_2\boldsymbol{\tau}_1 + \beta^{-1}\boldsymbol{\tau}_1 = T\mathbf{R}_d^\dagger\mathbf{H}_{11}^\dagger. \quad (39)$$

Multiplying (38) and (39) on the left by $\mathbf{R}_d^\dagger$ and $\mathbf{T}_d^\dagger$, respectively, we obtain

$$\mathbf{R}_d^\dagger\mathbf{T}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{T}_d\mathbf{r}_1 + \mathbf{R}_d^\dagger\mathbf{K}_1^\dagger\mathbf{K}_1\mathbf{r}_1 + \eta^{-1}\mathbf{R}_d^\dagger\mathbf{r}_1 = T\mathbf{R}_d^\dagger\mathbf{T}_d^\dagger\mathbf{H}_{11}^\dagger \quad (40)$$

$$\mathbf{T}_d^\dagger\mathbf{R}_d^\dagger\mathbf{H}_{11}^\dagger\mathbf{H}_{11}\mathbf{R}_d\boldsymbol{\tau}_1 + \mathbf{T}_d^\dagger\mathbf{K}_2^\dagger\mathbf{K}_2\boldsymbol{\tau}_1 + \beta^{-1}\mathbf{T}_d^\dagger\tau_1 = T\mathbf{T}_d^\dagger\mathbf{R}_d^\dagger\mathbf{H}_{11}^\dagger. \quad (41)$$

Recognizing that $\mathbf{T}_d\mathbf{R}_d = \mathbf{R}_d\mathbf{T}_d$ and $\mathbf{T}_d\mathbf{r}_1 = \mathbf{R}_d\boldsymbol{\tau}_1$, and equating (40) and (41), we obtain

$$\mathbf{R}_d^\dagger\mathbf{K}_1^\dagger\mathbf{K}_1\mathbf{r}_1 + \eta^{-1}\mathbf{R}_d^\dagger\mathbf{r}_1 = \mathbf{T}_d^\dagger\mathbf{K}_2^\dagger\mathbf{K}_2\boldsymbol{\tau}_1 + \beta^{-1}\mathbf{T}_d^\dagger\tau_1. \quad (42)$$

For the scenario considered in [5], where joint optimization of *all* interfering users is possible, the analogous condition to (42) is $\eta^{-1}\mathbf{R}_d^\dagger\mathbf{r}_1 = \beta^{-1}\mathbf{T}_d^\dagger\tau_1$. Comparing, it is apparent that (42) contains an additional term on both sides that indicate the further constraints on the optimization here, since the presence of legacy system implies fewer degrees of freedom, as compared with [5]. The additional term on the left-hand side of (42) effectively subjects the optimization to a fixed legacy receiver, where as the additional term on the right-hand side of (42) effectively subjects the optimization to a fixed legacy transmitter.

A. MMSE Solution: One Nyquist Zone Case

We consider the special case of systems that are confined to a bandwidth of $1/T$. For this case, (42) now becomes

$$\left[\alpha|R_2|^2|H_{12}|^2 + 1\right]\beta^{-1}|T_1|^2 = \left[|T_2|^2|H_{21}|^2 + \eta^{-1}\right]|R_1|^2 \quad (43)$$

and the expressions (36) and (37) can be written as

$$R_1 = \frac{TT_1^*H_{11}^*}{|T_1|^2|H_{11}|^2 + |T_2|^2|H_{21}|^2 + \eta^{-1}} \quad (44)$$

$$T_1 = \frac{TR_1^*H_{11}^*}{|R_1|^2|H_{11}|^2 + (\alpha|R_2|^2|H_{12}|^2 + 1)\beta^{-1}}. \quad (45)$$

Obtaining R_1^* from (44), and substituting in the numerator of (45) and simplifying, results in

$$T^2|H_{11}|^2 = \left[|H_{11}|^2|R_1|^2 + \left(\alpha|H_{12}|^2|R_2|^2 + 1\right)\beta^{-1}\right] \times \left[|T_1|^2|H_{11}|^2 + |T_2|^2|H_{21}|^2 + \eta^{-1}\right]. \quad (46)$$

$$\mathbf{A}_1 = \begin{bmatrix} T_{1,1}H_{11,1} & T_{1,2}H_{11,2} & T_{1,3}H_{11,3} & \ldots \\ T_{2,1}H_{21,1} & T_{2,2}H_{21,2} & T_{2,3}H_{21,3} & \ldots \end{bmatrix} \quad (21)$$

$$\mathbf{A}_2 = \begin{bmatrix} R_{1,1}H_{11,1} & R_{1,2}H_{11,2} & R_{1,3}H_{11,3} & \ldots \\ \sqrt{\lambda\alpha}R_{2,1}H_{12,1} & \sqrt{\lambda\alpha}R_{2,2}H_{12,2} & \sqrt{\lambda\alpha}R_{2,3}H_{12,3} & \ldots \end{bmatrix} \quad (22)$$

$$\mathbf{b} = [1, 0]^T \quad (23)$$

TABLE I
CHANNEL SETTINGS

Parameter	Value
N_0	10^{-8}
σ^2	1
k	$7.744\text{x}10^{-21}$
$Y(f)$	$0.737\sqrt{f} + 0.0092f$

Substituting for $|T_1|^2$ from (43) as $|T_1|^2 = k|R_1|^2$, where

$$k = \frac{|T_2|^2|H_{21}|^2 + \eta^{-1}}{(\alpha|R_2|^2|H_{12}|^2 + 1)\,\beta^{-1}} \tag{47}$$

gives

$$a|R_1|^4 + b|R_1|^2 + c = 0 \tag{48}$$

where

$$a = k|H_{11}|^4 \tag{49}$$

$$b = |H_{11}|^2\left[|T_2|^2|H_{21}|^2 + \eta^{-1} + (\alpha|R_2|^2|H_{12}|^2 + 1)\beta^{-1}k\right] \tag{50}$$

and

$$c = \left[\alpha|H_{12}|^2|R_2|^2 + 1\right]\beta^{-1}\left[|T_2|^2|H_{21}|^2 + \eta^{-1}\right] - T^2|H_{11}|^2. \tag{51}$$

Since (48) is a quadratic in $|R_1|^2$, the solution can be obtained by directly solving for the roots; substituting for $|R_1|^2$ in (43) then yields the optimal transmitter.

IV. MMSE Solution: Algorithmic Approach

The choice of transfer functions in this section is based upon the work in [4] and [8], and was used for computational purposes in [2] and [5]. The transfer functions model high-rate communication over copper loops and are given by

$$\text{Direct Channel Response} = H_{11}(f) = H_{22}(f) = \exp(-dY(f)) \tag{52}$$

and

$$\text{FEXT Response} = H_{12}(f) = H_{21}(f) = f\sqrt{kd}H_{11}(f) \tag{53}$$

where d is the length of the loop in kft and f is in MHz. The parameters k and $Y(f)$ are given in Table I. For simplicity, we will restrict our attention to *real* (absolute-valued) channels, i.e., the phase response is assumed to be zero for computational ease, as was done in [2] and [5]. A plot of the direct and crosstalk response is shown in Fig. 2. It is worth noting that for the case of coordinated users (as considered in [10]), the necessary condition for joint transmitter/receiver optimality takes on a product form. Hence, the transmitter phase may be simply chosen as the conjugate of the channel, so as to provide a system with overall zero-phase response. For our scenario (as well as the scenarios in [2] and [5]), the condition for joint transmitter/receiver optimality does not takes on a product form. Thus, the results in this section correspond to an upper bound on the performance of complex channels (nonzero phase response).

In [5], joint optimization of interfering users with symmetric responses using the MMSE criterion was investigated. It was assumed that all systems employ the same transmitter and receiver filters, and the figure of merit used was the MSE attainable for *each* system. The computational examples in our paper assume that the system for one (i.e., the legacy) user is fixed, which is anticipated to lead to inferior performance, compared with the situation addressed in [5], where all systems are subject to optimization. Consequently, we will compare the attainable MSE for each system in [5] to the relevant MSE values for the overlay and legacy systems. The overlay MSE [as defined in (2)] and the excess legacy MSE [as defined in (3)].

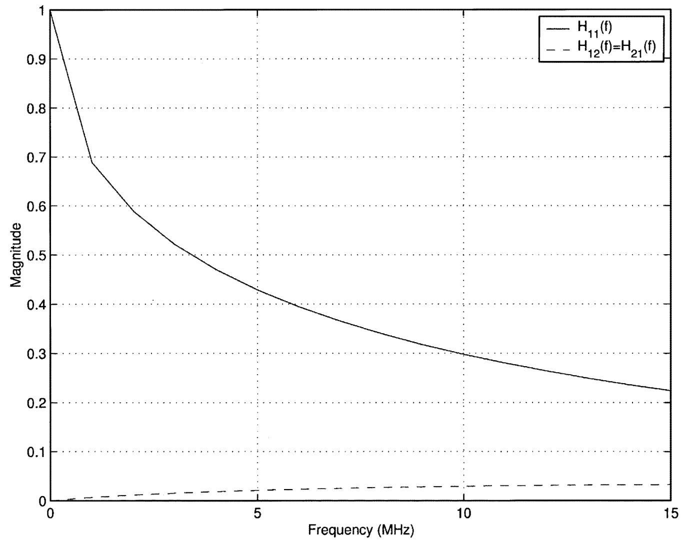

Fig. 2. Channel responses.

Our computations assume a legacy system that has been designed to perform optimally in the presence of AWGN only (i.e., $H_{12}(f) = 0$). For direct channels ($H_{ii}(f)$) with a lowpass characteristic (as is the case for copper loops), the Nyquist set corresponds to the basic Nyquist zone, considerably simplifying the problem. Hence, in this section, we will only concern ourselves with an overlay system that occupies the same bandwidth as the legacy system. Additionally, in fairness to the legacy system, α is set equal to one.

A. Computational Complexity

A uniformly spaced frequency grid is chosen and (48) is solved for each point to obtain a solution for the overlay transmitter/receiver. By iterating over the Lagrange multiplier λ, the constraint on the overlay transmitter is satisfied. The computational algorithm can be summarized as follows.

1) Choose a value for λ.
2) Solve (48).
3) Decrease (Increase) λ if the transmitter power is too low (high). If the transmitter power constraint is met for a given tolerance, go to the next step.
4) Evaluate (14) and (15) to determine the overlay and excess legacy MSE, respectively.

For a frequency grid with N points, the computational cost associated with finding the optimal overlay receiver response for a *given* transmitter is $O(N^2)$. Transmitter optimization is an iterative process which requires checking the power constraint at each iteration; the computational cost for each iteration is

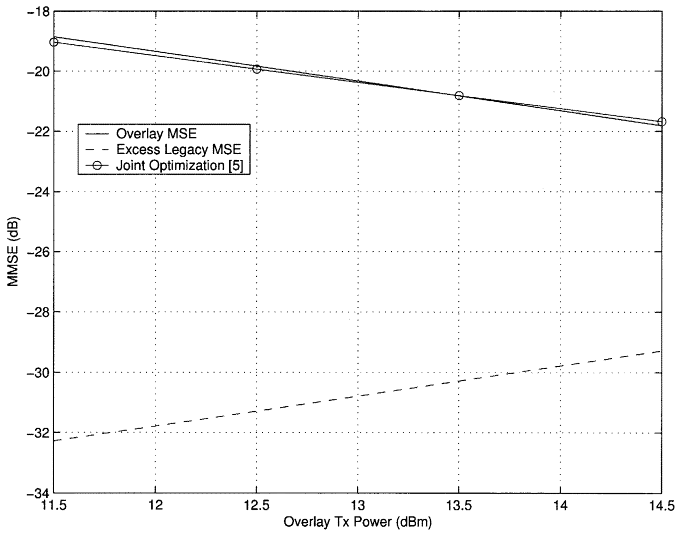

Fig. 3. Effect of varying overlay power when legacy power is fixed at -13.5 dBm (for all curves, $1/T = 15$ MHz, $d = 1.5$ kft).

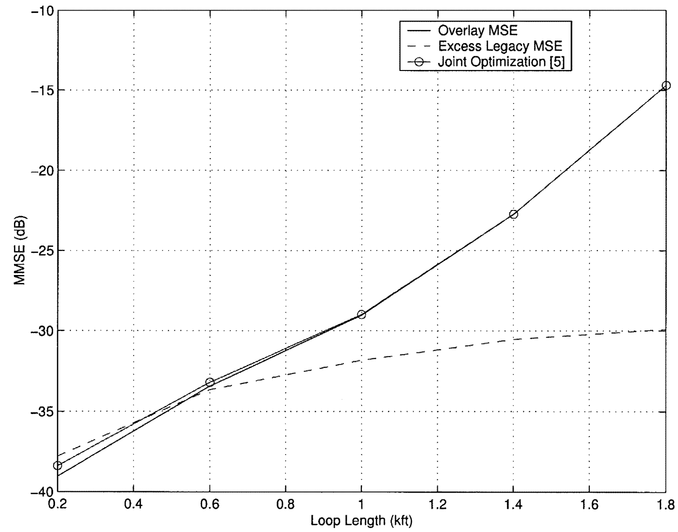

Fig. 4. Effect of varying loop length (for all curves, $1/T = 15$ MHz, Tx Power $= -13.5$ dBm).

$O(N^2)$. It was observed that in general, less than 10 iterations are required to satisfy the power constraint to within 0.01 dBm.

B. Results & Discussions

The performance of the overlay system as a function of its transmitter power for a fixed transmitter power of the legacy system and a (common) bandwidth of 15 MHz is investigated in Fig. 3. For all computations, we have assumed that the transmitter power of the legacy system is fixed at -13.5 dBm, while the overlay transmitter power may be varied.

It can be seen from Fig. 3 that the excess legacy MSE is quite small, compared with the overlay MSE, indicating minimal degradation in the legacy-system performance. As expected, however, increasing the power of the overlay system (while keeping the legacy power fixed) does indeed improve overlay system performance, though this is at the cost of the legacy-system performance, which is degraded due to increased interference levels caused by a larger overlay transmit power. Note that the dashed line in Fig. 3 should not be interpreted as having superior performance to the joint optimization/overlay MSE curve. The dashed line represents *excess* legacy MSE, which when added to the original legacy MSE (the MSE of the legacy system in the presence of AWGN only), will exhibit poorer performance than the joint optimization/overlay MSE curves.

Fig. 4 depicts performance as a function of loop length with the transmit power fixed at -13.5 dBm, and a (common) bandwidth of 15 MHz is assumed. Clearly, extended range causes greater attenuation and, therefore, reduces performance. Interestingly, it appears that for longer loop lengths, the excess legacy MSE stabilizes, whereas the overlay system performance continues to rapidly degrade.

By examining Figs. 3 and 4 together, we arrive at an interesting (and intuitively pleasing) result. Apparently, the MSE resulting from joint optimization (as in [5]) closely follows the overlay MSE. Thus, the loss due to the legacy restriction is obvious. The MSE that can be achieved for all users if the legacy restriction is absent can now only be achieved for the new (overlay) system.

V. Conclusion

The scenario of an existing legacy communication system being overlaid by a new system was investigated. Necessary conditions for the jointly optimal design of the overlay (new) transmitter and receiver subject to a fixed legacy system were derived. A composite MSE that minimizes the MSE of the new system, while simultaneously minimizing the excess MSE introduced into the legacy system due to the presence of the new system, was employed as an objective function. Performance loss due to the legacy restriction was investigated. It was determined that the MSE that can be achieved for both the overlay and legacy systems, in absence of the legacy restriction, can be achieved only for the overlay system if the legacy restriction is present.

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